(D-3)

: only two groups,
= SSB/1 = MSB.

ɔted, the LSD test
α-probability; and
t the expense of a

mplete description
like to point out,
technique running
highest to lowest.
using a particular
ıasses, comparisons
ːst with the second
s using successively

(D-1)

Note that since this correlation is over $J$ pairs of means and weights, $J$ is the $n$ in the equation, and all summations are from $j = 1$ to $J$.

Since $\Sigma\, w_j$ is stipulated to be zero, Equation D-1 can be simplified considerably:

$$r^2 = \frac{(J\,\Sigma\, M_j w_j)^2}{[J\,\Sigma\, w_j^2][J\,\Sigma\, M_j^2 - (\Sigma\, M_j)^2]}$$

Multiplying numerator and denominator of Equation D-2 by $n/J^2$ and noting that $M_j = T_j/n$,

$$r^2 = \frac{n(\Sigma\, M_j w_j)^2}{[\Sigma\, w_j^2][\Sigma(T_j^2/n) - (\Sigma\, T_j)^2/Jn]}$$

Since $\Sigma\, T_j = T$ and $Jn = N$,

$$\frac{\Sigma\, T_j^2}{n} - \frac{(\Sigma\, T_j)^2}{Jn} = \frac{\Sigma\, T_j^2}{n} - \frac{T^2}{N} = \text{SSB}$$

And thus,

$$r^2 = \frac{n(\Sigma\, M_j w_j)^2}{(\Sigma\, w_j^2)(\text{SSB})}$$

# PROBLEMS

1. An experiment is done with four groups of 10 subjects per group. The following means are obtained:

| Group 1 | Group 2 | Group 3 | Group 4 |
|---|---|---|---|
| $M_1 = 3$ | $M_2 = 2$ | $M_3 = 5$ | $M_4 = 10$ |

Assume SSW = 120.

a. Plot the four means along with 95% confidence intervals.
b. By how much must two means differ to be significant at the 0.01 level using an LSD test?
c. By how much must two means differ to be significant at the 0.01 level using a Scheffé test?
d. Test the hypothesis that the means increase linearly. What percentage of SSB is accounted for by this hypothesis? Is the residual significant?

2. An experiment is done to test the hypothesis that Miracugrow causes increases in the heights of Merkin plants. Four groups of 10 plants per group are given varying amounts of Miracugrow, and the heights of plants after a year are measured. Here are the data for the four groups:

| Group 1: 0 grams | Group 2: 4 grams | Group 3: 8 grams | Group 4: 12 grams |
|---|---|---|---|
| $n_1 = 10$ | $n_2 = 10$ | $n_3 = 10$ | $n_4 = 10$ |
| $M_1 = 2$ | $M_2 = 5$ | $M_3 = 5$ | $M_4 = 8$ |

Assume that MSW = 10.

a. Do an ANOVA on these data. Plot the means with 95% confidence intervals.
b. Test the hypothesis that the means increase linearly with amount of Miracugrow. How much of the variance between groups does this hypothesis account for?
c. Test the residual from this hypothesis.
d. Determine the weights corresponding to a second hypothesis orthogonal to that of question b.
e. Test the significance of this hypothesis.
f. Determine the weights corresponding to one hypothesis that will account for the *entire* sum of squares between.
g. Determine which groups are significantly different from one another using the Scheffé method.
h. Determine which groups are significantly different from one another using the LSD method.

3. Assume an experiment is run with three conditions (that is, three levels of the independent variable). There are four observations in each condition, and the following data are obtained:

| Condition I | Condition II | Condition III |
|---|---|---|
| 2 | 4 | 1 |
| 8 | 10 | 8 |
| 14 | 16 | 15 |
| 10 | 12 | 9 |

Consider this to be a between-subjects design.

a. Graph the means, showing the dispersion of data points around them. Include the 95% confidence intervals. Does it look as if there's an effect of condition?
b. Do a one-way ANOVA. Put your findings in an ANOVA table. Is there a significant effect due to condition?
c. What are the weights corresponding to the hypothesis: Condition II is better than the other two?
d. What is the sum of squares corresponding to the hypothesis? What is the residual SS?
e. What percentage of the SSB is accounted for by the hypothesis?
f. Does the hypothesis account for a significant proportion of the variance?

4. Suppose you do an experiment with $n = 10$ subjects in each of four conditions. The four means are as follows:

$$M_1 = 5 \qquad M_2 = 8 \qquad M_3 = 6 \qquad M_4 = 10$$

Assume that SSW = 180.

a. Do an ANOVA on these data. Plot the means along with 95% confidence intervals.
b. By how much does a pair of means have to differ to be significantly different by an LSD test?
c. By how much does a pair of means have to differ to be significantly different by a Scheffé test?
d. Test the hypothesis that these means increase linearly. What percentage of variance does this hypothesis account for?
e. Is the residual from the hypothesis tested in question d significant?

5. An experiment is done by the University of Washington Physical Education Department to test the effect of altitude on the amount of time it takes to run the 100-yard dash. Four cities are selected: Seattle (sea level), Boise (1000 feet above sea level), Denver (5000 feet above sea level), and Taos (10,000 feet above sea level). Twelve runners are randomly divided into four groups of three runners per group, with one group running in each of the four cities. The times it takes (in seconds) are as follows:

| City | | | |
|---|---|---|---|
| Group 1 Seattle (0 feet) | Group 2 Boise (1000 feet) | Group 3 Denver (5000 feet) | Group 4 Taos (10,000 feet) |
| 9 | 14 | 14 | 16 |
| 11 | 16 | 13 | 16 |
| 10 | 12 | 15 | 16 |

a. Is there a significant difference among the four groups? Plot the means along with 95% confidence intervals.
b. What is the smallest difference needed to declare two means to be significantly different using an LSD test? (Use a 0.01 $\alpha$-level.)
c. What is the smallest difference needed to declare two means to be significantly different using a Scheffé test? (Use a 0.01 $\alpha$-level.)
d. Test the hypothesis that running time increases *linearly* with altitude.
e. Test the residual from this hypothesis.
f. Is the variance in running time in Denver significantly different from the variance in running time in Boise?

6. Consider the data from Chapter 11, problem 4. Which of the groups differ significantly by a Scheffé test? By an LSD test?

7. Consider the data from Chapter 11, problem 6.

a. Which groups differ significantly by a Scheffé test? By an LSD test?

b. Test the hypothesis that number of errors decreases linearly with presentation rate. What percentage of SSB does this hypothesis account for? Test the residual from the hypothesis.

8. Consider the data from Chapter 11, problem 1.
   a. Which groups differ significantly by a Scheffé test? By an LSD test?
   b. Test the hypothesis that memory ability increases with age. Test the residual from this hypothesis.
   c. What do you think would be a more reasonable hypothesis?

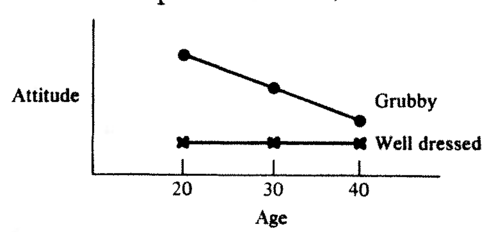
9. Consider both sets of data shown in Chapter 11, problem 2. For both sets answer the following questions:
   a. Which groups are different from one another by a Scheffé test? By an LSD test?
   b. Test the hypothesis that marijuana makes people higher (leads to longer reaction times) than bananas or tea, which do not differ from one another.
   c. Test the hypothesis (orthogonal to the one in question b) that bananas made you higher (lead to higher reaction times) than tea.

10. Consider the data from Chapter 13, problem 4. Test the hypothesis that number of words recalled decreases linearly with retention interval.

11. Consider the data from Chapter 13, problem 3.
    a. Test the hypothesis that the new technique and DDT are both more effective in mosquito control than nothing at all.
    b. Test the hypothesis (orthogonal to that of question a) that the new technique is more effective than DDT.

12. Consider the data from Chapter 13, problem 1.
    a. Test the hypothesis that stage fright increases linearly with audience size. What proportion of the sum of squares due to condition is accounted for by this hypothesis?
    b. Test the residual from this hypothesis.

13. Use the information from problem 2 of Chapter 12 to solve the following:
    a. Which of the cells differ significantly from one another by an LSD test?
    b. Suppose that you have an a priori theory that, with grubbily dressed speakers, attitudes should decrease with age; however, with well-dressed speakers, attitudes should remain constant over age. Furthermore, for a given age level attitudes should always be higher with grubbily dressed than with well-dressed speakers. That is, the data should look like this:

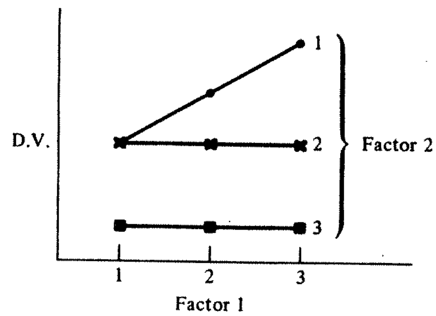



Generate the weights corresponding to this hypothesis.

c. Test the significance of the hypothesis. How much of the between-cell variance does it account for? Test the significance of the residual.

14. Use the information from problem 1 of Chapter 12 to solve the following: (Assume it to be a one-way design with four conditions and 10 subjects per condition.) Assume SSW = 3600.
   a. What are the weights corresponding to the hypothesis: The high-motivation groups do better than the low-motivation groups?
   b. Compute a sum of squares corresponding to this hypothesis. What are the dfs corresponding to this sum of squares?
   c. Compute an $F$ corresponding to the hypothesis. What are the dfs for this $F$? Is it significant?
   d. What are the residual sum of squares and mean squares? Compute an $F$ corresponding to the residual. What are the dfs for this $F$? Is the $F$ significant?

15. Consider a $3 \times 3$ design.
   a. Make up a set of weights to reflect this hypothesis:




   b. Suppose there are seven scores per cell and the means are as follows:

| | Factor 1 | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| 1 | 2 | 2 | 7 |
| 2 | 3 | 2 | 2 |
| 3 | 1 | 2 | 1 |

   Test the hypothesis and the residual (assume MSW = 2).

16. Here are two hypotheses for a four-group experiment:

$$H_1: \{3 \quad 1 \quad -1 \quad -3\}$$
$$H_2: \{1 \quad -1 \quad -1 \quad 1\}$$

   Make up a third hypothesis that is orthogonal to these two.

17. Here is a hypothesis for a four-group experiment:

$$H_1: \{1 \quad 1 \quad 1 \quad -3\}$$

Make up two more hypotheses such that all three are mutually independent.

18. Consider the data from Chapter 11, problem 5 (note that $\sigma^2$ is known). Which groups differ by a Scheffé test? By an LSD test?

19. Consider the data from Chapter 12, problem 6. Suppose Gazelle, Inc. has the following hypothesis about their braking systems. The new system is such that it will stop any size motorcycle in the same distance. However, with the old system stopping distance increases *linearly* with engine size.
    a. Generate the weights corresponding to this hypothesis (*Caution*!). Note that 125, 500, and 1000 do not form a linear sequence.
    b. Is this hypothesis significant? What proportion of SSB does it account for?
    c. Test the residual from this hypothesis.

20. Suppose an experiment produces the following four means, each mean based on $n = 10$ scores:

$$M_1 = 2 \qquad M_2 = 4 \qquad M_3 = 2 \qquad M_4 = 0$$

Note that SSB = 80. Assume that SSW = 360.
    a. Do an ANOVA on these data.
    b. What percentage of variance ($\omega^2$) is accounted for by the conditions?

21. A sociologist has the hypothesis that the average height of males living west of the Mississippi River is different from the average height of males living east of the Mississippi River. He takes random samples of 10,000 easterners and 9500 westerners and gets the following data (height is in feet):

| Easterners | Westerners |
|---|---|
| Mean height = 5.75 | Mean height = 5.74 |
| $\Sigma(X - M)^2 = 599$ | $\Sigma(X - M)^2 = 632$ |

Test the hypothesis that the average height of easterners differs from that of westerners. What is $\omega^2$ in this problem?

22. Consider the data from Chapter 10, problems 5–8, 10–14, 16, and 17. Compute $\omega^2$ in each case.

23. Consider the data from Chapter 11, problems 1, 2 (both sets of data), and 3–6. Compute $\omega^2$ in each case.

24. Consider the data from Chapter 12, problems 1–7. Compute $\omega^2$ for rows, columns, and interaction in each case.

25. Consider the data from Chapter 13, problems 1–5 and 9. Compute $\omega^2$ for conditions for each set of data.

26. Consider the data from this chapter, problems 1–5. Compute $\omega^2$ for each set of data.