

Notice that the two terms in the brackets of Equation D-3 are

$$E(x_A^2) - \mu_A^2 = \sigma_A^2$$

$$E(x_B^2) - \mu_B^2 = \sigma_B^2$$

Therefore,

$$\sigma_{B-A}^2 = \sigma_A^2 + \sigma_B^2$$

### PROBLEMS

1. Acme lightbulbs have a mean lifetime  $\mu = 150$  hours and a variance  $\sigma^2 = 75$ . Suppose random samples of Acme bulbs are tested. How probable is it that the mean lifetime of a sample will be:
  - a. Greater than 155 hours if the sample size is 1.
  - b. Greater than 155 hours if the sample size is 5.
  - c. Greater than 155 hours if the sample size is 10.
  - d. Less than 150 hours if the sample size is 1 billion.
  - e. Less than 149.9 hours if the sample size is 1 billion.
  - f. Between 149 and 151 hours if the sample size is 100.
2. The distribution of scores on a driving skill test has a mean of 80 and a standard deviation of 9.
  - a. Find the mean and standard deviation for a distribution of means of random samples of size 100.
  - b. Find the percentage of random samples of size 100 that would be expected to have a mean above 87.
  - c. Find the percentage of random samples of size 30 that would be expected to have a mean below 82.
  - d. Find an interval centered at the mean that would be expected to include the means of 50% of random samples of size 50.
3. Weights of Mississippi sheriffs are known to be distributed with a mean  $\mu = 250$  pounds. An experiment is done to test whether Texas sheriffs weigh less in general than Mississippi sheriffs. Thus, two Texas sheriffs are randomly selected and have the following weights:
 

Sheriff	Weight
1	$x_1 = 240$
2	$x_2 = 250$

Assume that the weights of Texas sheriffs are distributed with a standard deviation  $\sigma = 20$  pounds. Test the hypothesis that Texas sheriffs come from a population whose mean is less than that of Mississippi sheriffs.
4. A string manufacturer claims his product has a mean breaking strength of 60 pounds with a standard deviation of 3.5 pounds. Test the hypothesis that

$\mu = 6$   
test to  
0.01.

5. It is kn  
a mea  
whe  
robber  
hypoth  
armed

6. It is k  
Franci  
hours  
747s. T  
new w  
the hyp

7. Two hu  
by Dr.  
200 st  
mean c  
A's cla  
class ge  
a. Test  
b. Test  
c. Test  
stud

8. Scores  
populat  
Univer:  
MAT p  
are give  
UW  
85  
82  
84  
88  
86  
Do Har

9. A test  
Groups  
take the  
whereas  
that left

$\mu = 60$  pounds if a random sample of 49 pieces of this string is found under test to have a mean breaking strength of 55 pounds. Use an  $\alpha$  probability of 0.01.

5. It is known that armed robbers receive sentences normally distributed with a mean  $\mu$  of 7 years and a variance of 5. A criminologist is interested in whether rapists tend to receive sentences different from those of armed robbers. A sample of 10 rapists has a mean sentence  $M$  of 5 years. Test the hypothesis that rapists tend to receive sentences different from those of armed robbers.
6. It is known from long experience that Boeing 747 airliners fly from San Francisco to New York in times that are normally distributed with  $\mu = 4.7$  hours and  $\sigma = 0.5$  hour. Jupiter Airlines tries a new brand of wax on its 747s. The wax is supposed to increase air speed. After 100 flights with the new wax, the mean San Francisco/New York flying time is 4.45 hours. Test the hypothesis that the new wax really does increase air speed.
7. Two hundred students are randomly assigned to two statistics classes taught by Dr. A and Dr. B (100 students per class). At the end of the course all 200 students take a standard statistics exam known to have a population mean of 500 and a standard deviation of 100. The 100 students from Dr. A's class get an average score of 520, while the 100 students from Dr. B's class get an average score of 490.
- Test the hypothesis that Dr. A's students are different from the average.
  - Test the hypothesis that Dr. B's students are different from the average.
  - Test the hypothesis that Dr. A's students are different from Dr. B's students.
8. Scores on the Miller Analogies Test (MAT) are normally distributed with a population variance  $\sigma^2 = 22.5$ . An experiment is done to determine whether University of Washington students and Harvard students differ in terms of MAT performance. A sample of five Harvard students and five UW students are given the MAT and the following scores are obtained:

UW	Harvard
85	81
82	83
84	77
88	79
86	80

Do Harvard and UW students differ significantly?

9. A test of spatial abilities is known to have a standard deviation  $\sigma = 6$ . Groups of  $n_1 = 10$  left-handed children and  $n_2 = 15$  right-handed children take the test. The left-handed children get a mean  $M_1 = 78$  on the test, whereas the right-handed children get a mean  $M_2 = 69$ . Can it be concluded that left- and right-handed children differ in terms of spatial ability?

10. The time it takes aardvarks to run down a runway is known to be distributed with  $\sigma^2 = 4$ . It is of interest to see whether aardvarks run faster to peanut butter or to jelly. A group of  $n_1 = 3$  aardvarks is put in a peanut butter condition and a second group of  $n_2 = 5$  aardvarks is put in a jelly condition. The mean times for the aardvarks are as follows:

Peanut Butter ( $n_1 = 3$ )	Jelly ( $n_2 = 5$ )
$x_{11} = 3$	$x_{12} = 7$
$x_{21} = 2$	$x_{22} = 11$
$x_{31} = 4$	$x_{32} = 9$
	$x_{42} = 5$
	$x_{52} = 8$

Test whether the two groups differ.

11. Joe Smith is employed as a dental researcher and is interested in whether Ultra-Brite toothpaste produces more sex appeal than does Crest. Joe thus gets one group of two people to brush with Ultra-Brite and a second group of three people to brush with Crest. Sex appeal is then measured using a rating scale. The ratings are as follows:

Crest	Ultra-Brite
5	2
3	2
7	

(Note: The greater the sex appeal, the higher the rating.) It is known that rating scores are distributed with a population variance  $\sigma^2 = 1.5$ . Test the hypothesis that Ultra-Brite leads to more sex appeal than does Crest. List all hypothesis-testing steps. Use the 0.01  $\alpha$ -level.

12. Joe Smith is working to try to make his Merkin plants grow taller. He suspects that one type of plant food, Miracugrow, leads to taller Merkin plants than does another kind, Amazofood. To test this hypothesis, he gives Miracugrow to one group of 10 Merkin plants and Amazofood to another group of 10 Merkin plants. Unfortunately, 9 of the 10 Amazofood plants die. At adulthood, the one remaining Amazofood plant is 48 inches tall, and the 10 Miracugrow plants have a mean height of 52 inches. It is known that heights of Merkin plants are distributed with a variance  $\sigma^2 = 4$ . Carry out hypothesis-testing steps to decide on the original issue (whether Miracugrow leads to taller Merkin plants than does Amazofood). Use all hypothesis-testing steps and use the 0.01  $\alpha$ -level.
13. It is known that the amount of beer served in Dante's Tavern is normally distributed with a variance of 4. Of interest is whether the mean amount served in men's mugs differs from the mean amount served in women's mugs. Suppose we have samples of

$$n_1 = 1 \text{ male whose amount of beer is } M_1$$

$$n_2 = 4 \text{ females whose mean amount of beer is } M_2$$

- a. How  
we  
fem  
tail  
b. Rec
14. Assum  
Belling  
from S  
Belling  
sample  
that fav  
favor m  
be  $P_B$ .  
 $P_B - I$
15. A musi  
The me  
scores v  
each, sc  
a. Draw  
sampl  
b. In w  
as hi  
c. In w
16. Consid  
a. Find  
b. Find  
3, an  
and  
c. Find  
d. Find  
e. Find  
all th

- a. How big a difference between  $M_1$  and  $M_2$  would be required such that we could reject (at the 0.05 level) the null hypothesis that males and females do not differ in terms of amount of beer served. Assume a one-tailed test with an  $\alpha$ -level of 0.05.
  - b. Recompute the above, but assume a two-tailed test.
- 14 Assume 80% of Seattle voters favor legalization of marijuana and 50% of Bellingham voters favor legalization. A random sample of 25 voters is drawn from Seattle, and another random sample of 25 voters is drawn from Bellingham. What is the probability that the percentage of the Bellingham sample that favor marijuana will *exceed* the percentage of the Seattle sample that favor marijuana? (*Hint*: Let the proportion of the Seattle sample that favor marijuana be  $P_S$ . Likewise, let the proportion of the Bellingham sample be  $P_B$ . Now the question may be rephrased: What is the probability that  $P_B - P_S$  exceeds zero?)
- 15 A musical aptitude test was given to all third-grade classes in New York City. The mean score on this test was 75, and the standard deviation of children's scores was 15 points. Suppose that the classes are composed of 36 children each, so that in effect there are a vast number of samples of 36 scores each.
- a. Draw and label the graph of the distribution of the means of these samples (assuming that each sample is a random collection of 36 scores).
  - b. In what percentage of the classes would we expect the class mean to be as high as 80?
  - c. In what percentage would the mean be as low as 70?
- 16 Consider a population consisting of only the scores 3, 6, 8, 11, and 15.
- a. Find the mean and standard deviation of these scores.
  - b. Find all the possible samples of size 2 (that is, 3 and 6, 3 and 8, 3 and 3, and so on) which can be drawn from this population *with replacement* and list these (there are 25 such samples).
  - c. Find the means of all the samples of size 2.
  - d. Find the mean of the sample means.
  - e. Find the standard deviation of the sample means. Compare the mean of all the samples of size 2 and the standard deviation with the results in a.