DIGRESSION 6-3

Statement of the Null Hypothesis

Referring to the mean lifetime of treated bulbs as μ, we have stated our null and alternative hypotheses as

$$H_0\text{: } \mu = 100$$

$$H_1\text{: } \mu > 100$$

Note that these two hypotheses are not exhaustive in the sense that they exclude the possibility that $\mu < 100$. The very notion of a directional alternative hypothesis renders this possibility unlikely. That is, we set up a directional hypothesis only if we consider it impossible that the process would reduce lightbulb lifetime.

However, some statisticians take the view that hypotheses should be constructed so as to exhaust all possible states of reality and therefore that the null hypothesis should be set up as

$$H_0\text{: } \mu \leq 100$$

What does this way of stating the null hypothesis do to the probability of a type I error? Note that if we set up the criterion such that the probability of a type I error is 0.05 when $\mu = 100$, then when $\mu < 100$, the probability of a type I error must be less than 0.05. Therefore, stating the null hypothesis in this way, the *maximum* probability of a type I error is 0.05.

PROBLEMS

1. Graduate Record Exam (GRE) scores are normally distributed with a mean (μ) of 500 and a standard deviation (σ) of 100.
   a. Write the equation for $p(x)$, where $p(x)$ is the probability density function for GRE scores.

   Suppose a random student takes the GRE. Compute the probabilities that x, the student's exam score, will fall in the following ranges:
   b. $x > 500$
   c. $x < 400$
   d. $x > 620$
   e. $490 < x < 530$
   f. $305 < x < 480$
   g. $530 < x < 600$
   h. $x > 460$
   i. $x < 590$
2. The time lengths of films produced by a French film company are found to be normally distributed with a mean length of 92 minutes and a standard

deviation of 23 minutes. Sketch the normal curve depicting the distribution of film lengths. Find the percentage of films that last:

a. Less than 65 minutes.
b. More than 77 minutes.
c. Less than 112 minutes.
d. More than 120 minutes.
e. Between 69 and 115 minutes.
f. Between 92 and 100 minutes.
g. Between 100 and 108 minutes.

Speculate as to *why* the films might be normally distributed. Be specific in your speculations.

3. It is known that the amount of tartar sauce served in tartar sauce cups at Ghoti's Fish House is normally distributed with a mean of 50 cubic centimeters and a *variance* of 25. Find the probability that a random cup of Ghoti's Fish House tartar sauce contains:
   a. Between 45 and 55 cubic centimeters of tartar sauce.
   b. Between 39 and 48 cubic centimeters of tartar sauce.
   c. Between 52 and 61 cubic centimeters of tartar sauce.
   d. Less than 43 cubic centimeters of tartar sauce.

4. Joe Smith is growing a Merkin plant. His *Handbook of Merkin Plants* tells him that the heights of adult Merkin plants are normally distributed with a mean of 65 inches and a standard deviation of 3 inches. Find the probability that Joe's Merkin plant will grow to be:
   a. Between 64 and 67 inches tall.
   b. Between 61 and 63 inches tall.
   c. Between 66 and 67 inches tall.
   d. Less than 64 inches tall.
   e. Greater than 60 inches tall.
   f. Less than 40 inches tall.
   g. Less than 100 inches tall.

5. The Acme Rope Company manufactures climbing ropes. The breaking strength of these ropes is normally distributed with a mean of 300 pounds and a standard deviation of 76 pounds. Find the probability that an Acme Rope will break if used for climbing by:
   a. A 150-pound climber.
   b. A 200-pound climber.
   c. A 200-pound climber with his 80-pound backpack.
   d. A 150-pound climber with her 160-pound boyfriend hanging onto her.
   e. A 50-pound macaque monkey.

6. The lifetime of Acme lightbulbs is normally distributed with a mean of 10 hours and a variance of 6. Suppose I buy an Acme lightbulb. Find the probability that it will last:

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b. Bet
c. Mo
d. Few
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a. Between 9 and 12 hours.
b. Between 7 and 8 hours.
c. More than 11 hours.
d. Fewer than 9.5 hours.
e. Exactly 10 hours.

7. Assume that the amount of beer served in beer mugs at Russell's Tavern is normally distributed with a mean μ of 25 ounces and a standard deviation of 1.6 ounces. Find the probability that a beer purchased at Russell's will consist of:
   a. More than 24.2 ounces.
   b. Between 24 and 25.5 ounces.
   c. Exactly 25 ounces.

8. Suppose that each day in the month of October the probability of rain in Seattle is $\frac{7}{10}$. Suppose further that the weather on one day is independent of the weather on any other day. What is the probability that there will be *at least one* sunny day during the month of October? Answer this question in two ways: using the binomial distribution and the normal approximation to the binomial distribution.

9. Joe Smith has taken up squash. Every day for 900 days Joe drives to the squash courts and parks in front of a parking meter. Joe never puts any money in the meter, and every day the probability is 0.1 that a policeman will check the meter and give Joe a ticket.

   Over the 900 days:
   a. How many tickets does Joe expect to get?
   b. What is the *standard deviation* of the distribution of the number of tickets Joe expects to get?
   c. What is the probability that Joe will get 87 or more tickets?
   d. What is the probability that Joe will get between 95 and 100 tickets (inclusive)?
   e. What is the probability that Joe will get *exactly* 90 tickets?

10. Joe Smith is shooting fish in a Penny Arcade at the fair. Each time he shoots, he wins

    a rag doll with probability 0.30,
    a medallion with probability 0.20, or
    nothing with probability 0.50.

    Suppose Joe has 20 shots.
    a. What is the probability that Joe wins between 5 and 7 (that is, 5, 6, or 7) rag dolls? Compute using the binomial distribution.
    b. Compute the probability that Joe wins between 5 and 7 rag dolls using the normal approximation to the binomial *without* the correction for continuity.
    c. Recompute the probability using the correction for continuity.
    d. Suppose Joe has 50 shots. What is the probability that he wins exactly 15 rag dolls?

11. A very nasty professor decides to flunk roughly a quarter of his class on their final examination. If the exam scores are *normally distributed* with a *mean* of 75 and a *variance* of 25, what score does a student need to pass?

12. It is known that the amount of liquor drunk by college students on weekends is normally distributed with

$$\mu = 10 \text{ ounces (per student)}$$

$$\sigma = 2 \text{ ounces}$$

It is hypothesized that on the weekend of final exams students will drink *more* than they ordinarily would. A random student is selected, and the amount she drinks on the weekend during finals turns out to be 13.5 ounces. We now wish to test the hypothesis that this is more than the average of 10.0 ounces.

a. What is the null hypothesis?
b. What is the alternative hypothesis?
c. Assume the null hypothesis is correct. Characterize the distribution that our observed sample of 13.5 comes from. What are the *mean*, *variance*, and *standard* deviation of this distribution?
d. What is the z-score corresponding to our observed score of 13.5? What probability does this z-score correspond to?
e. Should we reject the null hypothesis? Why or why not?

13. Suppose you are in Las Vegas playing roulette. You decide you will play 100 times, and each time you will bet on the numbers 1–9. (Thus, if the wheel stops at 1–9, you win; if it stops at 10–36, you lose. Assume there are *only* the numbers 1–36 on the wheel.) After 100 plays you have won 20 times and lost 80 times. You now wish to test the alternative hypothesis that the wheel is biased against you against the null hypothesis that the wheel is fair.

a. What are four possible outcomes of your decision? Which outcome corresponds to a type I error, and which corresponds to a type II error?
b. What is the null hypothesis?
c. What is the alternative hypothesis?
d. Assume the null hypothesis is true. How should the number of your wins in 100 spins of the wheel be distributed? What is the mean and standard deviation of this distribution?
e. Should you reject the null hypothesis at the 0.001 α-level?

14. Suppose you are tutoring a student and decide to give him a test to find out if he's learned anything. You give him a 100-question test; each question is multiple choice, with four possible answers. His score on the test turns out to be 30 out of 100. You want to test the alternative hypothesis that he has learned something against the null hypothesis that he has learned nothing.

a. What are four possible outcomes of your decision? Which outcome corresponds to a type I error and which corresponds to a type II error?
b. What is the null hypothesis?
c. What is the alternative hypothesis?

d. Assume the null hypothesis is true. How should the number of correct answers in this 100-question test be distributed? What are the mean and standard deviation of the distribution?

e. Can you reject the null hypothesis at the 0.005 α-level?

15. Suppose you are doing an experiment in ESP. You sit in front of a subject with an opaque screen separating you, and you begin to throw a die. Each time you throw a die, the subject has to guess how the die came up. After 1296 throws, you stop and compute how many correct answers the subject got. You now wish to answer the question: Does the subject have ESP?

a. What are four possible outcomes of your decision? Which outcome corresponds to a type I error, and which corresponds to a type II error?

b. What is the null hypothesis?

c. What is the alternative hypothesis?

d. Assume the null hypothesis is true. Characterize the distribution of correct answers in the sample of 1296. (What are the mean and the standard deviation?)

e. You want to make sure that your probability of incorrectly rejecting the null hypothesis is 0.05. How do you set your criterion (or criteria)?

16. It is known that 53% of the pupils in a very large school system are male. Use a two-tailed test at the 0.05 significance level to test the hypothesis that a *random sample* of 450 pupils was obtained if the number of males it contains is (a) 180, (b) 168, (c) 220, (d) 210, (e) 225, (f) 300. (*Hint:* Remember that for a binomial distribution the mean is Np and standard deviation is $\sqrt{Npq}$.)

17. A student is taking a test with 1000 questions; each question has four alternatives. Unfortunately, the student knows absolutely nothing about the material, so she guesses randomly on each question. What is the probability that her proportion of correct answers will be between 0.26 and 0.35?

18. The Muy Munchy Mixed Nut Company has three types of nuts—cashews, peanuts, and pecans—in its cans of mixed nuts. There are equal quantities of each type of nut, and they are thoroughly mixed. Joe Smith draws 200 nuts from a Muy Munchy can. Compute the following probabilities.

a. Joe will get between 68 and 75 (inclusive) pecans.

b. The sum of Joe's peanuts and cashews will be 133.

c. Suppose Joe draws three nuts. What is the probability that he will get one cashew, one peanut, and one pecan?

19. A thousand babies are born in the Sheikdom of Shirk. Suppose that the probability is 0.5 that any given baby is a boy.

a. What is the probability that exactly 500 babies will be boys?

b. What is the probability that between 495 and 510 babies will be boys?

c. Suppose that 470 of the babies are boys. Test the null hypothesis that p, the probability of a boy, is 0.5 against the alternative hypothesis that $p \neq 0.5$.

20. Suppose that you are buying banjo strings for your five-string banjo. You know that the manufacturer says each banjo string has a mean breaking tension of 12 pounds with a standard deviation of 1 pound. You also know that your strings will be under the following tensions:
    String 1: 11.26 pounds.
    String 2: 11.15 pounds.
    String 3: 10.72 pounds.
    String 4: 10.72 pounds.
    String 5: 10.72 pounds.
    a. What is the probability that each of the five strings will break?
    b. Assuming that strings break independently, what is the probability that *at least* one string will break?
    c. What is the probability that *all* the strings will break?

21. Gail Gershwin comes in for her statistics final exam knowing 80% of the material. The exam consists of n questions, and thus on any given question Gail has a probability of 0.8 of being correct. Naturally, the ideal outcome of the exam is for Gail to achieve a score of around 80% so that her performance will perfectly reflect her knowledge of the course material.

    Suppose $n = 10$:
    a. What is the probability that Gail will get *exactly* 80% on the exam?
    b. What is the probability that Gail will get between 70 and 90% on the exam?

    Suppose $n = 100$:
    a. What is the probability that Gail will get exactly 80% on the exam?
    b. What is the probability that Gail will get between 70 and 90% on the exam?

    (*Hint:* If you use the normal approximation, use the correction for continuity.)