perverse) alternate technique of computing areas under curves was relatively easy, since calculating areas of rectangles is relatively straightforward. Now it appears we have to be concerned with calculating very oddly shaped areas. How do we do it?

There are two answers to this question. First the relevant areas of all probability distributions with which we will be concerned have already been computed and put into tables by various industrious individuals. (Some of these tables are reproduced in Appendix E of this book). Second, the branch of mathematics known as *integral calculus* is specifically concerned with computation of areas under curves. It was by using integral calculus that the aforementioned industrious individuals managed to construct the various tables of relevant areas. Since we have the tables, we don't need to know about integral calculus. For convenience of discourse, however, we will borrow a piece of notation from integral calculus called an *integral sign*. An integral sign is used to denote the area under some curve $p(x)$ within the interval whose limits are a and b. In general,

$$\text{Area under } p(x) \text{ between } a \text{ and } b = \int_a^b p(x)\,dx$$

Thus, to represent that the shaded-in area of Figure 2-8 is equal to 0.78, we would write

$$\int_{68}^{72} p(h)\,dh = 0.78 \qquad \text{(D-1)}$$

Notice the similarity between Equation D-1 and Equation 2-1, which gave the probability that a value of a discrete distribution fell within some interval. In fact, there is a great deal of similarity between a summation sign and an integral sign. The expression

$$\sum_{i=2}^{4} p(i)(1)$$

means we should sum the areas of a finite number of bars whose heights are $p(i)$ and whose widths are 1. Likewise, the expression

$$\int_{68}^{72} p(h)\,dh$$

means we should sum an infinite number of "bars" whose heights are $p(h)$ and whose widths (dh) are infinitely skinny.

PROBLEMS

1. A political incumbent commissions a survey on how well she is doing in office. Each participant in the survey rates the incumbent on a scale from 1

to 5, where 1 signifies "abominable" and 5 signifies "fantastic."

The results of the 77 respondents produce the following ratings:

2	1	3	3	2	1	3	4	2	1	4
1	4	1	5	3	4	1	1	2	1	2
2	3	1	1	1	2	1	3	4	4	5
1	4	1	4	4	4	2	4	2	3	5
3	1	1	1	5	5	3	2	5	5	3
4	1	3	4	4	3	3	4	3	3	1
4	5	2	3	5	5	4	5	3	4	4

a. Represent these numbers as a frequency distribution.
b. Represent them as probability distributions.

2. Perusing the want ads in a newspaper, we find 50 houses with the following prices (in thousands of dollars):

250	260	130	260	810	480	630	950	110	980
540	130	290	820	750	680	290	940	920	590
530	480	370	960	350	910	160	850	430	110
140	370	960	510	260	190	730	600	110	90
930	820	100	770	650	120	60	550	760	410

a. Plot a frequency distribution of house prices.
b. Convert this distribution to a probability distribution.

3. A die is thrown and its value is noted. Denote the value a, which can range from 1 to 6.

A random variable X is created as follows:

$$X = 3a^2 + 2a - 1$$

a. What are the members of V, where V consists of all values that X can take on?
b. Compute the probability distribution of V.

4. A family plans to have three children. Suppose that each child has a probability 0.5 of being a boy, and suppose the sex of each child is independent of the sex of every other child.
a. What are the members of V, where V is the set of possible numbers of boys the family could have?
b. Compute the probability distribution over the members of V.

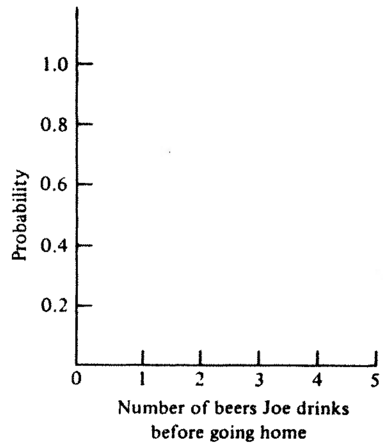
5. Suppose a player bats twice in a baseball game. On the first at-bat his probability P of getting a hit is 0.25. On the second at-bat, his probability of getting a hit is determined as follows:

If he got a hit on the first at-bat, $P = 0.35$.

If he did not get a hit on the first at-bat, $P = 0.25$.

a. What are the members of V, where V is the set of possible numbers of hits the player could get during his two at-bats?
b. Compute the probability distribution of the members of V.

6. Joe Smith has gone out for beer to celebrate his safe return from a scuba-diving expedition. *After drinking his first beer*, Joe decides that the number of subsequent ~~beers~~ he will drink that night will be determined in the following way. Before each potential round of beer, Joe will throw a die. If the die comes up 1, 2, 3, or 4 Joe will drink that round of beer. If the die comes up a 5 or 6, Joe will not drink the round of beer but will walk home instead.
   a. What are the members of V, where V is the set of possible numbers of beers Joe could drink that night before walking home?
   b. Is V a finite, countably infinite or uncountably infinite set?
   c. Fill in the probability distribution below of the number of beers Joe will drink before walking home.
   d. What is the probability that Joe will drink *more than* four beers before walking home?



7. A professor decides to grade papers according to the following scheme. For any given paper:
   She rolls a die. If the die comes up 6, the paper receives an A. Otherwise:
   She rolls the die again. If the die comes up 5 or 6, the paper receives a B. Otherwise:
   She rolls the die again. If the die comes up 4, 5, or 6, the paper receives a C. Otherwise:
   She rolls the die again. If the die comes up 3, 4, 5, or 6, the paper receives a D. Otherwise the paper receives an F.
   a. Plot the probability distribution of paper grades.
   b. Suppose that 1296 students turn in papers. Plot the expected frequency distribution for grades.

8. Antoine the accountant sits down one night to fill out three tax returns. He decides that the number of tax returns he will fill out before quitting that night will be determined in the following way: Before each tax return, he will draw a card from a well-shuffled deck. If the card comes up a spade, he will quit; otherwise, he'll fill out the tax return. (Naturally, if he fills out all three tax returns, he'll quit.)
   a. What are the members of V, where V is the set of possible number of tax returns Antoine could fill out that night?
   b. What is the probability of each member of V?

9. Two dice are thrown and the sum s is observed.
   a. Another random variable, x, is created using the equation

   $$x = s$$

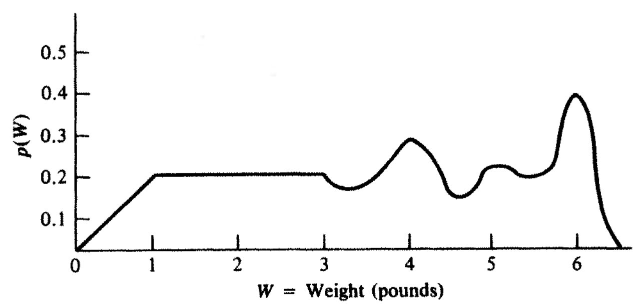
   Plot the probability distribution of this random variable.
   b. Another random variable is created as follows:

   $$y = 2s^2 \text{ if at least one 6 shows up}$$

   $$y = s^2 \text{ if no 6 shows up}$$

   Plot the probability distribution of this random variable.

10. Below is a probability distribution for weight of salmon caught in the state of Washington. (Assume that weight is a continuous random variable.)
   a. Calculate the probability that a Washington salmon will weigh between 1 and 3 pounds.
   b. Calculate the probability that a Washington salmon will weigh less than 2 pounds.
   c. Calculate the probability that a Washington salmon will be greater than or equal to 2.5 pounds.
   (*Hint:* You shouldn't have to compute any strange areas.)



11. A probability distribution is represented by the following function:

   $$p(x) = 0 \quad \text{for } x < 0$$

   $$p(x) = x \quad \text{for } 0 \le x < 0.50$$

$$p(x) = 0.5 \quad \text{for } 0.5 \leq x < 2.00$$

$$p(x) = 2.50 - x \quad \text{for } 2.00 \leq x < 2.50$$

$$p(x) = 0 \quad \text{for } x \geq 2.50$$

a. Plot $p(x)$ as a function of x.
b. What is $p(0.1 \leq x \leq 0.25)$?
c. What is $p(0 \leq x \leq 0.5)$?
d. What is $p(x < 1.0)$?
e. What is $p(x > 1.0)$?
f. What is $p(0 \leq x \leq 1.25)$?
g. What is $p(x > 0)$?

12. In a strange version of basketball, a shooter shoots foul shots (each successful shot worth one point). A shooter keeps shooting until he or she fails, up to five shots.

Plot the probability distribution for the number of points a shooter will get if the shooter's probability of making any given shot is

a. $\frac{9}{10}$ c. $\frac{1}{4}$
b. $\frac{1}{2}$ d. $\frac{1}{8}$

13. Each trial of the game of Dazzle involves throwing a black die, then a white die, and finally flipping a coin. The score x obtained on the trial is then obtained by the following formula:

x = number on white die plus number on black die plus 1 if coin turns up heads

x = number on white die plus number on black die if coin turns up tails

a. List the members of V, where V is the set of scores that a player could potentially get on each trial.
b. Compute the probability of all members of V.

In Superdazzle the rules are the same, but the dice and coin used are such that if the white die comes up a 1, it exerts a magic forcefield that causes the coin to always turn up heads.

a. List the members of V for Superdazzle.
b. Compute the probability of all members of V.

14. George the pollster plans to interview two individuals and inquire about their attitudes toward birth control. George gives each individual a score of 0 if he or she is unalterably opposed to birth control, a score of 2 if he or she firmly believes in birth control, and a score of 1 if he or she doesn't care. George will then sum the scores from the two individuals to obtain a "sample opinion" score.

a. What are the members of V, where V is the set of possible sample opinions?

b. Suppose that each individual has probabilities of 0.10, 0.50, and 0.40 of producing scores of 0, 1, and 2. What is the probability distribution over the members of V?

15. Suppose a coin is flipped. Let $a = 4$ if the coin comes up heads and $a = 2$ if the coin comes up tails. Now a die is thrown. Let b equal the number showing on the die; thus, b can range from 1 to 6.
    a. Letting a and b be random variables, compute their probability distributions.
    b. Suppose a new random variable x is created such that $x = b - a$. Determine the probability distribution of x.
    c. Suppose a new random variable y is created such that $y = b + x$. Determine the probability distribution of y.