$$\sum_{i=1}^{n_j} \epsilon_{ij} = 0$$

The null and alternative hypotheses may now be formulated as

$$H_0: \alpha_1 = \alpha_2 = \cdots = \alpha_j = 0$$

$$H_1: \text{not } H_0$$

We will not be discussing the linear model framework in great detail in this book, as we prefer (somewhat arbitrarily) the partition-of-sum-of-squares framework. Hays 1973; (see the general statistics section of the bibliography) provides an excellent description of the linear model.

PROBLEMS

1. An experiment is done to test the effect of age on memory in a free-recall paradigm. Three groups of subjects are chosen—10-year-olds, 21-year-olds, and 75-year-olds—each with 5 subjects in it. Each of the 15 subjects is shown a list of 20 words and then asked to write down as many words as possible. The results are as follows:

Number of Words Recalled

10-year-olds	21-year-olds	75-year-olds
$x_{11} = 1$	$x_{12} = 9$	$x_{13} = 3$
$x_{21} = 4$	$x_{22} = 8$	$x_{23} = 5$
$x_{31} = 5$	$x_{32} = 7$	$x_{33} = 7$
$x_{41} = 6$	$x_{42} = 10$	$x_{43} = 7$
$x_{51} = 4$	$x_{52} = 6$	$x_{53} = 8$

a. Plot the three means.
b. State the null hypothesis and alternative hypothesis.
c. For each of the three groups, compute an estimate of σ^2.
d. Take the average of these three estimates to get MSW.
e. Now get MSW by determining the sum of squares within and dividing it by the degrees of freedom within. Do you get the same answer as you do for question c?
f. Use the three means to estimate σ_M^2.
g. Now use your answer from question e plus the fact that est σ_M^2 = est σ^2/n to get the mean square between.
h. What is your F-score? How many degrees of freedom? What is the criterion F for the 0.05 level? Does your obtained F exceed this? Should you reject H_0? What does it mean if you reject H_0?
i. Draw 95% confidence intervals around the three means you plotted.

2. Some years ago there was a controversy involving whether smoking banana skins made one high, and a test was made. Three randomly selected groups of six subjects per group were given a dried-up substance to smoke and told it was banana skins. In fact, for the first group the substance was tea; for the second group the substance was marijuana; for the third group the substance was indeed banana. After smoking the substance, each subject rated how high he or she was on a scale from 7 (meaning "very high") to 1 (meaning "not high at all"). The following ratings were obtained:

Tea	**Marijuana**	**Banana**
5	2	3
6	3	4
3	6	4
5	6	5
4	4	5
4	5	1

   a. Test the hypothesis that these three substances differ from each other in terms of how high they make the subjects feel.
   b. Plot these three means along with 95% confidence intervals.
   c. During the experiment simple reaction times are measured. Average reaction times for the three groups are as follows:

Reaction Time (tenths of a second)		
Tea	**Marijuana**	**Bananas**
1.0	1.5	0.8
1.1	2.0	1.0
0.9	1.8	0.9
1.2	1.8	1.2
1.0	1.9	1.1
1.3	1.7	1.0

   d. Plot these three means along with 95% confidence intervals.
   e. Test the hypothesis that the three substances differ with respect to their effects on reaction time.

3. The following two "conditions" each contain seven random numbers between 0 and 10. (Random numbers were obtained from an SR-51A calculator.)

Condition 1	**Condition 2**
$x_{11} = 9$	$x_{12} = 3$
$x_{21} = 6$	$x_{22} = 4$
$x_{31} = 4$	$x_{32} = 7$
$x_{41} = 2$	$x_{42} = 4$
$x_{51} = 6$	$x_{52} = 2$
$x_{61} = 6$	$x_{62} = 0$
$x_{71} = 0$	$x_{72} = 3$

a. Compute an estimate of σ^2 by considering variation within each of the two groups (compute MSW).
b. Compute a second estimate of σ^2 by considering variation between the means of the two groups (compute MSB).
c. Do these two estimates of σ^2 differ significantly? Should they? Why or why not?

4. A psychologist was interested in the effects of different types of liquor on a mirror drawing task. He administered five types of liquor (beer, wine, vodka, gin, and scotch). Each of the five types of liquor was drunk by five subjects (that is, five *S*'s drank wine, five drank beer, and so on). Each group then performed the mirror drawing task, and the number of errors was recorded in the following table:

Wine	Scotch	Gin	Beer	Vodka
5	9	8	4	7
7	13	11	8	10
6	10	12	10	9
8	11	7	6	8
4	7	7	7	6

a. Plot the five means along with 95% confidence intervals.
b. Test the hypothesis that the five liquors differ with respect to effect on mirror drawing.

5. Joe Smith is interested in whether different brands of mouthwash lead to differences in breath freshness. He has three different groups of subjects. He gives Scope to one group, Listerine to another, and Lavoris to a third. Freshness of breath is then measured with a breathometer. Higher breathometer scores indicate fresher breaths, and breathometer scores are known to be distributed in the population with a variance $\sigma^2 = 2.0$. The data are as follows:

Group 1 (Scope) ($n_1 = 4$)	Group 2 (Listerine) ($n_2 = 2$)	Group 3 (Lavoris) ($n_3 = 3$)
$x_{11} = 2$	$x_{12} = 7$	$x_{13} = 5$
$x_{21} = 4$	$x_{22} = 10$	$x_{23} = 6$
$x_{31} = 4$		$x_{33} = 4$
$x_{41} = 2$		

a. Plot the three means along with 95% confidence intervals.
b. Perform the appropriate test on these data. Make sure you make use of the fact that the population variance is known.
c. Suppose you wanted to test whether MSW obtained from your data is a faithful estimate of σ^2. Perform such a test. Use the 0.05 α-level.
d. Suppose you did not know σ^2. Perform a standard ANOVA on these data.

6. In a verbal learning task nonsense syllables are presented for later recall. Three different groups of subjects see the nonsense syllables at a 1-, 5-, or

10-second presentation rate. The data (number of errors) for the three groups are as follows:

1-second Group	5-second Group	10-second Group
13	11	3
15	14	5
15	13	6
12	12	6
13	16	9
12	12	7
9	11	2
8	9	4
15	10	
	8	
	8	

a. Plot the three means along with 95% confidence intervals.
b. Perform an ANOVA on the data.

7. Consider the data of Chapter 10, problems 6, 7, 8, 10, and 11. Redo these problems using an ANOVA rather than a *t*-test. In each case compare the *t* you obtained with the *F* you now obtain. What is the relationship between the *t* and the *F* in each case?

8. For problem 6 of this chapter, evaluate the question: Do the scores in group 3 come from a population whose *variance* is greater than the variance of populations of groups 1 and 2 scores? (Assume the populations of groups 1 and 2 have the same variance.)

9. Use the information from Chapter 7, problem 10, to solve the following:
   a. Compute the 82% confidence intervals around the means of the two groups.
   b. Compute the power corresponding to the alternative hypothesis:

   $$\mu_1 - \mu_2 = 2$$

   Parts 3–6 of this problem assume that you do not trust the known variance estimate, and you plan to estimate the variance instead.
   c. Retest whether the groups differ.
   d. Compute the 95% confidence interval around each mean.
   e. Test whether your estimated variance differs from the "known" σ^2 of 4. (Use $\alpha = 0.10$.)
   f. Test whether the variance estimate from group 1 differs from the variance estimate of group 2. (Use $\alpha = 0.10$.)