Online Appendix:
Why Are University Endowments Large and Risky?

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Abstract

This is the online appendix to our paper "Why are university endowments large and risky?" It contains a number of additional details and results that are referenced in the body of the paper. In particular, the details of the time-varying expected returns extension to the non-rivalrous model as well as a complete set and analysis of impulse response functions for the non-rivalrous model (unconstrained and constrained). It also contains additional comparative statics of the constrained non-rivalrous model and results of the non-rivalrous model with a higher coefficient of relative risk aversion. Lastly, it contains results of the rivalrous model with a lower outside option and details on the multiple steady states in the non-rivalrous model with low average cash flow productivity from donations.

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O.1 Predictability in Expected Returns

In this appendix, we summarize the model of expected returns of Campbell and Viceira (1999), which is further analyzed in Campbell and Viceira (2002) and Campbell, Chan, and Viceira (2003). The log return of the risky asset is

$$r_{t+1} - E_t\left[r_{t+1}\right] = u_{t+1} \tag{O-1}$$

where u_{t+1} is the innovation to the return and is normally distributed with mean zero and variance σ_u^2. The expected excess stock return is a state variable (x_t) and is defined as

$$E_t\left[r_{t+1}\right] - r_f + \frac{\sigma_u^2}{2} = x_t \tag{O-2}$$

The state variable is modeled as an AR(1) process with mean μ and persistence ϕ

$$x_{t+1} = \mu + \phi(x_t - \mu) + \eta_{t+1} \tag{O-3}$$

where the innovation η_{t+1} is normally distributed with mean zero and variance σ_η^2.

The model's key assumption relies in the covariance between the two innovations, η_{t+1} and u_{t+1}, which is labeled as $\sigma_{\eta\mu}$. It is this covariance that generates intertemporal hedging and x_t represents the investment opportunity set at time t. Expected returns are allowed to be mean-reverting by making $\sigma_{\eta\mu} < 0$: high returns today are followed by low expected returns tomorrow.

Empirically, the state variable x_t is taken to be the log dividend-price ratio $(d_t - p_t)$, which is known to be a good predictor of stock returns. Using post-war quarterly U.S. financial data, Campbell and Viceira (1999) estimate the following restricted VAR(1) model

$$\begin{pmatrix} r_{1,t+1} - r_f \\ d_{t+1} - p_{t+1} \end{pmatrix} = \begin{pmatrix} \theta_0 \\ \beta_0 \end{pmatrix} + \begin{pmatrix} \theta_1 \\ \beta_1 \end{pmatrix} (d_t - p_t) + \begin{pmatrix} \varepsilon_{1,t+1} \\ \varepsilon_{1,t+1} \end{pmatrix} \tag{O-4}$$

where the innovations are normally distributed with mean zero and covariance matrix Ω. From the estimated coefficients of the VAR(1) model, the coefficients in equations (O-1), (O-2) and (O-3) can be recovered.

In our calibration, we use the same assumptions and estimates as in Campbell and Viceira (1999). The unconditional expected log excess return μ is estimated to be 5% per year and the log real risk-free rate r_f is 0.28% per year. The persistence parameter ϕ of the state variable process is 0.957 and the correlation between the η and u innovations is -0.737. The annual volatility of the stock return σ_u is 14.55% and the annual volatility of the state variable σ_η is 0.75%.

O.2 Impulse Response Functions

In this appendix, we analyze impulse response functions (IRF) for the unconstrained and constrained non-rivalrous models. The results are presented in Figure O.1. In Panel A, we consider the case where the university receives a large positive shock to its endowment in the form of a large donation equal to a 20% of a year's budget (+0.2). In Panel B, we consider the case of an equivalent dollar loss of -0.2 following large negative returns.[1] In both panels, the first column is the $\mathcal{D} = 10\%$ case and the second column is the $\mathcal{D} = 25\%$ case. The light (grey) lines are for the unconstrained case and the dark (black) lines are for the constrained case. We plot the equilibrium values as solid lines and the IRFs as dashed lines for four characteristics: endowment size, allocation to the risky asset, dollar spending on new internal projects, and university size (capital).

First, we first discuss the impact of a large positive donation on the university and its endowment (Panel A). We see that in the unconstrained case, both universities immediately increase the amount spent on new internal projects via an increased endowment payout. The capital stock grows, leaving the university permanently larger, and the university returns to its equilibrium endowment-to-capital ratio in two periods.[2] When the constrained universities receive the donation, they also increase their payout on new internal investment, causing their capital stock to increase, but they can only do so very slowly and it takes many years for them to fully deploy the donation. In the mean time, they are forced to hold the excess funds that they would rather invest internally in their endowment. The constrained universities increase their endowment's allocation to the risky asset, thereby taking risk in financial markets rather than internally.

As previously shown, the constraint actually leads to an increase in the growth rate of the less productive universities ($\mathcal{D} = 10\%$), but this increased growth comes at the cost of reduced precautionary savings and reduced ability to buffer shocks, which we analyze

[1] We analyze a fixed dollar loss, rather than a percentage loss, to facilitate comparison across models which have different equilibrium values for their endowment-to-capital ratio.

[2] The endowment's asset allocation is unchanged as the donation is rapidly spent internally on new capital.

in Panel B. However, the constraint significantly decreases the growth rate of the most productive universities ($\mathcal{D} = 25\%$) and it severely impedes their ability to rapidly invest on new internal projects. The gap between the IRF and the equilibrium university size is much larger in the unconstrained case, leaving the constrained university ever further behind.

Second, we discuss the impact of a large dollar loss (Panel B). In the case of a less productive university ($\mathcal{D} = 10\%$), we see that the constraint limits the university's ability to use its endowment to buffer the shock to such an extent that it shrinks its capital stock. While the unconstrained university also curtails new internal investment, it does not shrink its capital. This inability to buffer due to the constraint causes the constrained university to have a larger decrease in its long run size than the unconstrained university, i.e., the (negative) gap between the IRF and the equilibrium university size is much larger in the constrained case. Both universities shift their endowment asset allocation away from the risky asset but the unconstrained university does so for longer, which is one way it avoids shrinking its capital – the other being increased payout from its endowment which is used as a precautionary savings buffer.

In the case of the most productive universities ($\mathcal{D} = 25\%$), we see that the university is willing to shrink its capital stock following a negative shock. This occurs because the university maintains a very small endowment which cannot buffer the shock. However, due to its level of productivity, the cost of shrinking is low and the capital recovers very rapidly, as the amount of new internal investment is back to its equilibrium level within two periods. When constrained, the university does not shrink its capital since it holds a much larger endowment to buffer the shock and the cost of shrinking is high. Indeed, the constraint limits the ability of the university to invest in new projects quickly.

O.3 Tables and Figures

Table O.1
Non-Rivalrous Production Model with Higher Risk Aversion

For the unconstrained non-rivalrous production model, this table presents various characteristics of the university at its stable stochastic steady-state when the coefficient of relative risk aversion of the university stakeholders is increased to 7. The results are presented for different values of the cash flow (CF) productivity of its internal projects from donations, namely $\mathcal{D}$, and different values for the correlation of the donation process with returns. The reported characteristics are the endowment size (E_t/K_t), the endowment's allocation to the risky asset (α_t), the endowment's payout rate (P_t/E_t), the ratio of the internal investment (I_t) over the endowment's total payout (P_t), the growth rate of capital at the steady-state. The volatility of the donations is $\sigma_D = 35\%$ and the marginal cost of internal projects is $A = 1$ (medium).

Average CF productivity ($\mathcal{D}$)	8%	10%	10%	10%	12%	17%	25%
Correlation (ρ)	0.18	0.03	0.18	0.32	0.18	0.18	0.18
Endowment size	16.91	14.52	15.18	15.93	10.64	0.23	0.21
Allocation to risky asset	17.11%	15.51%	14.86%	14.62%	13.19%	100.00%	100.00%
Payout rate	0.66%	0.77%	0.73%	0.72%	1.05%	78.65%	95.42%
Internal investment / payout	10.06%	10.51%	10.21%	10.56%	10.60%	37.98%	55.21%
Local capital growth rate	1.06%	1.17%	1.13%	1.11%	1.18%	5.91%	11.49%

Table O.2
Rivalrous Production Model with Lower Outside Option

For the unconstrained rivalrous production model, this table presents various characteristics of the university at five selected endowment sizes (no stochastic steady state exists). The outside option is decreased to 25% of the net maintenance cost ($L = 0.75$). We report the endowment size (E_t/K_t), the endowment's allocation to the risky asset (α_t), the endowment's payout rate (P_t/E_t), the ratio of either the personal payment ($\mathcal{P}_t$) or the internal investment (I_t) over the endowment's total payout (P_t), and the growth rate of capital at the selected endowment size. The productivity of the internal projects from donations is equal to $\mathcal{D} =$ 10%; the volatility of the donations is $\sigma_D = 35\%$; the correlation between donations and market returns is $\rho = 0.18$; and the marginal cost of internal projects is $A = 1$ (medium).

Selected endowment size	0.11	0.98	2	5	10	20
Allocation to risky asset	18.36%	100.00%	100.00%	98.73%	85.36%	75.77%
Payout rate	90.91%	13.93%	9.58%	6.27%	4.87%	4.01%
Personal payment / payout	0.00%	26.77%	47.81%	68.11%	79.45%	87.54%
Internal investment / payout	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Local growth rate	-8.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table O.3
Non-Rivalrous Production Model with Constrained Endowment Payout

For the constrained non-rivalrous production model, this table presents various characteristics of the university at its stable stochastic steady-state for different values of the volatility of donations, correlation of donations with market returns, or marginal cost of new internal projects, where $\mathcal{D}$ is fixed at 10%. The correlation is 0.18 when the volatility is changed. The volatility is 0.35 when the correlation is changed. The correlation is 0.18 and the volatility is 0.35 when the marginal cost is changed. The marginal cost is 1 (medium) in all cases but the last two columns of Panel A. The reported characteristics are the endowment size (E_t/K_t), the endowment's allocation to the risky asset (α_t), the endowment's payout rate (P_t/E_t), the ratio of the internal investment (I_t) over the endowment's total payout (P_t), the growth rate of capital at the steady-state, as well as the average growth rate of capital and the volatility of that growth rate from 10,000 Monte Carlo (MC) simulations of 500 years beginning at the university's steady-state and using the optimal policies.

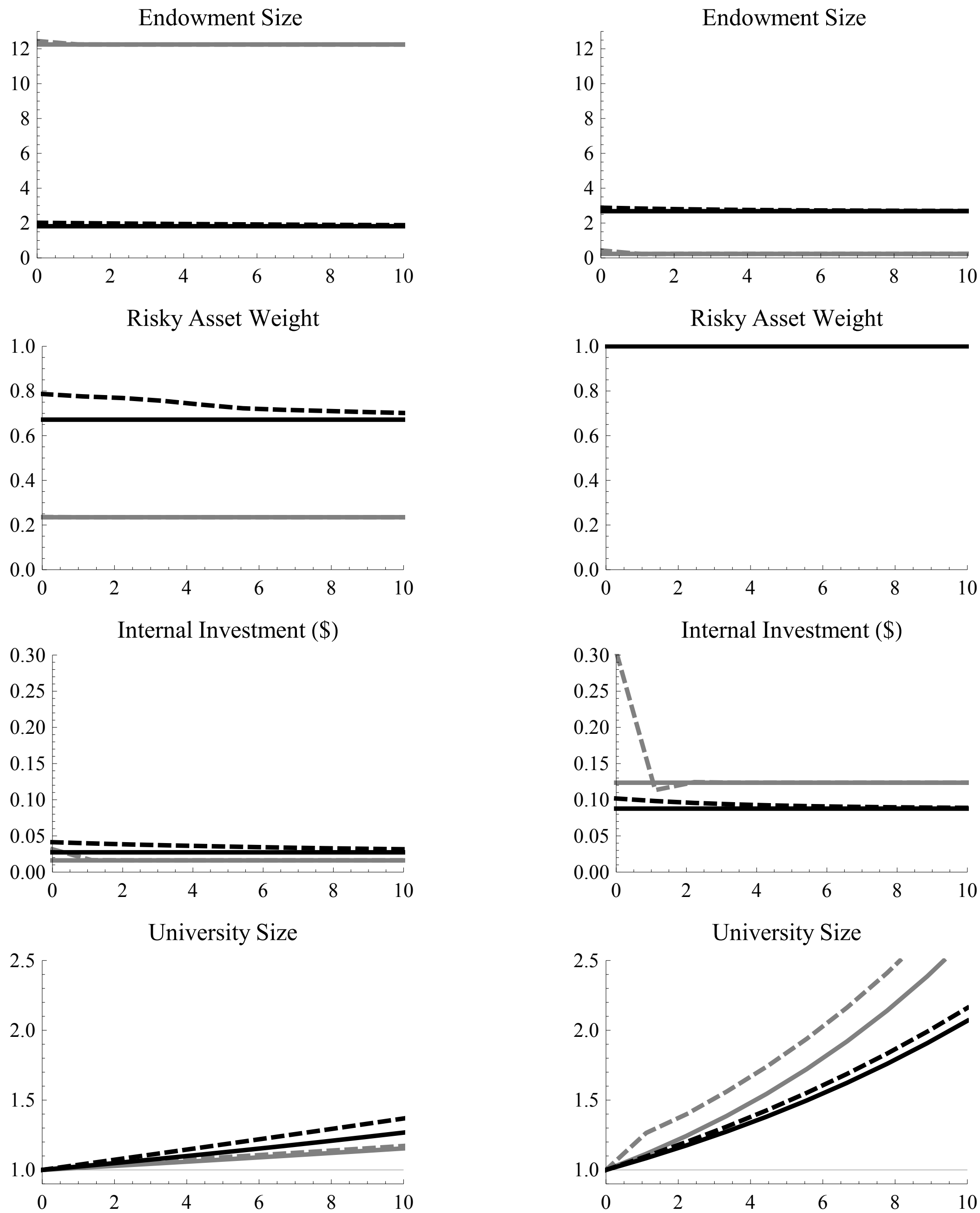
	Volatility (σ_D)		Correlation (ρ)		Marginal cost (A)	
	0.25	0.43	0.03	0.32	High	Low
Endowment size	1.81	1.83	1.80	1.84	2.12	1.69
Allocation to risky asset	68.22%	65.57%	70.69%	63.37%	88.67%	60.14%
Payout rate	7.00%	7.00%	7.00%	7.00%	7.00%	7.00%
Internal investment / payout	21.07%	21.72%	20.50%	22.36%	32.52%	15.62%
Local capital growth rate	2.71%	2.60%	2.81%	2.51%	2.66%	3.15%
MC average growth rate	2.21%	2.11%	2.28%	2.05%	2.14%	2.20%
MC growth rate volatility	2.71%	2.78%	2.72%	2.77%	2.52%	3.03%

Figure O.1
IRFs in Non-Rivalrous Model: Effect of the Payout Constraint

The graphs show the non-rivalrous steady state (solid lines) and the Impulse Response Functions (IRF, dashed lines) over 10 years for four characteristics of the university: endowment size, risky asset allocation, dollar payout for internal investment, and university size (capital). The gray (lighter) lines are for the unconstrained model and the black (darker) lines are for the constrained model. All results are from the non-rivalrous model with medium production cost ($A = 1$). Panel A is for a +0.2 shock in the endowment and Panel B is for a -0.2 shock in the endowment. The first column shows results for $\mathcal{D} = 10\%$ and the second column shows results for $\mathcal{D} = 25\%$.

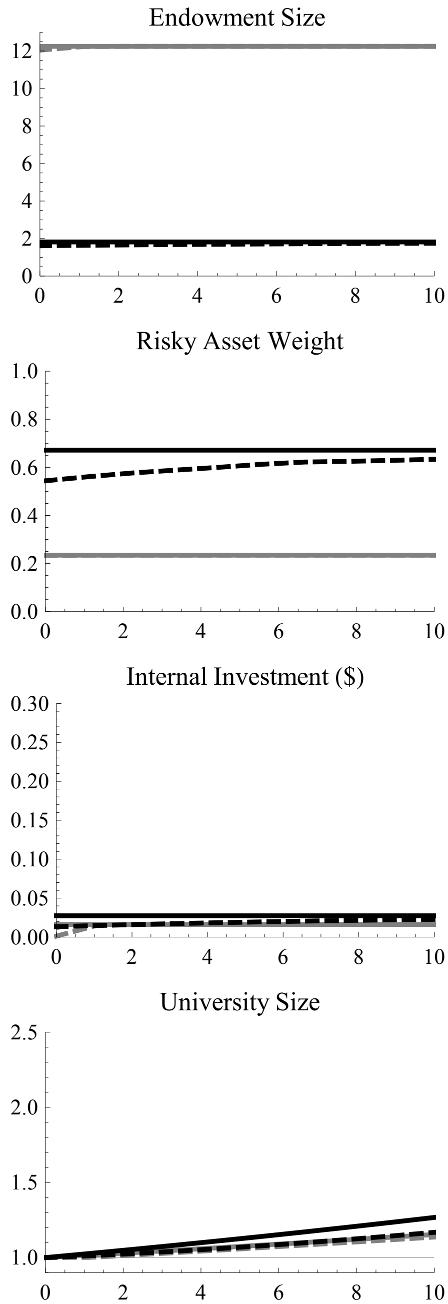
Panel A: +0.2 Shock in Endowment

Col. 1: CF Productivity $\mathcal{D} = 10\%$ | Col. 2: CF Productivity $\mathcal{D} = 25\%$



Panel B: -0.2 Shock in Endowment

Col. 1: CF Productivity $\mathcal{D} = 10\%$



Col. 2: CF Productivity $\mathcal{D} = 25\%$

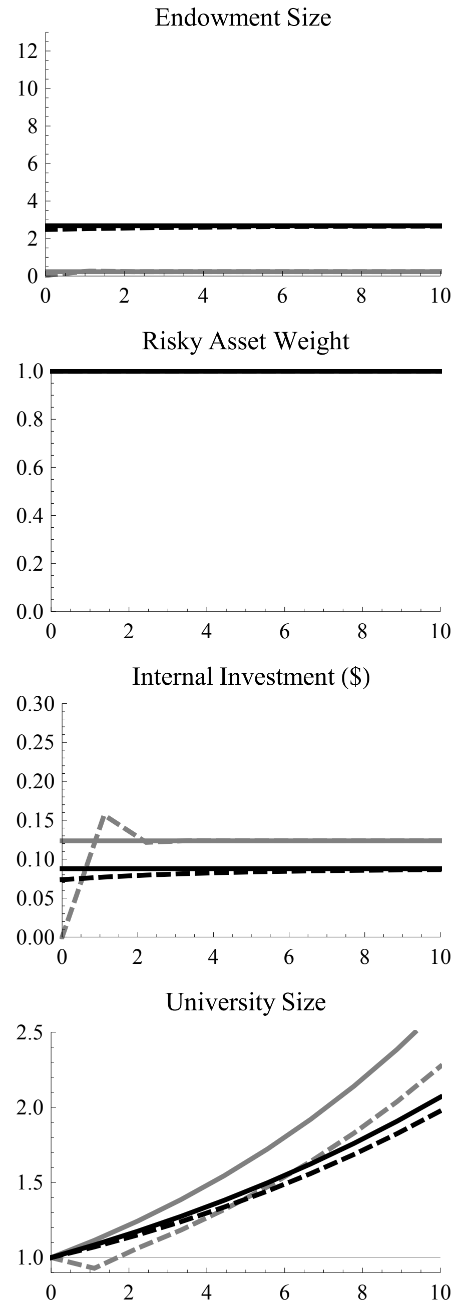
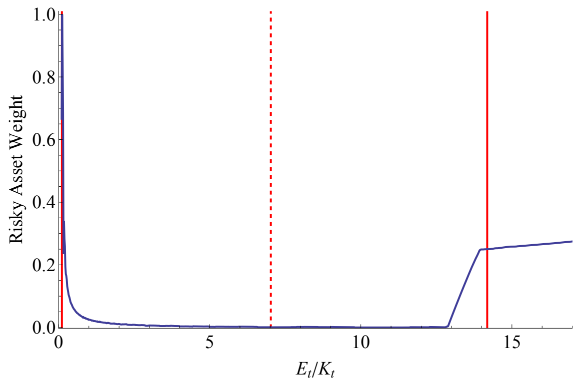


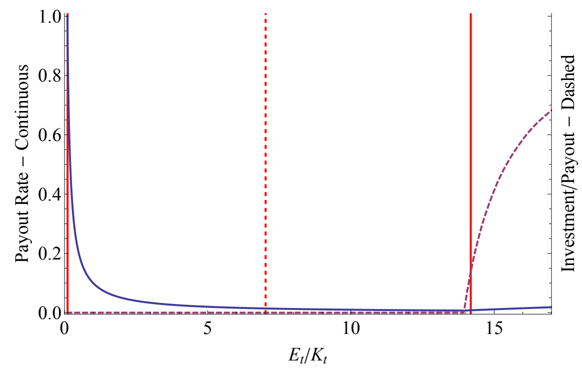
Figure O.2
Multiple Steady States in Non-Rivalrous Production Model with Low Average Cash Flow Productivity

This figure and accompanying table report the comparable results to Figure 2 and Table 2 for the unconstrained non-rivalrous model with average cash flow productivity ($\mathcal{D}$) of 8%. See Figure 2 and Table 2 for detailed captions. The continuous (red) vertical lines represent the stable stochastic steady states and the dashed (red) vertical line represent the unstable steady state.

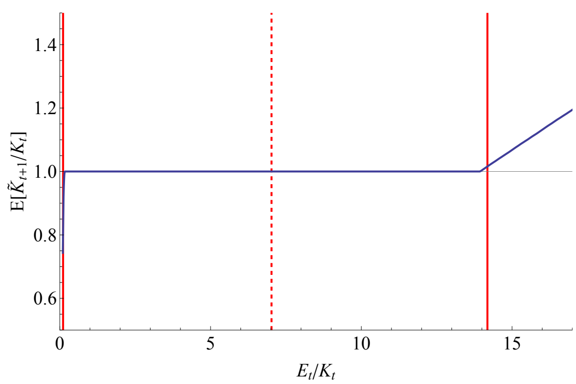
Panel A: Risky asset weight



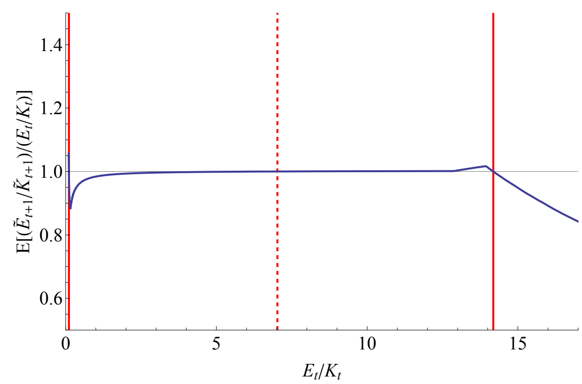
Panel B: Payout and internal investment



Panel C: Capital growth



Panel D: Endowment/capital growth



Panel E: Characteristics of stochastic steady states

Steady state type	Stable	Unstable	Stable
Endowment size	0.11	7.02	14.18
Allocation to risky asset	100.00%	0.07%	24.99%
Payout rate	92.59%	1.42%	0.82%
Internal investment / payout	0.00%	0.00%	13.74%
Local growth rate	-19.12%	0.00%	1.58%
MC average growth rate	-19.59%	0.00%	1.09%
MC growth rate volatility	19.61%	0.00%	2.10%

References

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