Internet Appendix for "Precautionary Savings with Risky Assets: When Cash is Not Cash"

Ran Duchin, Thomas Gilbert, Jarrad Harford and Christopher Hrdlicka*

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ABSTRACT

This Internet Appendix presents two supplementary tables as well as a theoretical model of a firm's financial investment decisions. We consider a world with two states and three dates in which the firm has access to liquid or illiquid safe assets and a liquid risky asset. We derive the optimal investment policy under both risk neutrality and risk aversion. We also show how the model is robust to covariation between the risky asset returns and (1) the productivity of the firm's real investment projects and (2) the ability to access external capital.

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INTERNET APPENDIX: TABLE IV*

Tax Costs of Repatriating Foreign Earnings – Alternative Measure

This table reports OLS and 2SLS estimates from panel regressions explaining firms' financial investment policy. In the first stage of the 2SLS model, we estimate the following equation:

$$(1)\ Total\ investments_{i,t} = \alpha_0 + \alpha_1 Unexpected\ cash\ flow_{i,t} + \alpha_2 Foreign\ income_{i,t} + \alpha_3 Domestic\ income_{i,t} + \alpha_4 Tax\ cost\ of\ repatriating\ earnings_{i,t} + \beta' X + \sum_t year_t + \sum_t ind_t + \varepsilon^T_{i,t}$$

where: *Unexpected cash flow* is the residual from regressing a firm's annual change in cash flow on the annual cash flow changes over the past three years, and *X* is a vector of baseline regressors, which include a firm's market-to-book ratio, size, cash flow, net working capital (excluding cash), capital expenditure, leverage, industry cash flow volatility, a dividend dummy, R&D expenditure, and acquisition expenditure (Bates, Kahle, and Stulz, 2009). *Tax cost of repatriating earnings* is computed by first subtracting foreign taxes paid from the product of a firm's foreign pre-tax income and its marginal effective tax rate as calculated in Graham (1996). We then scale the maximum of this difference or zero by total firm assets. *Alternative tax cost of repatriating earnings* is computed in the same way as the *Tax cost of repatriating earnings*, but with US statutory tax rates used in place of marginal tax rates. Table B.3 in Appendix B gives definitions and summary statistics for all variables. In the second stage, we estimate the following equation:

$$(2)\ Risky\ investments_{i,t} = \alpha_0 + \alpha_1 Total\ investments^*{}_{i,t} + \alpha_2 Foreign\ income_{i,t} + \alpha_3 Domestic\ income_{i,t} + \alpha_4 Tax\ cost\ of\ repatriating\ earnings_{i,t} + \beta' X + \sum_t year_t + \sum_t ind_t + \varepsilon^R_{i,t}$$

where: $Total\ investments^*$ is the predicted value from the first stage estimation. Risky investments are defined as assets that fall into the following asset classes: government debt excluding U.S. Treasuries, corporate debt, asset- and mortgage-backed securities (ABS & MBS), other debt, equity, and other securities. Safe investments are defined as assets that fall into the following asset classes: cash, cash equivalents, and U.S. Treasuries. Each investment is manually assigned into a unique asset class based on hand-collected data from footnotes of annual reports. The sample comprises all firms in the S&P 500 Index from 2009-2012, excluding financial firms (SIC 6000-6999) and regulated utilities (SIC 4900-4999). The regressions include year and industry fixed effects, which are not shown. The standard errors (in brackets) are heteroskedasticity consistent and clustered at the firm level. Significance levels are indicated as follows: * = 10%, ** = 5%, *** = 1%.

Model	OLS	2SLS		OLS	2SLS		OLS	2SLS	
		First stage	Second stage		First stage	Second stage		First stage	Second stage
Dependent variable	Risky investments / investments	Total investments / assets	Risky investments / investments	Risky investments / investments	Total investments / assets	Risky investments / investments	Risky investments / investments	Total investments / assets	Risky investments / investments
Column	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Total investments	0.722*** [0.103]			0.708*** [0.107]			0.719*** [0.109]		
Unexpected cash flow		0.175** [0.071]			0.144** [0.062]			0.144** [0.062]	
Total investments*			0.295*** [0.093]			0.280*** [0.084]			0.286*** [0.092]
Pretax foreign income	-0.187 [0.165]	0.514*** [0.133]	0.168 [0.395]	-0.252 [0.188]	-0.081 [0.116]	-0.296 [0.200]	-0.201 [0.208]	-0.269** [0.125]	-0.393 [0.262]
Pretax domestic income	0.176 [0.132]	0.109 [0.090]	0.262 [0.162]	0.173 [0.132]	0.075 [0.080]	0.249 [0.158]	0.176 [0.132]	0.094 [0.080]	0.265 [0.165]
Tax cost of repatriating earnings				0.587 [1.300]	4.916*** [0.769]	4.544 [3.834]			
Alternative tax cost of repatriating earnings							0.083 [1.114]	4.545*** [0.675]	3.693 [3.470]
Control variables?	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year fixed effects?	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry fixed effects?	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	0.398	0.565	0.315	0.399	0.613	0.284	0.398	0.615	0.287
N_obs	1,727	1,727	1,727	1,727	1,727	1,727	1,727	1,727	1,727

INTERNET APPENDIX: TABLE V*

Agency Problems – Controlling for Pretax Foreign Income

This table reports OLS and 2SLS estimates from panel regressions explaining firms' financial investment policy. Model specifications augment those in Table V with pretax foreign income. *Unexpected cash flow* is the residual from regressing a firm's annual change in cash flow on the annual cash flow changes over the past three years, The control variables include a firm's market-to-book ratio, size, cash flow, net working capital (excluding cash), capital expenditure, leverage, industry cash flow volatility, a dividend dummy, R&D expenditure, and acquisition expenditure (Bates, Kahle, and Stulz (2009)). *Governance* is measured using three proxies following Dittmar and Mahrt-Smith (2007): the *E-index*, the sum of the 5% institutional *block holdings*, and the sum of public *pension fund holdings*. $Total\ investments^{*}$ is the predicted value from the first stage estimation. Risky investments are defined as assets that fall into the following asset classes: government debt excluding U.S. Treasuries, corporate debt, asset- and mortgage-backed securities (ABS & MBS), other debt, equity, and other securities. Safe investments are defined as assets that fall into the following asset classes: cash, cash equivalents, and U.S. Treasuries. Each investment is manually assigned into a unique asset class based on hand-collected data from footnotes of annual reports. The sample comprises all firms in the S&P 500 Index from 2009-2012, excluding financial firms (SIC 6000-6999) and regulated utilities (SIC 4900-4999). The regressions include year and industry fixed effects, which are not shown. The standard errors (in brackets) are heteroskedasticity consistent and clustered at the firm level. Significance levels are indicated as follows: * = 10%, ** = 5%, *** = 1%.

Model	OLS	2SLS		OLS	2SLS		OLS	2SLS	
		First stage	Second stage		First stage	Second stage		First stage	Second stage
Dependent variable	Risky investments / investments	Total investments / assets	Risky investments / investments	Risky investments / investments	Total investments / assets	Risky investments / investments	Risky investments / investments	Total investments / assets	Risky investments / investments
Column	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Total investments	0.754*** [0.120]			0.723*** [0.108]			0.672*** [0.117]		
Unexpected cash flow		0.218*** [0.080]			0.178** [0.073]			0.150** [0.071]	
Total investments*			0.272*** [0.081]			0.274*** [0.083]			0.283*** [0.081]
E-index	0.022* [0.013]	-0.012** [0.006]	0.022** [0.011]						
Block holdings				-0.049*** [0.018]	0.012 [0.011]	-0.041** [0.019]			
Pension holdings							-0.069*** [0.026]	0.005 [0.020]	-0.066** [0.029]
Pretax foreign income	-0.355* [0.207]	0.485*** [0.152]	-0.193 [0.346]	-0.243 [0.193]	0.450*** [0.148]	0.056 [0.381]	-0.245 [0.204]	0.465*** [0.159]	0.035 [0.439]
Control variables?	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year fixed effects?	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry fixed effects?	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	0.390	0.584	0.371	0.392	0.552	0.306	0.367	0.539	0.291
N_obs	1,436	1,436	1,436	1,502	1,502	1,502	1,553	1,553	1,553

In this Internet Appendix we present a parsimonious model to consider how a firm facing real investment projects and limited access to financial markets chooses (1) the size of its financial portfolio and (2) the allocation of that portfolio. This model abstracts away from taxes as we discuss the potential effects of those more fully in Appendix C. Our model can be viewed as an extension of the models in Almedia, Campello, and Weisbach (2004) and Gilbert and Hrdlicka (2015).[1] The main innovation in our model is allowing firms to invest in an array of financial assets.

We consider different types of available financial assets, interacted with the firm's production technology, financial frictions, and agency problems. In Section II we consider the case of only safe assets, both liquid and illiquid. In Section III we consider risky financial assets. We first model the manager and shareholders as being risk neutral. We then model them as being risk averse, having the market wide discount factor. In Section IV we derive extensions in which our results are robust to state-dependent real projects and financial frictions. Finally, in Section V we consider agency frictions in both financial and real investments.

I. Basic Setup

Consider a world with two states and three dates. Due to financial frictions (unmodelled) the firm cannot freely access financial markets. For simplicity, we initially assume infinite costs of issuing equity and borrowing, though we later relax the borrowing constraint in Section IV.

The firm begins with an exogenous amount of capital C *inside* the firm. We take C as given and do not explicitly model the initial motivations or mechanisms for holding and raising financial capital. The firm uses C to invest in both real projects and financial assets. We denote the terminal value of the firm's financial assets as F. Importantly, the firm always has access to a risk-free asset with rate of return, $R_f = 1$.

[1]Gilbert and Hrdlicka (2015) study the size and allocation of financial portfolios in the context of university endowments.

The firm has two real investment projects, a date 0 project and a date 1 project, both paying off at date 2. Our results apply to general projects with decreasing returns to scale, but we focus on the log case for tractability. The first project requires a date 0 investment I_0 and pays off

$$Y_0 \equiv R_0 \log(1 + I_0). \tag{1}$$

The second project requires a date 1 investment I_1 and pays off

$$Y_1 \equiv R_1 \log(1 + I_1). \tag{2}$$

R_0 and R_1 are the initial marginal productivity of the two projects.

The manager maximizes firm value, which is the discounted sum of the expected payoffs from the projects and the financial portfolio. We assume that financial markets are perfectly competitive such that the firm behaves as a price taker when transacting financial assets.

II. Safe Financial Assets

A. Liquid Asset

The firm only has access to the risk-free asset. The manager and the shareholders are risk neutral. The manager maximizes the expected present value of the firm, i.e., the sum of the projects' payoffs and the value of the financial portfolio. The manager solves the following problem

$$\max_{I_0, I_1} R_0 \log(1 + I_0) + R_1 \log(1 + I_1) + F \tag{3}$$

subject to the budget constraints

$$0 \leq I_0 \leq C, \tag{4}$$

$$0 \leq I_1 \leq (C - I_0)R_f, \quad \text{and} \tag{5}$$

$$F = [(C - I_0)R_f - I_1]R_f. \tag{6}$$

The payoffs in Equation (3) are all at the same date and therefore need no discounting.[2]

The solution to the manager's problem equates the benefit from additional investment in the date 0 project with the benefit from investing those resources in financial markets and using them to fund the date 1 project. The marginal benefit of the date 0 project is the marginal cost of investing in the date 1 project.

We need to consider two cases: (1) the firm is unconstrained, i.e., it has sufficient initial capital to fully exploit both projects; and (2) the firm is constrained, i.e., it has insufficient initial capital to fully exploit both projects.

A.1. Unconstrained Case

The firm is unconstrained when the initial capital within the firm, C, is sufficient to support first-best investment in both projects. The first-best investment in both projects occurs when their marginal return is driven to the risk-free rate ($R_f = 1$). The amount of real investment that delivers this equality is

$$I_0 = R_0 - 1 \quad \text{and} \tag{7}$$

$$I_1 = R_1 - 1. \tag{8}$$

Thus the firm is unconstrained when

$$C \geq R_0 + R_1 - 2. \tag{9}$$

[2] $R_f = 1$ and therefore could be suppressed in the budget constraints.

At the optimum, when firms make first-best investments in both projects, the opportunity cost of excess retained capital is zero. In Figure 1, we present the marginal cost and benefit curves from all retained capital choices. The intersection of these curves captures the optimum, while other points represent suboptimal decisions. Notice that the value of the firm (up to a constant) is the integral of the upper envelope of the marginal benefit and cost curves joined at their intersection.

When the firm is unconstrained, the opportunity cost is positive only when the firm chooses to retain so much capital at date 0 such that it cannot fully exploit its date 0 project. Similarly, the marginal benefit of retained capital is zero once the firm has retained enough to fully exploit its date 1 project. Importantly, these are measured relative to the benchmark outside option of the risk-free rate.

Panels (a) and (b) of Figure 1 correspond to the unconstrained case. In panel (a), where $C > R_0 + R_1 - 2$, there exists a region in which the firm is indifferent about how much capital it retains. In panel (b), where $C = R_0 + R_1 - 2$, the firm has only one optimal amount of capital to retain. The firm invests in its real projects identically across both cases.

A.2. Constrained Case

In the case of a constrained firm, where the initial capital within the firm, C, is insufficient to support first best investments, the budget constraint in Equation (5) binds. The firm invests all its capital in real projects and the value of the financial portfolio, F, is 0 at date 2.

The firm invests to equate the marginal productivity of both projects. The solution for this problem is

$$I_0 = \frac{R_0(1+C) - R_1}{R_0 + R_1} \quad \text{and} \tag{10}$$

$$I_1 = C - I_0 \tag{11}$$

The firm's marginal cost and benefit of retained capital are plotted in panels (c) and (d) of Figure 1. The marginal cost of retained capital may be zero at first, and eventually becomes that of forgoing some of the date 0 project. Similarly, the marginal benefit of retained capital is at first that of undertaking the date 1 project but can also eventually fall to zero as that project is exhausted. The curves intersect when the marginal productivity of the projects is equalized. This result, though deterministic, is similar to that in standard models of precautionary savings (see Almeida, Campello, Cunha, and Weisbach, 2014, and references therein).

B. Liquid and Illiquid Assets

Now suppose the firm can also invest at date 0 in an illiquid asset that matures at date 2. It pays a rate R_{IL} which is higher than the risk-free rate (i.e., the firm receives R_{IL} at date 2). The illiquid security pays a lower return than both projects ($R_{IL} < R_0$ and $R_{IL} < R_1$). The illiquidity arises because investment in the illiquid asset can only be liquidated at date 1 for a loss relative to investing in the risk-free liquid asset $R^*_{IL} < R_f$ (i.e., the firm receives R^*_{IL} at date 1 and nothing at date 2).

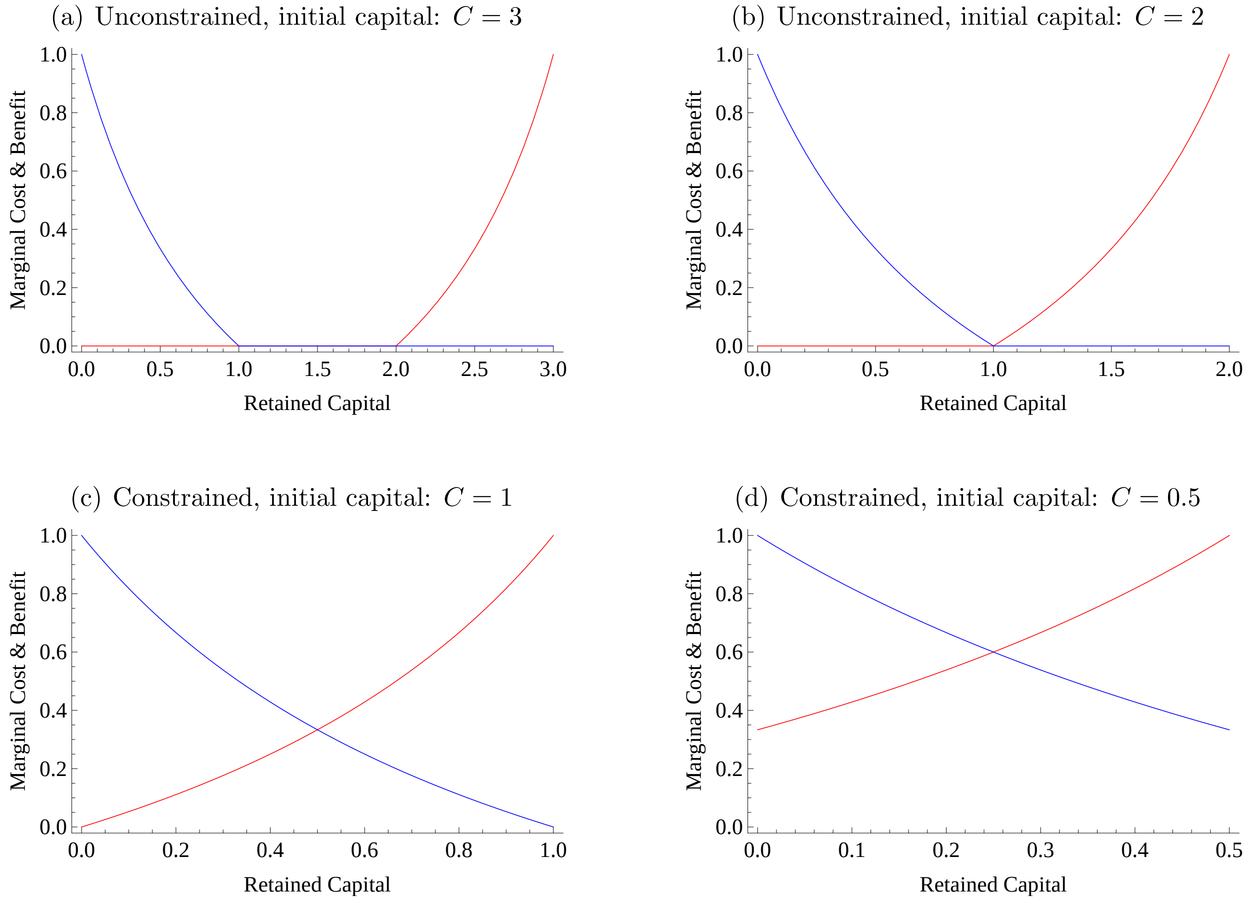

Figure 1. Safe Liquid Financial Asset: This figure shows the marginal cost (red) and benefit (blue) of the firm for retaining internal capital not invested in the date 0 project (or the best financial alternative). The marginal cost curve is

$$\max\left\{ \left.\frac{\partial Y_0}{\partial I_0}\right|_{C-x},\ R_f \right\} - R_f.$$

The marginal benefit curve is

$$\max\left\{ \left.\frac{\partial Y_1}{\partial I_1}\right|_{x},\ R_f \right\} - R_f.$$

The parameters are $R_f = 1$, $R_0 = 2$, $R_1 = 2$. Panels (a) through (d) show decreasing levels of initial capital within the firm $C = 3,\ 2,\ 1$ and 0.5. Note that the marginal cost and benefit are in excess of the risk-free rate.

The manager solves

$$\max_{I_0, I_1, w_{IL,0}, w_{IL,1}} R_0 \log(1 + I_0) + R_1 \log(1 + I_1) + F \tag{12}$$

subject to the following budget constraints

$$0 \leq I_0 \leq C, \tag{13}$$

$$0 \leq I_1 \leq (C - I_0) \left[(1 - w_{IL,0}) R_f + w_{IL,1} R^*_{IL}\right] \quad \text{and} \tag{14}$$

$$F = \left\{(C - I_0) \left[(1 - w_{IL,0}) R_f + w_{IL,1} R^*_{IL}\right] - I_1\right\} R_f + (C - I_0)(w_{IL,0} - w_{IL,1}) R_{IL} \tag{15}$$

where $w_{IL,0}$ is the fraction of the portfolio invested in illiquid asset at date 0 and $w_{IL,1}$ is the fraction of the portfolio the was invested in illiquid assets, but is liquidated at date 1. $w_{IL,1}$ must be less than or equal to the initial fraction invested in illiquid assets. Of the funds invested in the illiquid asset at date 0, only those liquidated at date 1 can be used for real investment.

Because there is no uncertainty in the problem, the manager can perfectly forecast that the amount they need in liquid assets equals the amount they plan to invest in project 1 at date 1. Therefore the manager will never liquidate the illiquid asset at date 1.

Immediately following from this, the firm only invests in the illiquid asset if it is unconstrained. However, the level of first-best investment changes given that the outside option is now $R_{IL} > R_f$. This change in first-best investment lowers the level of capital necessary for the firm to be unconstrained.

In all other respects, the solution to the manager's problem is essentially identical to the solution in the previous section. Thus, the unconstrained solution is the amount of investment which equalizes the marginal productivity of the real projects with the rate of return on the illiquid asset:

$$I_0 = R_0 - R_{IL} \tag{16}$$

$$I_1 = R_1 - R_{IL}. \tag{17}$$

Figure 2 plots the marginal cost and benefit curves (dashed lines). To facilitate comparisons with Figure 1, we measure these relative to the risk-free rate. The marginal cost curve is defined as in the previous case. Note that the opportunity to hold the capital in the illiquid asset, which earns a higher rate of return, raises the marginal cost curve (higher opportunity costs). The marginal benefit curve is defined analogously. For ease of comparison, throughout the remainder of this document, we also plot the marginal benefit and marginal cost curves (solid lines) of the base case of investing solely in the liquid risk-free asset (Figure 1).

Panels (a) and (b) demonstrate the lower first-best (real) investment levels which make the firm more unconstrained. The flat portion of the curve represents the retained capital not invested in either project, which the firm optimally invests in the illiquid asset.

Panels (c) and (d) demonstrate that when the firm is constrained, being unable to reach first-best investment levels, it behaves exactly as in the liquid asset only case. This identical behavior arises because constrained firms never use the illiquid asset.

Panels (a) and (b) show that when the firm is unconstrained, it is better off when the illiquid asset is available since that asset raises its outside option. We see the firm is better off as the area under the marginal benefit and cost curves is larger. Panels (c) and (d) show that when the firm is constrained, it is indifferent to having access to the illiquid asset: the areas under the curves are identical with or without the illiquid asset.[3]

[3] In Panel (c), the differences between the solid and dashed curves only occur in sections irrelevant to the upper envelope. In contrast, in Panels (a) and (b) the upper envelope differs across the solid and dashed curves.

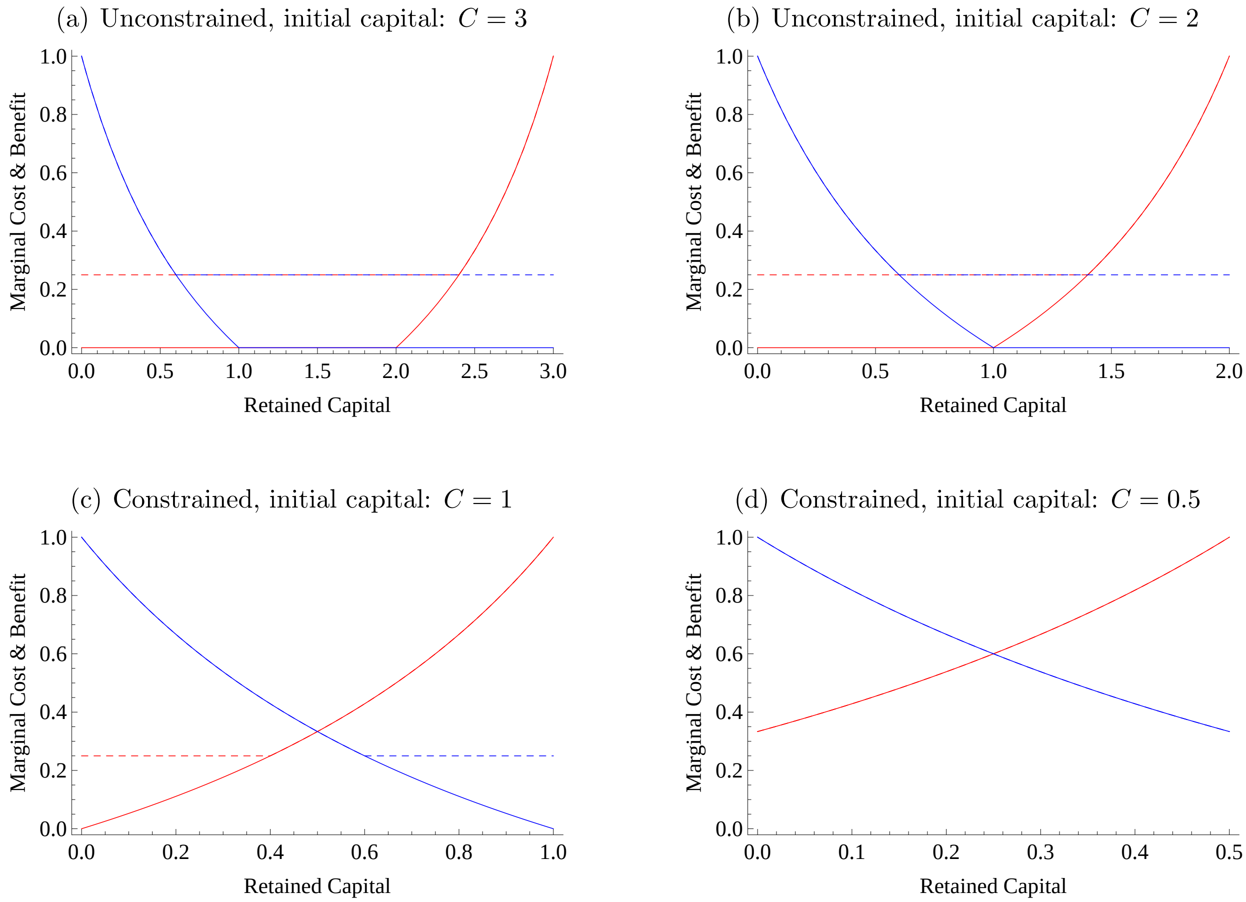

Figure 2. Safe Liquid and Safe Illiquid Financial Assets: This figure shows the marginal cost (red) and benefit (blue) of the firm for retaining internal capital uninvested in the date 0 project. The marginal cost curve is

$$\max\left\{ \left.\frac{\partial Y_0}{\partial I_0}\right|_{C-x}, R_{IL} \right\} - R_f.$$

The marginal benefit curve is

$$\max\left\{ \left.\frac{\partial Y_1}{\partial I_1}\right|_{x}, R_{IL} \right\} - R_f.$$

The dashed lines show the optimal policy of investing the retained capital in the illiquid asset only when the firm will not use the capital at date 1. The solid lines show the preceding case (Figure 1) where the retained capital is fully invested in the liquid asset. The parameters are $R_f = 1$, $R_{IL} = 1.25$, $R^*_{IL} < R_f$, $R_0 = 2$, $R_1 = 2$. Panels (a) through (d) show decreasing levels of initial capital within the firm $C = 3$, 2, 1 and 0.5.

III. Safe and Risky Financial Assets

In this section, we consider the case where the firm has access to the risk-free liquid asset at dates 0 and 1 as well as to a state-dependent risky financial asset at date 0.[4] At date 1, there are two possible states: u and d with probability p and $1-p$. The risky asset pays off at date 1. The payoffs in the two states are R_u and R_d, where $R_u > R_f > R_d$. The risky asset's risk premium λ is:

$$\lambda \equiv p\,R_u + (1-p)R_d - R_f > 0. \tag{18}$$

A. *Risk Neutrality*

In this subsection, the manager and shareholders are risk neutral. They therefore maximize the expected value of the firm without discounting payoffs for risk. Consequently, investing in the risky asset does not increase the cost of capital in the eyes of the manager and shareholders.

If the firm invests a fraction w of its retained capital in the risky asset, the amount it chooses to invest in its date 1 project may vary across the states: $I_{1,u}$ and $I_{1,d}$. The payoff to the firm's financial portfolio may also vary across states: F_u and F_d.

The manager's problem becomes

$$\max_{I_0, I_{1,u}, I_{1,d}, w} \; R_0 \log(1+I_0) + \left[p\,R_1 \log(1+I_{1,u}) + (1-p)\,R_1 \log(1+I_{1,d})\right] + \left[p\,F_u + (1-p)F_d\right] \tag{19}$$

[4] Allowing the firm to access a risky financial asset at date 1 does not change our main results.

subject to the state-date budget constraints

$$0 \leq I_0 \leq C, \tag{20}$$

$$0 \leq I_{1,u} \leq (C - I_0)\left[R_f + w(R_u - R_f)\right], \tag{21}$$

$$0 \leq I_{1,d} \leq (C - I_0)\left[R_f + w(R_d - R_f)\right], \tag{22}$$

$$F_u = (C - I_0)\left[R_f + w(R_u - R_f)\right] - I_{1,u}, \tag{23}$$

$$F_d = (C - I_0)\left[R_f + w(R_d - R_f)\right] - I_{1,d} \tag{24}$$

and the constraint that the firm cannot short or take leverage.

There are three cases to consider: (1) *fully unconstrained* – the firm can fully exhaust both projects in both states; (2) *partially constrained* – the firm can only fully exhaust both projects in the good state but not the bad state; and (3) *fully constrained* – the firm cannot exhaust both projects in either state. These constraints are defined relative to the firm's optimal choice of investment in the risky asset. However, the firm might be unconstrained if it were limited to the risk-free asset.

As before, the solution to the manager's problem equates the benefit from additional investment in the date 0 project with the benefit from investing those resources in financial markets and using them to fund the date 1 project. Importantly, now the manager must consider the payoffs across states for both the financial portfolio and the real investment. The solution is characterized by

$$\begin{aligned} \left.\frac{\partial Y_0}{\partial I_0}\right|_{I_0} &= p\left(R_f + (R_u - R_f)w\right)\max\left\{\left.\frac{\partial Y_1}{\partial I_1}\right|_{(C-I_0)\left(R_f+(R_u-R_f)w\right)}, R_f\right\} \\ &+ (1-p)\left(R_f + (R_d - R_f)w\right)\max\left\{\left.\frac{\partial Y_1}{\partial I_1}\right|_{(C-I_0)\left(R_f+(R_d-R_f)w\right)}, R_f\right\}. \end{aligned} \tag{25}$$

In each state, the investment in the date 1 project is the lesser of the available retained capital (including the payoff from the safe and risky assets) or the amount that drives the marginal return on the date 1 project to the risk-free rate.[5]

The firm becomes constrained first in the down state where it has fewer resources. Therefore the boundary between the *fully unconstrained* case and the *partially constrained* case comes from the second (down state) maximization in Equation (25). This boundary satisfies

$$\left.\frac{\partial Y_1}{\partial I_1}\right|_{(C-I_0)\left(R_f+(R_d-R_f)w\right)} = R_f. \tag{26}$$

If one continues past that boundary, one eventually encounters the boundary between the *partially constrained* case and the *fully constrained* case. This boundary condition comes from the first (up state) maximization in Equation (25) and satisfies

$$\left.\frac{\partial Y_1}{\partial I_1}\right|_{(C-I_0)\left(R_f+(R_u-R_f)w\right)} = R_f. \tag{27}$$

Whether the manager finds investing in the risky asset optimal depends upon the size of the risk premium and the concavity (decreasing returns to scale) of the firm's date 1 real project. If the risk premium is large relative to the concavity, the manager will invest in the risky asset. If the risk premium is small relative to the concavity, the manager will not invest in the risky asset.

To see this result, let $\tilde{I}_1^*$ be the distribution of the firm's date 1 real investment, given its risky financial investment, and I_1 be the investment if the firm only invests in the safe asset. Jensen's inequality gives

$$Y_1(E[\tilde{I}_1^*]) \geq E[Y_1(\tilde{I}_1^*)]. \tag{28}$$

Notice that

$$E[\tilde{I}_1^*] = I_1 + I_1\lambda. \tag{29}$$

[5] If the firm had access to a risky asset at date 1, risk neutrality implies that the firm's outside option would be the risk-free rate plus that asset's risk premium. With this higher outside option, the firm would stop exploiting the date 1 project earlier. This behavior would resemble that when the firm has access to the illiquid asset discussed previously.

Holding the concavity of the production function (and the variance of the risky asset) fixed, consider the effects of varying the risk premium λ ($\lambda_S \ll \lambda_B$). If the risk premium is small relative to the concavity, we get

$$Y_1(\underbrace{E[\tilde{I}_1^*] - I_1\lambda_S}_{I_1}) > E[Y_1(\tilde{I}_1^*)]. \tag{30}$$

The manager therefore avoids the risky asset.

If the risk premium is big relative to the concavity, we get the reverse result:

$$Y_1(\underbrace{E[\tilde{I}_1^*] - I_1\lambda_B}_{I_1}) < E[Y_1(\tilde{I}_1^*)]. \tag{31}$$

The manager therefore chooses to invest in the risky asset.

In Figure 3 we illustrate this result. The black dots show the real investment ($I_{1,d}^*$ and $I_{1,u}^*$) and corresponding output levels in the two states when the firm invests in the risky financial asset. The blue dot, which lies on the blue dashed line connecting the two black dots, shows the expected real investment level $E[\tilde{I}_1^*]$ and corresponding expected output $E[Y_1(\tilde{I}_1^*)]$. Panels (a) and (b) illustrate Equations (30) and (31). In both panels, the red dot shows the real investment level and corresponding output when the firm invests in the safe financial asset. In Panel (a), where the risk premium λ_S is small, the red dot is above the blue dot, showing that the firm prefers the safe financial asset. In Panel (b), where the risk premium λ_B is big, the red dot is below the blue dot, showing that the firm prefers the risky financial asset.

Thus, even in the case of risk neutrality, the concavity of the date 1 production function introduces concavity into the overall objective function of the manager. This concavity operates as implicit risk aversion, affecting the manager's evaluation of the risky asset's risk-return trade-off.

In Figure 4 we plot the marginal cost and benefit under parameters that satisfy the case

(a) Small Risk Premium λ_S

(b) Big Risk Premium λ_B

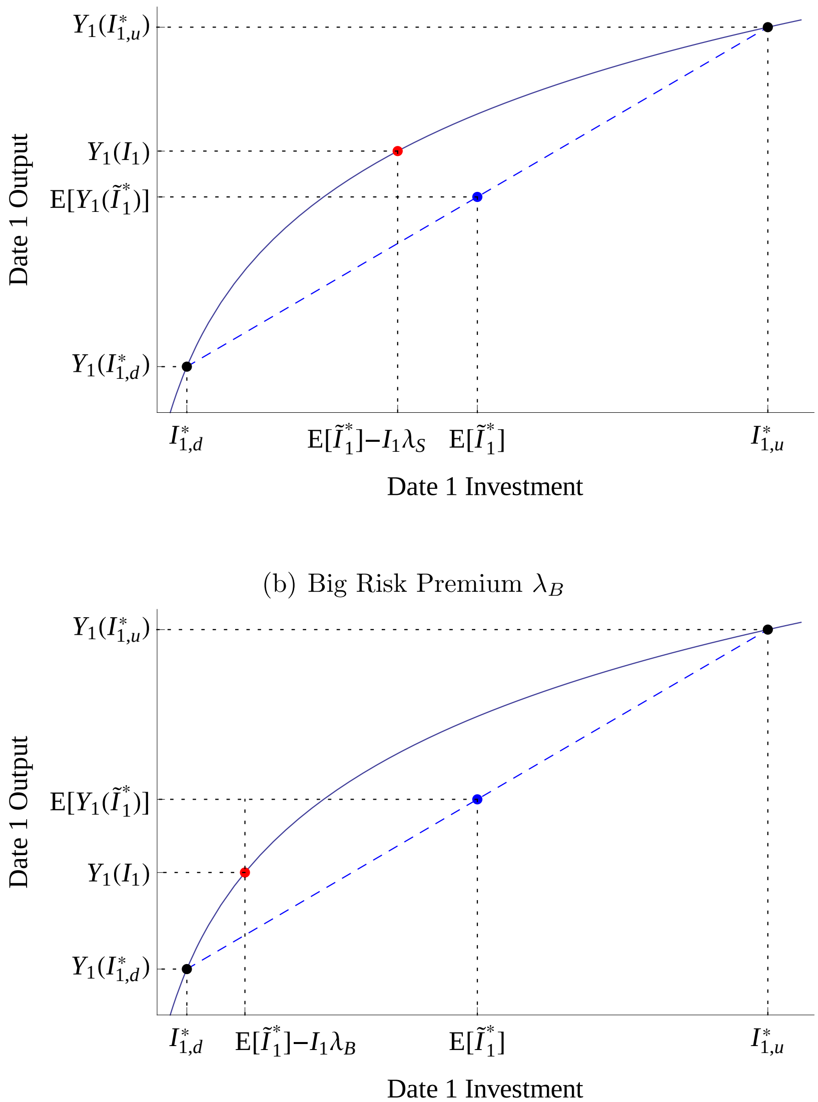

Figure 3. Risky Financial Asset with Risk Neutrality: This figure shows the optimality, or lack thereof, of investing in the risky financial asset as the size of the risk premium varies. The black dots show the real investment and corresponding output levels in the two states when the firm invests in the risky financial asset. The blue dot, which lies on the blue dashed line connecting the two black dots, shows the expected real investment level and corresponding expected output. Panel (a) illustrates Equation (30) where the risk premium λ_S is small. The red dot shows the real investment and corresponding output level when the firm invests in the safe financial asset. Panel (b) illustrates Equation (31) where the risk premium λ_B is big. The red dot shows the real investment and corresponding output level when the firm invests in the safe financial asset.

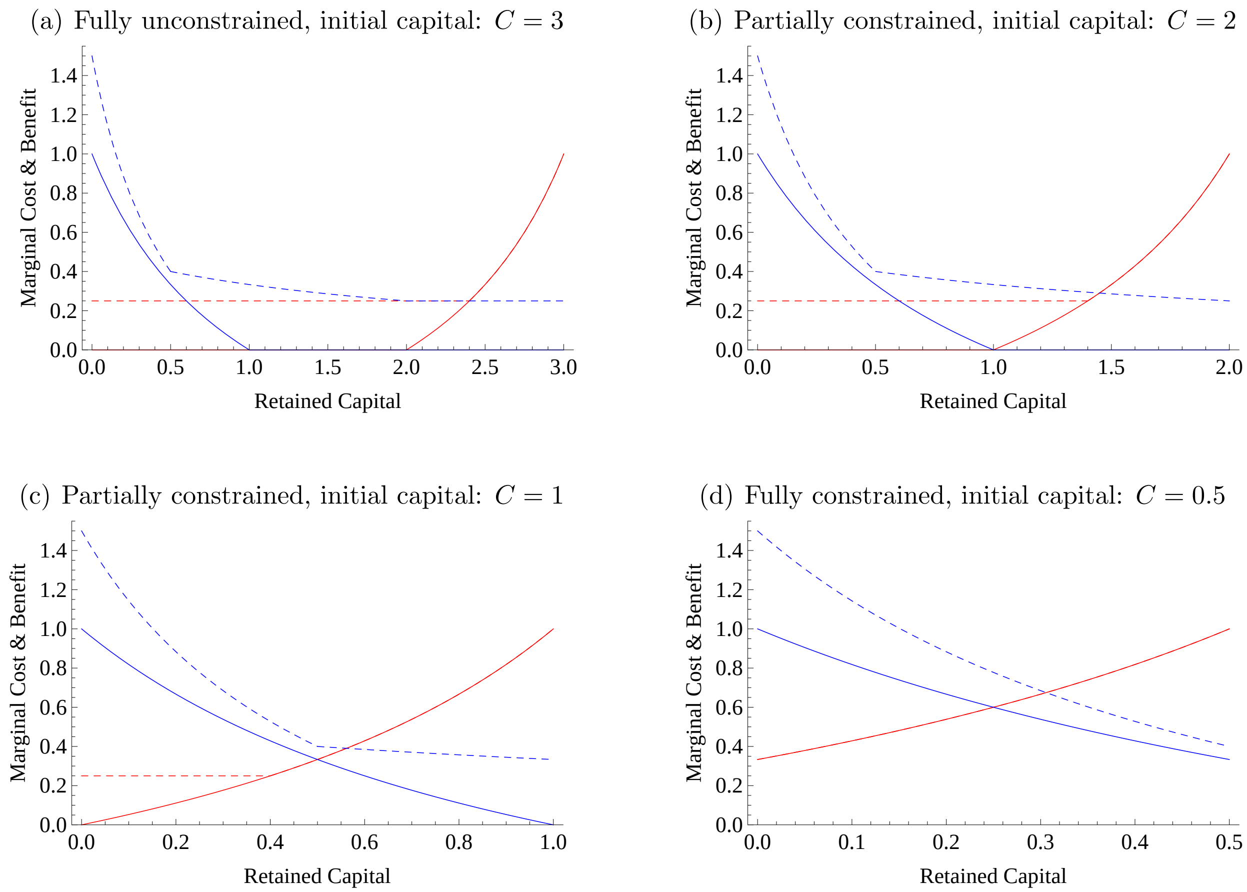

Figure 4. Risky Financial Asset with Risk Neutrality: This figure shows the marginal cost (red) and benefit (blue) of the firm for retaining internal capital uninvested in the date 0 project. The marginal cost curve is

$$\max \left\{ \left. \frac{\partial Y_0}{\partial I_0} \right|_{C-x}, \, R_f + \lambda \right\} - R_f.$$

The marginal benefit curve is

$$p \left(R_f + (u - R_f) w \right) \max \left\{ \left. \frac{\partial Y_1}{\partial I_1} \right|_{(x)\left(R_f + (R_u - R_f) w\right)}, R_f \right\}$$

$$+ (1-p) \left(R_f + (d - R_f) w \right) \max \left\{ \left. \frac{\partial Y_1}{\partial I_1} \right|_{(x)\left(R_f + (R_d - R_f) w\right)}, R_f \right\} - R_f.$$

The solid lines show the suboptimal policy of investing the retained capital in only the safe asset. The dashed lines show the optimal policy of investing the retained capital in the risky financial asset. The parameters are $R_f = 1$, $R_0 = 2$, $R_1 = 2, R_u = 2, R_d = .5, p = .5$. Panels (a) through (d) show decreasing levels of initial capital within the firm $C = 3$, 2, 1 and 0.5. The panel case labels refer to the optimal financial investment policy.

where the manager prefers the risky asset (Equation (31)). The solid lines show the cost and benefit when the firm only invests in the safe asset. The dashed lines show the case of only investing in the risky asset. Panel (a) corresponds to the *fully unconstrained* case. Panels (b) and (c) correspond to the *partially constrained* case. Panel (d) corresponds to the *fully constrained* case. The kinks in the dashed marginal benefit curves show where the constraints across the states bind. Across all panels, the outward shift of the dashed curves indicates that the firm is better off investing in the risky financial asset.

Taken together, our analyses indicate that under risk neutrality, there exist cases where the firm is strictly better off investing in the risky asset. Firms benefit from the risky asset because their cost of capital is not adjusted for the additional risk the firm undertakes by investing in the risky financial asset. This failure to adjust the cost of capital could also arise in the case of ignorant risk averse managers or shareholders.

B. *Risk Aversion*

We now depart from the assumption that the manager and the shareholders are risk neutral. When the manager and shareholders are risk averse, investing in the risky financial asset increases the firm's cost of capital, thereby discouraging such investment.

In what follows, the manager maximizes the expected discounted value of the firm with a stochastic discount factor m, which is consistent with the returns of the risky financial asset. The stochastic discount factor is given by the following state prices

$$m_u = \frac{1}{p} \frac{1}{R_f} \frac{R_f - R_d}{R_f (R_u - R_d)} \quad \text{and} \tag{32}$$

$$m_d = \frac{1}{1-p} \frac{1}{R_f} \frac{R_u - R_f}{R_f (R_u - R_d)}. \tag{33}$$

The manager solves the following problem

$$\max_{I_0, I_{1,u}, I_{1,d}, w} \; R_0 I_0 + \left[p\, m_u\, R_1 \log(1+I_{1,u}) + (1-p) m_d\, R_1 \log(1+I_{1,d})\right] + \left[p\, m_u F_u + (1-p) m_d F_d\right] \tag{34}$$

subject to the state-date budget constraints

$$0 \leq I_0 \leq C, \tag{35}$$

$$0 \leq I_{1,u} \leq (C - I_0)\left[R_f + w(R_u - R_f)\right], \tag{36}$$

$$0 \leq I_{1,d} \leq (C - I_0)\left[R_f + w(R_d - R_f)\right], \tag{37}$$

$$F_u = (C - I_0)\left[R_f + w(R_u - R_f)\right] - I_{1,u}, \tag{38}$$

$$F_d = (C - I_0)\left[R_f + w(R_d - R_f)\right] - I_{1,d} \tag{39}$$

and the constraint that the firm cannot short or take leverage.

The solution is characterized by an equation similar to that of Equation (25); however, state prices are now included

$$\begin{aligned} \left.\frac{\partial Y_0}{\partial I_0}\right|_{I_0} &= p\, m_u \left(R_f + (R_u - R_f) w\right) \max\left\{ \left.\frac{\partial Y_1}{\partial I_1}\right|_{(C-I_0)\left(R_f + (R_u - R_f) w\right)}, R_f \right\} \\ &+ (1-p) m_d \left(R_f + (R_d - R_f) w\right) \max\left\{ \left.\frac{\partial Y_1}{\partial I_1}\right|_{(C-I_0)\left(R_f + (R_d - R_f) w\right)}, R_f \right\}. \end{aligned} \tag{40}$$

The equations describing the boundaries between the three cases (*fully unconstrained, partially constrained* and *fully constrained*) are identical to those from the preceding subsection (Equations (26) and (27)).

PROPOSITION 1: *When the following assumptions hold:*

(1) managers and shareholders are risk averse, having a stochastic discount factor consistent with the returns of the risky financial asset,

(2) the firm has decreasing returns to scale production and

(3) the firm is either partially constrained or fully constrained,

the manager will never invest in the risky asset.

COROLLARY 1: *When the the firm is fully unconstrained the manager is indifferent to investing in the risky asset.*

Proof of Proposition 1. The proof follows from Jensen's inequality. Let $Y_1^*(I_1)$ be the firm's output from the optimal allocation of I_1 across its date 1 project and its best outside financial option, i.e., the risk-free rate. Note that the function Y_1^* is concave in the region where the firm is constrained and linear (with slope R_f) when it is unconstrained.

As before, $\tilde{I}_1^*$ is the distribution of the firm's date 1 real investment given its risky financial asset and I_1 is the investment available if the firm only invests in the safe asset. Under the risk-neutral measure, where all assets earn the risk-free rate of return,

$$E^Q[\tilde{I}_1^*] = E^Q[I_1] = I_1. \tag{41}$$

Applying Jensen's inequality to the production function under the risk neutral measure gives

$$E^Q[Y_1^*(\tilde{I}_1^*)] \leq Y_1^*(E^Q[\tilde{I}_1^*]) = Y_1^*(E^Q[I_1]) = Y_1^*(I_1). \tag{42}$$

This inequality holds strictly as long as the firm is *partially constrained* or *fully constrained* as Y_1^* is concave in these regions. □

Proof of Corollary 1. Equation (42) becomes an equality when the firm is *fully unconstrained* as Y_1^* is linear in that region. □

Thus, when the firm's cost of capital is adjusted for the risk of the financial portfolio, the cost of capital exactly offsets the benefit from earning the risk premium. Thus if the firm is constrained, i.e., produces in the concave region of its production function, it avoids the risky financial asset. However, when the firm is *fully unconstrained*, it is indifferent to investing in the risky asset.

We illustrate this proof in Figure 5. Figure 5 builds on Figure 3 with the addition of a purple dot, which also lies on the blue dashed line. The purple dot shows the expected real investment level $E^Q[\tilde{I}_1^*]$ and corresponding expected output $E^Q[Y_1^*(\tilde{I}_1^*)]$ under the risk-neutral measure. The purple dot has shifted toward the down state's output compared to the blue dot, because the risk neutral measure weighs the down state more heavily compared to the physical measure. The purple dot and the red dot (representing the safe financial investment) lie on the same vertical line, corresponding to the same expected investment level, because all assets earn the risk-free rate under the risk neutral measure. Regardless of the risk premium, the red dot lies above the purple dot, showing that the firm always prefers investing in the safe financial asset.

Figure 6 shows this in the marginal cost and benefit framework. The solid lines plot the optimal policy of investing in only the safe asset and the dashed lines plot the suboptimal policy of investing 100% of retained capital in the risky financial asset. In Panels (a) through (d) we vary the amount of initial capital from the *fully unconstrained* case to the *fully constrained* case.

Panel (a) shows a case where the firm is unconstrained across both states when investing in the risky asset. The firm is indifferent in this case because it is producing on the implied linear portion of the production function achieved by exploiting the outside option of the risk-free asset. We can see this by the way the marginal benefit curve for the risky investment case (dashed) first shifts down below the safe investment case but then kinks and rises above it. The areas in the two triangles between the two curves are identical. Thus the total benefit to the firm from either investment strategy is identical.

As the initial amount of capital decreases and the firm becomes increasingly constrained,

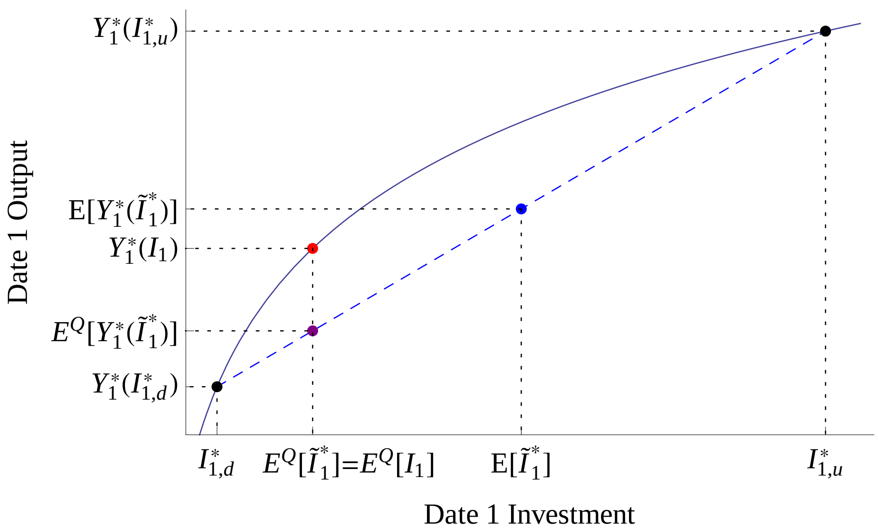

Figure 5. Risky Financial Asset with Risk Aversion: This figure illustrates the proof of Proposition 1, in particular Equation (42). Just as in Figure 3, the black dots show the real investment and corresponding output levels in the two states when the firm invests in the risky financial asset. The blue dot, which lies on the blue dashed line connecting the two black dots, shows the expected real investment level and corresponding expected output. The purple dot, which also lies on the blue dashed line, shows the expected real investment level and corresponding expected output under the risk-neutral measure. The red dot shows the real investment and corresponding output level when the firm invests in the safe financial asset.

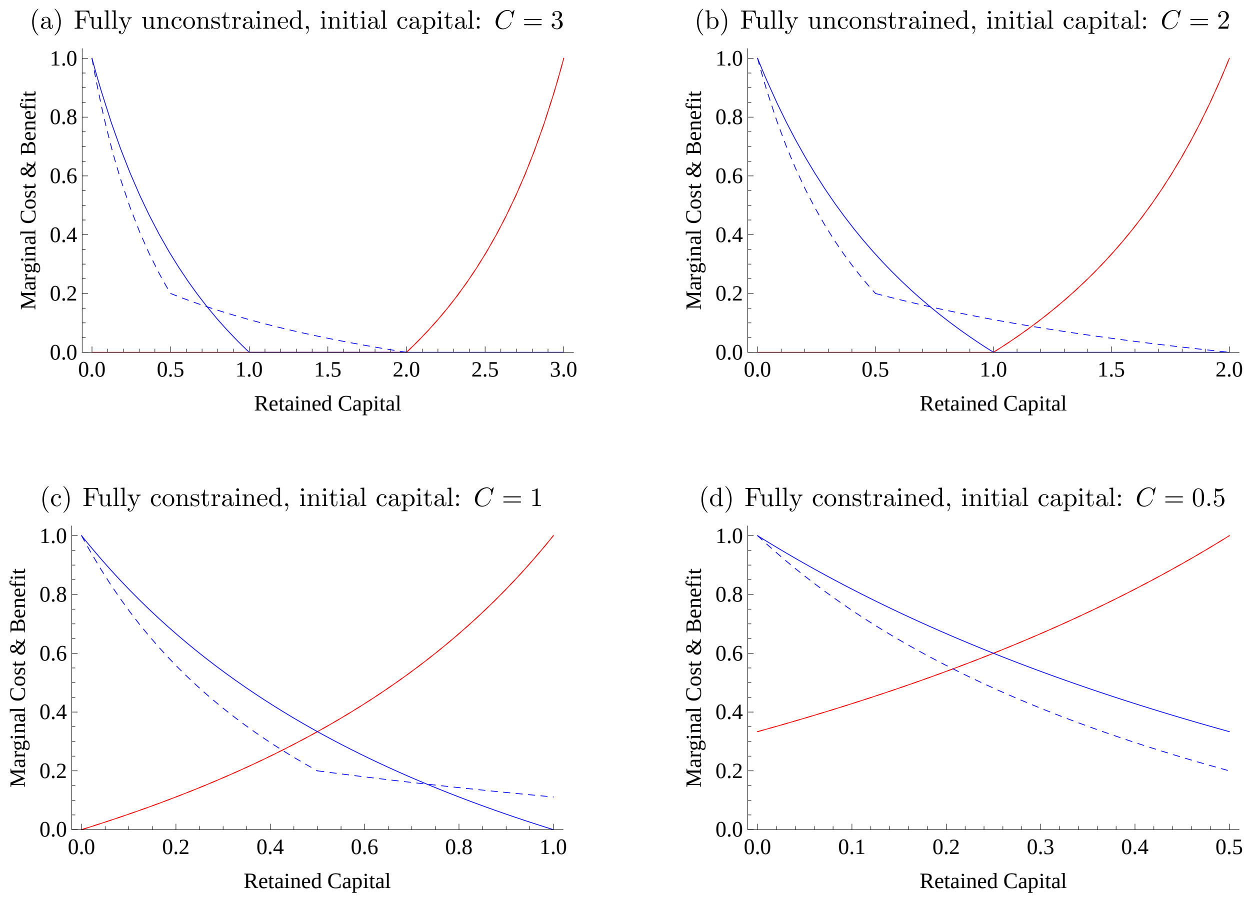

Figure 6. Risky Financial Asset with Risk Aversion: This figure shows the marginal cost (red) and benefit (blue) of the firm for retaining internal capital uninvested in the date 0 project. The marginal cost curve is

$$\max\left\{ \left.\frac{\partial Y_0}{\partial I_0}\right|_{C-x},\ R_f + \lambda \right\} - R_f.$$

The marginal benefit curve is

$$p\, m_u\left(R_f + (R_u - R_f)w\right) \max\left\{ \left.\frac{\partial Y_1}{\partial I_1}\right|_{(x)\left(R_f+(R_u-R_f)w\right)}, R_f \right\}$$

$$+(1-p)\, m_d\left(R_f + (R_d - R_f)w\right) \max\left\{ \left.\frac{\partial Y_1}{\partial I_1}\right|_{(x)\left(R_f+(R_d-R_f)w\right)}, R_f \right\} - R_f.$$

The solid lines show the optimal policy of investing the retained capital in only the safe asset. The dashed lines show the optimal policy of investing the retained capital in the risky financial asset. The parameters are $R_f = 1$, $R_0 = 2$, $R_1 = 2, R_u = 2, R_d = .5, p = .5$. Panels (a) through (d) show decreasing levels of initial capital within the firm $C = 3$, 2, 1 and 0.5. The panel case labels refer to the optimal financial investment policy.

as in panels (b), (c) and (d), the firm strictly prefers the safe asset over the risky asset. In these cases, transferring resources across states makes the firm worse off. We see this from the shift of the marginal cost curve. The gain from the lower triangle fails to offset the loss from the initial downward shift that marks the upper triangle. Thus the firm is worse off investing in the risky financial asset than investing only in the safe asset.

In Figure 7 we extend the four cases presented in Figure 6 to a continuum of retained capital values covering all cases from *fully constrained* to *fully unconstrained*. We plot the firm's optimal financial investment policy versus the optimal retained capital (financial portfolio size) of the firm. The blue line marks the optimal choice or the boundary of the region of choices over which the manager is indifferent. The region of indifference is shaded. The different levels of retained capital are driven by different levels of initial capital C within the firm at date 0. For comparison between the two figures notice that the retained capital level of 2 in Figure 7 corresponds with the case presented in Panel (a) of Figure 6.

Again we see that when the firm has low retained capital (low initial capital) the firm chooses not to invest in the risky asset. As the firm has more retained capital, it is able to fully exhaust its projects and becomes indifferent to how this additional capital is invested. We see that the region of indifference grows as the firm has more retained capital.

One empirical implication is that firms invest more in risky financial assets as they have larger financial portfolios. This larger risky investment would occur if firms randomly allocate across the safe and risky assets within the region of indifference. If managers have even a slight additional preference for risky assets that breaks this indifference, we would see an even stronger positive relation. A second empirical implication is that the increasing indifference generates increased cross-sectional variation in the allocation of financial assets as the financial portfolio grows.

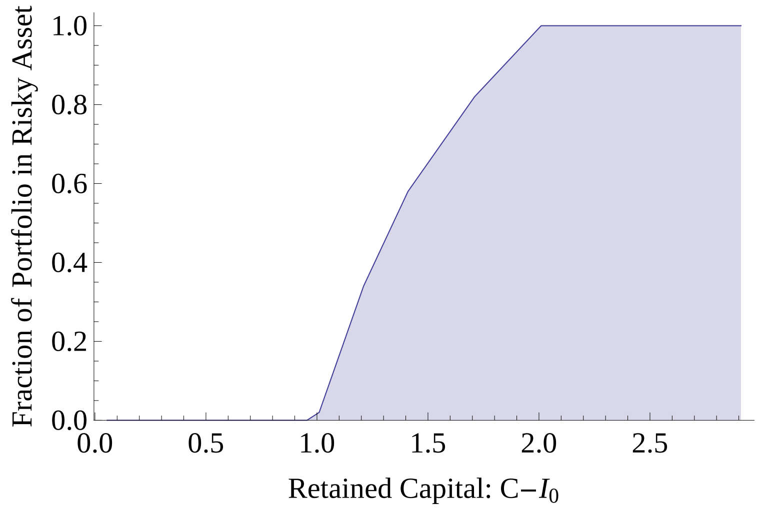

Figure 7. Risky Investment Fraction v Retained Capital: This figure shows the optimal fraction of the firm's retained capital that it chooses to invest in the risky asset as a function of its retained capital. The solid blue line marks the optimal investment fraction or the bound of the region of indifference. The region of indifference is shaded light blue. The parameters are $R_f = 1$, $R_0 = 2$, $R_1 = 2$, $R_u = 2$, $R_d = .5$, $p = .5$.

IV. Robustness

In this section we consider the covariation of financial returns with the firm's real investment opportunities and its ability to access external capital. We show that our main results hold when the firm faces positive covariation in its real investment opportunities (Berk, Green, and Naik, 1999; Carlson, Fisher, and Giammarino, 2004) but negative covariation in its ability to access external capital.

Suppose the firm's projects vary positively with the state of the economy such that its date 1 project is state-dependent:

$$Y_{1.u} \equiv R_{1,u} \log(1 + I_{1,u}) \quad \text{and} \tag{43}$$

$$Y_{1.d} \equiv R_{1,d} \log(1 + I_{1,d}) \tag{44}$$

where $R_{1,u} > R_{1,d}$.

When the firm cannot access external capital, the *fully unconstrained* and *fully constrained* cases remain unchanged. However, the *partially constrained* case is more complex.

As before, the firm may exhaust its date 1 project in the up state first. Alternatively the firm may first exhaust the date 1 project in the down state. This alternative occurs if the project is sufficiently good in the up state such that the additional resources in that state are not enough to exhaust the project.

When the firm can access external capital, the exhaustion of projects depends primarily upon the cost of accessing external finance. In the previous sections, firms could not access external capital markets in any state. We now relax this assumption allowing the firm to borrow. We assume that financial frictions in debt markets covary negatively with the state of the economy (see for example Shleifer and Vishny, 1992; Pulvino, 1998; Eisfeldt and Rampini, 2006). For simplicity, we allow the firm to borrow at a fair rate (the risk-free rate since its internal projects are risk-free) in the up state but restrict borrowing completely in the down state.

The risk averse manager solves

$$\max_{I_0,I_{1,u},I_{1,d},D_{1,u},w} R_0 I_0 + \left[p\, m_u\, R_{1,u} \log(1+I_{1,u}) + (1-p)\, m_d\, R_{1,d} \log(1+I_{1,d})\right] + \left[p\, m_u\, F_u + (1-p)\, m_d\, F_d\right] \tag{45}$$

subject to the state-date budget constraints

$$0 \le I_0 \le C, \tag{46}$$

$$0 \le I_{1,u} \le (C - I_0)\left[R_f + w(R_u - R_f)\right] + D_{1,u}, \tag{47}$$

$$0 \le I_{1,d} \le (C - I_0)\left[R_f + w(R_d - R_f)\right], \tag{48}$$

$$F_u = (C - I_0)\left[R_f + w(R_u - R_f)\right] + D_{1,u} - I_{1,u} - D_{1,u}, \tag{49}$$

$$F_d = (C - I_0)\left[R_f + w(R_d - R_f)\right] - I_{1,d}, \tag{50}$$

and the constraint that the firm cannot short or lever its financial portfolio. Here $D_{1,u}$ is the amount the firm borrows in the up state.

Notice that the ability to borrow slackens the constraint on the investment in the up state, Equation (47), but must then be paid back at the end, and hence lowers the value of the financial portfolio in the up state, Equation (49). Any extra borrowing the firm undertakes but does not use for real investment simply nets out.

Because borrowing is available in the up state, the firm will always exhaust its date 1 project. This means there are now only two cases: when the firm exhausts the project in the down state and when it does not.

The manager again equates the marginal benefit of date 0 production with that of financial investments combined with date 1 production. The solution is characterized by an equation similar to that of Equation (40) except that the derivatives vary across states

$$\begin{aligned}\left.\frac{\partial Y_0}{\partial I_0}\right|_{I_0} &= p\, m_u \big(R_f + (R_u - R_f)w\big)\, R_f \\ &\quad + (1-p)\, m_d \big(R_f + (R_d - R_f)w\big) \max\left\{ \left.\frac{\partial Y_{1,d}}{\partial I_1}\right|_{(C-I_0)\left(R_f+(R_d-R_f)w\right)}, R_f \right\}.\end{aligned} \tag{51}$$

The boundary between the two cases is when the following equation is satisfied

$$\left.\frac{\partial Y_{1,d}}{\partial I_1}\right|_{(C-I_0)\left(R_f+(R_d-R_f)w\right)} = R_f. \tag{52}$$

The positive co-variation of the date 1 project with the aggregate state implies that the manager may find investing in the risky asset beneficial. Whether such investment is beneficial depends upon the magnitude of the covariation. For small positive (or any negative) covariation the manager still prefers the safe financial asset. For large positive covariation the manager prefers the risky financial asset.

The intuition for this effect is that adding positive covariation between the projects and the financial returns across states effectively lowers the concavity of the production function across states and can even make it effectively convex, thus reversing Jensen's inequality.

On the other hand, the negatively varying financial frictions make investing in the risky financial asset less appealing. The intuition for this effect is that when the firm has a greater ability to access to capital markets in good states than bad, the firm gains less from transferring resources across states for real investment. Following the argument from Jensen's inequality, the negative covariation in financial frictions effectively increases the concavity of the implementable production function across states. Whether these effects overcome the positive covariation of the real investment opportunities depends upon the relative size of the two effects.

In Figure 8 we plot the marginal cost and benefit for parameters under which the effect of the state-varying financial frictions dominates, making investments in the risky financial asset suboptimal. The solid lines plot the optimal policy of investing in only the safe asset and the dashed lines plot the suboptimal policy of investing in the risky financial asset.

When the firm is unconstrained, panels (a) and (b), it is indifferent among the investment strategies. In panels (c) and (d), when it has less capital, the shift down in the marginal benefit curve of the risky strategy implies that the firm prefers the safe investment strategy. In fact in panel (d) the marginal benefit curve shifts down so much that it does not intersect the marginal cost curve. This lack of intersection means that if the firm were forced to invest in the risky asset, it would choose to retain no capital, simply funding the date 1 project with borrowing in the up state and forgoing the project entirely in the down state.

V. Breaking the Indifference: Agency and Overconfidence

The model produces regions of indifference over the firm's financial portfolio allocation. This indifference occurs only when the firm is unconstrained. There are many ways to break this indifference from the point of view of the manager, and we sketch how to implement them in our model.

One way is to introduce a private benefit from investing in the risky asset. Such a private benefit would arise if experience managing multi-asset portfolios is valued in the

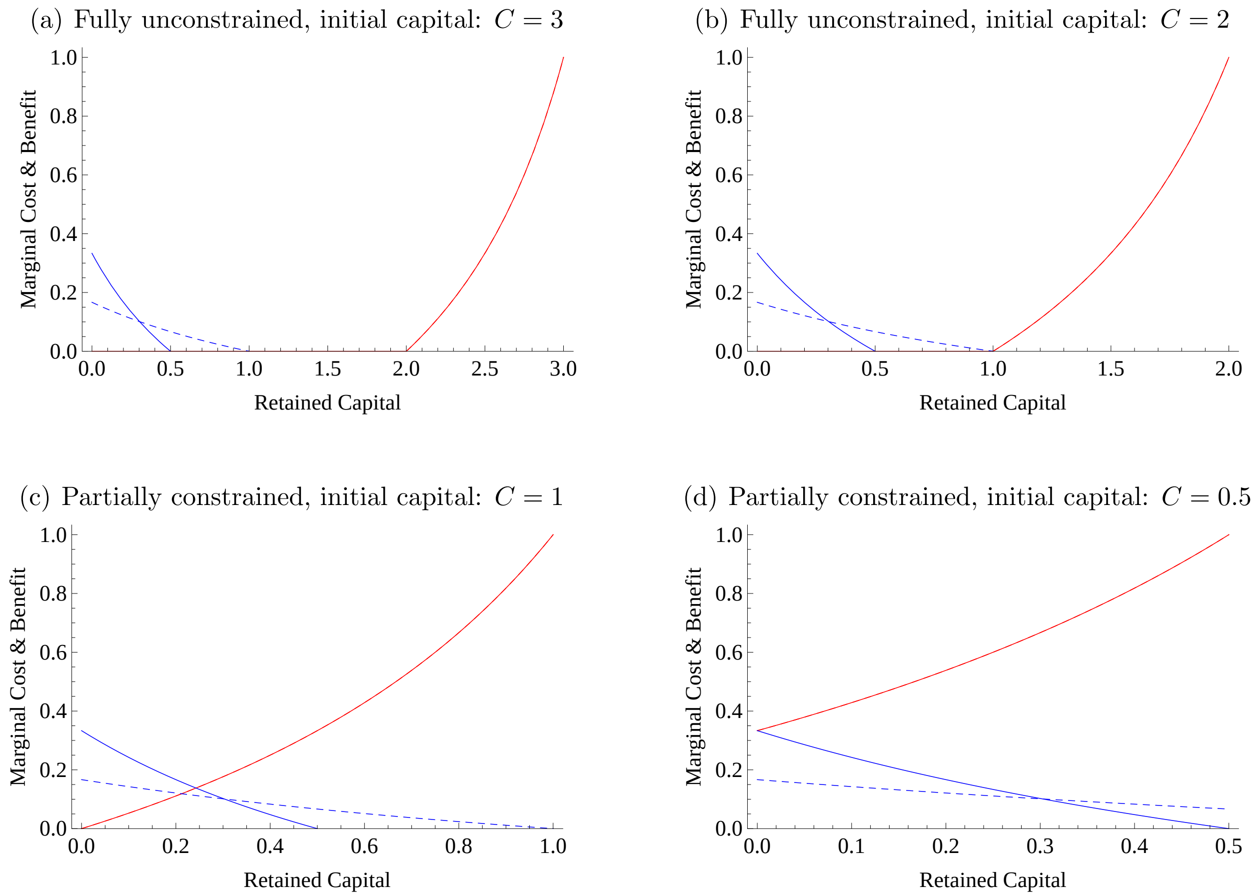

Figure 8. Risky Financial Asset with Positive Covariation of Real Investment Projects and Negative Covariation of Financial Frictions: This figure shows the marginal cost (red) and benefit (blue) of the firm for retaining internal capital uninvested in the date 0 project. The marginal cost curve is

$$\max\left\{ \left.\frac{\partial Y_0}{\partial I_0}\right|_{C-x},\ R_f + \lambda \right\} - R_f.$$

The marginal benefit curve is

$$p\, m_u \big(R_f + (R_u - R_f)w\big) R_f$$

$$+(1-p)\, m_d \big(R_f + (R_d - R_f)w\big) \max\left\{ \left.\frac{\partial Y_{1,d}}{\partial I_1}\right|_{(x)\left(R_f+(R_d-R_f)w\right)}, R_f \right\} - R_f.$$

The solid lines show the optimal policy of investing the retained capital in only the safe asset. The dashed lines show the suboptimal policy of investing the retained capital in the risky financial asset. The parameters are $R_f = 1$, $R_0 = 2$, $R_{1,u} = 3$, $R_{1,d} = 1.5$, $R_u = 2$, $R_d = .5$, $p = .5$. Panels (a) through (d) show decreasing levels of initial capital within the firm $C = 3$, 2, 1 and 0.5. The panel case labels refer to the optimal financial investment policy.

asset management labor market. If this private is big enough the manager will tilt the firm's financial portfolio toward the risky asset.

Additionally the manager may believe he can generate alpha but must take on systematic risk to do so. Such over confidence could be modeled as replacing the state returns with the manager's belief of state returns $R_u^* \geq R_u$ and $R_d^* \geq R_d$, where at least one of these inequalities is strict. This would obviously push the manager toward investing in the risky asset provided the manager believes he has enough alpha to overcome the concavity in the date 1 project. Of course this would be suboptimal for the firm because the firm will only receive the true state returns.

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