Online Appendix to: Investor Inattention and the Market Impact of Summary Statistics

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Abstract

This Online Appendix contains three sections of supporting materials to our paper "Investor Inattention and the Market Impact of Summary Statistics." First, we provide details on the construction methodology of our instrument of stale macroeconomic information, the U.S. Index of Leading Economic Indicators (LEI). Second, we provide the theoretical details and derivations of our stylized model of investor inattention. And third, we provide additional robustness checks to our main result that the release of the LEI impacts aggregate markets.

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1 Appendix: LEI Calculation

The indicator approach has a long history since the mid-1930s and was developed at the National Bureau of Economic Research (NBER), following the influential work of Wesley C. Mitchell and Arthur F. Burns. It has been a major component of the NBER program on economic growth and fluctuations. TCB took over the responsibility to publish and maintain the LEI and the Business Cycle Indicators database from the Bureau of Economic Analysis starting with the December 6, 1995 release.

Let ΔLEI_t denote the monthly change in the LEI that is published/released in month t but which refers to month t-1. This monthly change is calculated as the sum of component contributions which are derived from a symmetric percentage change formula:

$$\Delta LEI_t = \left(\sum_{i=1}^{10} \sigma_i \times 200 \times \frac{X_{i,t} - X_{i,t-1}}{X_{i,t} + X_{i,t-1}}\right)$$
(A.1)

where σ_i is the standardization factor calculated by dividing the inverse standard deviation of component *i* by the sum of the inverse standard deviations over all components. As the notation makes clear, the index published in month *t* refers to past data for t - 1 which has already been published.

Since January 2001, leading indicator components for month t-1 that are not available at the time of publication, month t, are estimated by TCB using a univariate autoregressive model to forecast each unavailable component. This procedure seeks to address the problem of varying availability in its components (publication lags). Without it, the index would contain incomplete components or it would not be available promptly under the current schedule.

In the publication schedule prior to January 2001, the index published in month t referred to the month t - 2. In the new schedule after January 2001, the index published in month t refers to the preceding month t - 1 (this information is available from The Conference Board). For example, in the old publication schedule the index would be calculated in the first week of March (t) for January (t - 2), and the January value of the LEI would use a complete set of components. According the new schedule, the index is calculated in the third week of March for February (t - 1), and the February value of the index uses 70% of the components which are already available and the remaining 30% are forecast. As seen in this example, users of the LEI would have had to wait for two more weeks until April for the February index.

The missing components (manufacturers new orders for consumer goods and materials, manufacturers new orders for nondefense capital goods, and the personal consumption expenditure used to deflate the money supply (M2)) are estimated using a time series regression that uses two lags (see McGuckin, Ozyildirim, and Zarnowitz (2001) for more on this model and a comparison with other alternative lag structures). The procedure used to estimate the current month's personal consumption expenditure deflator incorporates the current month's consumer price index (CPI) when it is available before the release of the LEI. When the unavailable data become available in the next month, the index is revised.

The missing components could be forecast through alternative means. However, The Conference Board has focused on simplicity, stability, and low costs of production and argues for concentrating on easily implementable autoregressive model. Note that under the pre-2001 release schedule of the LEI, it would have been possible to perfectly forecast the new value each month just by collecting the individual data components and following the index calculation methodology. In the post-2001 schedule, this is still possible, but the estimated components require one additional step.

2 Appendix: Details of Model Derivation

2.1 Model Setup

The model is a pure exchange economy with three dates, labeled as t = 1, 2, and 3. Figure A.1 shows a timeline of the model. Trading occurs at t = 1 and t = 2. There is one risky asset that distributes a dividend in each period. The dividends are i.i.d. normally distributed with mean 0 and variance σ_d^2 . The risky asset is in supply x and, for simplicity and without loss of generality, we assume that there is a risk-free asset that pays a zero rate of interest, and there is no time discounting.

There are two groups of investors in the model, attentive investors, labeled as A with risk aversion coefficient a_A , and inattentive investors, labeled as I with risk aversion coefficient a_I . The mass of attentive investors is labeled as m and the mass of inattentive investors is 1 - m. All investors have constant absolute risk aversion (CARA) utility and are assumed to myopically maximize their utility over wealth at each date in the model. In each trading period, all investors choose how much to invest in the risky and risk-free assets. Based on the myopic CARA assumption, the per-investor demand of the risky asset is $x_{i,t} = \frac{E_i[p_{t+1}+d_{t+1}|\Gamma_t]-p_t}{var_i[p_{t+1}+d_{t+1}|\Gamma_t]}$ where Γ_t represent the information set of investor i available at time t.

The structure, types, and interpretation of the signals released are the key features of this model. At t = 1, all investors observe a noisy public signal of d_3 : $s_1 = d_3 + \varepsilon_1$ where $\varepsilon_1 \sim N(0, \sigma_{\varepsilon}^2)$. At t = 2, the signal is re-released, $s_2 = s_1$, but the two types of investors interpret this second signal differently. In the next three subsections, we solve three versions

Figure A.1 Timeline of the Model

This figure shows the basic timeline of the model. The risky asset pays off a dividend d_2 at t = 2 and d_3 at t = 3. At t = 1, a noisy public signal of d_3 is released $(s_1 = d_3 + \varepsilon_1)$ and trading takes place at a market-clearing equilibrium price of p_1 . At t = 2, the first-period signal is re-released as $s_2 = s_1$, i.e., it is a stale release of information. Trading occurs at the equilibrium price p_2 .

t = 1	t = 2	t = 3		
		Dividendel is neelised		
<u>New Information</u>	Stale Information	Dividend d ₃ is realized.		
$s_1 = a_3 + \varepsilon_1$ is released.	Dividend d_2 is realized.			
Trading occurs at p ₁ .	s_1 is re-released as s_2 .			
	Irading occurs at p ₂ .			

of the model, each making a different assumption about the type of inattention suffered by the inattentive investors:

- 1. Benchmark: All investors know that the second period's signal s_2 is a stale release of the first period's signal s_1 , i.e., they are attentive and the second signal therefore provides no new information whatsoever.
- 2. Inattention 1: The inattentive investors know that the signal s_2 is a stale release of s_1 but they choose to learn in the second period only, i.e., they ignore s_1 . This learning delay could be due to un-modeled information processing costs (time or bandwidth constraint, for instance) in the first period. This type of inattention is present in DellaVigna and Pollet (2009), among others, who show that investors ignore valuable pieces of information because there is too much information revealed on certain days.
- 3. Inattention 2: The inattentive investors do not realize that the signal s_2 is stale and they confound it with additional new information. More precisely, they believe that it gives them new information about d_3 not already revealed by the first signal s_1 . This type of inattention is present in Tetlock (2011), among others.

Across the above cases, the attentive investors' information set is unchanged – they know that s_2 is simply a re-release of s_1 . For the three versions of the model, we analyze the price behavior surrounding the release of both the raw information and the stale information.

2.2 Benchmark Model: Efficient Market

In the benchmark case of the model, the inattentive investors are actually attentive and know that s_2 is simply a stale re-release of s_1 . As a result, all investors observe s_1 in the first period and there is no additional learning in the second period. The model is solved by backwards induction and at t = 3, we trivially have $p_3 = 0$, where, in the remainder of the model, p_t is the ex-dividend price of the risky asset at date t.

At t = 2, both types of agents have observed s_2 and they know that it is a stale release of s_1 , i.e., both agents know that $s_2 = s_1 = d_3 + \varepsilon_1$ where $\varepsilon_1 \sim N(0, \sigma_{\varepsilon}^2)$. As a result, both agents have the same information set and form the same beliefs about p_3 :

$$E[p_3 + d_3|s_2] = E[d_3|s_1] = \kappa_1 s_1 \tag{A.2}$$

where $\kappa_1 = \frac{\sigma_d^2}{\sigma_d^2 + \sigma_{\varepsilon}^2}$, and

$$var[p_3 + d_3|s_2] = var[d_3|s_1] = \sigma_d^2 - \frac{\sigma_d^4}{\sigma_d^2 + \sigma_\varepsilon^2} = \kappa_1 \sigma_\varepsilon^2$$
(A.3)

Since both types of agents have the same beliefs about $p_3 + d_3$, we can write the following market-clearing condition by equating total demand to total supply x:

$$m\frac{E_A[p_3+d_3|s_2]-p_2}{a_A var_A[p_3+d_3|s_2]} + (1-m)\frac{E_I[p_3+d_3|s_2]-p_2}{a_I var_A[p_3+d_3|s_2]} = x$$
(A.4)

in which we can plug in the above conditional expectation and variance:

$$m\frac{\kappa_1 s_1 - p_2}{a_A \kappa_1 \sigma_{\varepsilon}^2} + (1 - m)\frac{\kappa_1 s_1 - p_2}{a_I \kappa_1 \sigma_{\varepsilon}^2} = x \tag{A.5}$$

where m is the mass of attentive investors. For the remainder of the analysis, we assume that the risky asset is in zero net supply (x = 0).¹ As a result, we have in equilibrium that:

$$p_2 = \kappa_1 s_1 \tag{A.6}$$

¹The assumption that the risky asset is in zero net supply, as opposed to in positive supply, means there will be no trade in the risky asset unless the attentive and the inattentive have different expectations of the final period dividend. If the asset instead is in positive supply, the agents risk aversions come into play as a risk premium is required to hold the asset even if expectations are homogenous. Since the difference in information sets between the attentive and the inattentive agents are at the heart of the paper, the zero net supply assumption serves to elucidate the implications of the models along this dimension. Further, given that the maximum return interval analyzed in this paper is 30 hours, the per-period market risk premium is quantitatively much smaller than the price responses we document. The zero net supply assumption over such short horizons is therefore economically reasonable.

At t = 1, both agents observe s_1 and know the price function at t = 2 that was derived above. As a result, they form the following beliefs:

$$E[p_2 + d_2|s_1] = \kappa_1 s_1 \tag{A.7}$$

and

$$var[p_2 + d_2|s_1] = \sigma_d^2 \tag{A.8}$$

We can therefore write down the market-clearing condition:

$$m\frac{\kappa_1 s_1 - p_1}{a_A \sigma_d^2} + (1 - m)\frac{\kappa_1 s_1 - p_1}{a_I \sigma_d^2} = 0$$
(A.9)

which leads to the following equilibrium price:

$$p_1 = \kappa_1 s_1 \tag{A.10}$$

The average return dynamics of this benchmark model where all agents are attentive is shown as the solid line on Figure A.2 (for positive initial signals s_1). The ex-dividend price moves up to its efficient-market level at t = 1, resulting in (on average) a positive return in the first period and zero returns in the following two periods, since no new information is revealed.

2.3 Inattention Model 1: Ignoring the Initial Signal

In this version of the model, the inattentive agents are inattentive in the sense that they do not realize that the signal has been released before, i.e., they completely ignore s_1 . This inattention to the first release of the signal at t = 1 can be due to a number of un-modeled reasons, such as time constraint, information processing cost, bandwidth constraint, distraction, etc. Thus, after the re-release, all investors – attentive and inattentive – will agree on fundamentals, even though at the initial release, only the attentive take s_1 into account.

The price at t = 3 price is trivially as in the benchmark model. At t = 2, both types of investors have observed s_1 and agree on the fundamentals. This is equivalent to the second period of trading in the benchmark model and we therefore have $p_2 = \kappa_1 s_1$. At t = 1, only the attentive investors have observed s_1 . As a result, taking the expected price function p_2 into account, the attentive investors form the following beliefs:

$$E_A[p_2 + d_2|s_1] = \kappa_1 s_1 \tag{A.11}$$

Figure A.2 Return Responses under Different Models of Inattention

This figure shows the return responses to positive ("good news") information releases under the three versions of the model. At t = 1, the initial information is released. At t = 2, this information, now stale, is re-released. At t = 3, terminal values are realized and uncertainty is resolved. The models differ with respect to the inattentive investors' reading of the stale signal at t = 2.



and

$$var[p_2 + d_2|s_1] = \sigma_d^2$$
 (A.12)

Similarly, the inattentive investors take the price conjecture into account and form the following unconditional beliefs since they have not observed s_1 :

$$E_A[p_2 + d_2] = 0 \tag{A.13}$$

and

$$var[p_2 + d_2|s_1] = \sigma_d^2 + \kappa_1^2(\sigma_d^2 + \sigma_{\varepsilon}^2)$$
 (A.14)

Note that we have here implicitly assumed that the inattentive do not try to infer the information of the attentive from period 1 prices. In other words, they are inattentive not only to the initial signal, but also to the fact that prices move "too much" or "too little" in the first period relative to the dividend, d_1 , as a consequence of the actions of the attentive

investors. The market-clearing condition therefore is:

$$m\frac{\kappa_1 s_1 - p_1}{a_A \sigma_d^2} + (1 - m)\frac{0 - p_1}{a_I(\sigma_d^2 + \kappa_1^2(\sigma_d^2 + \sigma_\varepsilon^2))} = 0$$
(A.15)

which we can solve for the equilibrium price:

$$p_1 = \frac{1}{1 + \frac{(1-m)a_A\sigma_d^2}{a_I(\sigma_d^2 + \kappa_1^2(\sigma_d^2 + \sigma_\varepsilon^2))m}} \kappa_1 s_1 = \frac{1}{1+B} p_2$$
(A.16)

where B > 0. As a result, if $s_1 > 0$, then $p_1 < p_2$ and we have a price impact both at the initial release and at the re-release in the direction of the information release (s_2) , but no over-reaction and no reversal. The price slowly moves to its efficient-market level since the two types of investors learn from the public signal consecutively, rather than simultaneously as is the case in the benchmark model. The reason for this is that the attentive are risk averse. Since the period 2 dividend is risky, the attentive agents are not willing to take unbounded positions to front-run the trades of the inattentive. This price behavior is represented as the dotted-dashed line on Figure A.2.

2.4 Inattention Model 2: Confounding the Re-release with New Information

In this version of the model, both types of investors – attentive and inattentive – observe $s_1 = d_3 + \varepsilon_1$ during the first trading period. However, during the second trading period, the inattentive investors confound the stale re-release of s_1 with new information, i.e., they believe that the second-period signal actually is $s_2 = d_3 + \varepsilon_2$ where $\varepsilon_2 \sim N(0, \sigma_{\varepsilon}^2)$ and is independent of ε_1 . The attentive investors know that $s_2 = s_1$ and they also know that there are inattentive investors in the market who misinterpret the second signal, a fact that they can take advantage of.

The price at t = 3 is trivially as in the benchmark model. At t = 2, the attentive investors observe s_2 and know that it is simply a re-release of s_1 , i.e., $s_2 = s_1 = d_3 + \varepsilon_1$. Their beliefs therefore coincide with the benchmark case and the per-agent demand therefore is:

$$x_{A,2} = \gamma_{A,2} \left(\kappa_1 s_1 - p_2 \right) \tag{A.17}$$

where $\gamma_{A,2} = \frac{1}{a_A \kappa_1 \sigma_{\varepsilon}^2}$. Similarly, the inattentive investors have per-agent demand:

$$x_{I,2} = \gamma_{I,2} \left(\kappa_2 (s_1 + s_2) - p_2 \right) \tag{A.18}$$

where $\gamma_{I,2} = \frac{1}{a_I \kappa_2 \sigma_{\varepsilon}^2}$ and $\kappa_2 = \frac{\sigma_d^2}{2\sigma_d^2 + \sigma_{\varepsilon}^2}$. By equating total demand to the risky asset's supply, we can solve for the trading price in the second period:

$$p_2 = \frac{m\gamma_{A,2}\kappa_1 s_1 + (1-m)\gamma_{I,2}\kappa_2 (s_1 + s_2)}{m\gamma_{A,2} + (1-m)\gamma_{I,2}}$$
(A.19)

We make one additional assumption that simplifies the algebraic tractability of the model and allows a cleaner exposition of the economics underlying the model's equilibrium. We assume that the coefficients of risk aversion of the two sets of agents differ in exactly such a way so as to make $\gamma_{I,2} = \gamma_{A,2} = 1$. This assumption implies that $a_I > a_A$, or more precisely $a_I = \frac{\kappa_1}{\kappa_2} a_A$.² As a result, the trading price in the second period simplifies to:

$$p_2 = m\kappa_1 s_1 + (1 - m)\kappa_2 (s_1 + s_2) \tag{A.20}$$

Since we know that $s_2 = s_1$, something the attentive investors also know, we can write the above equilibrium price as:

$$p_2 = \kappa_1 s_1 + (1 - m)(2\kappa_2 - \kappa_1)s_1 \tag{A.21}$$

where it is easy to show that $2\kappa_2 - \kappa_1 > 0$. Together with the fact that $E[p_3 + d_3|s_1] = E[d_3|s_1] = \kappa_1 s_1$, the above equilibrium price yields the first important result of the model: There is a return reversal on average in the third period, i.e., $E[p_3 + d_3 - p_2|s_1 > 0] < 0$. The economic intuition is that the price must at the end of the model converge to its efficient-market level. As a result, since the price over-reacts in both periods one and two (as is shown below), then there will be a reversal in the third period.

While equation (A.21) represents the "true" equilibrium price as seen by the attentive investors, the inattentive investors do not know that there are other investors in the market who know that s_2 is a re-release. Instead, the inattentive investors believe that all market participants are like them and have demand given by equation (A.18). As a result, the inattentive investors believe that the t = 2 equilibrium price is given by:

$$p_2 = \kappa_2(s_1 + s_2) \tag{A.22}$$

In the first period, t = 1, each type of investors anticipates their believed pricing function as given above (equation (A.21) for the attentive and equation (A.22) for the inattentive) and

 $^{^{2}}$ The simplifying assumption on the coefficients of risk aversion turns out not to have any impact on the model's equilibrium outcome. A complete derivation of the model under the more standard assumption that both types of investors have risk aversion equal to one, as in Tetlock (2011), is available from the authors upon request.

take it into account when forming their beliefs. The attentive investors know that $s_2 = s_1$ and they also know that the inattentive investors will react to the re-release of the second signal. The attentive per-agent demand therefore is:

$$x_{A,1} = \frac{[m\kappa_1 + 2(1-m)\kappa_2]s_1 - p_1}{a_A \sigma_d^2}$$
(A.23)

Because the inattentive investors believe that $s_2 = d_3 + \varepsilon_2$, i.e., that the re-release actually provides new information about d_3 , their per-agent demand is:

$$x_{I,1} = \frac{\kappa_2 (1 + \kappa_1) s_1 - p_1}{a_I \left(\sigma_d^2 + \kappa_2^2 (1 + \kappa_1) \sigma_{\varepsilon}^2\right)}$$
(A.24)

where the conditional expectation and variance stem from plugging in $s_2 = d_3 + \varepsilon_2$ in equation (A.22).

By setting total demand equal to supply, we obtain the following equilibrium trading price in the first period:

$$p_1 = p_2 + \frac{(m-1)\kappa(m\kappa_1 + \kappa_2 - (2m+\kappa_1)\kappa_2)}{m+(1-m)\kappa}s_1$$
(A.25)

$$= p_2 - \frac{(m-1)^2 \sigma_d^2 \sigma_\varepsilon^2}{2\sigma_d^4 + (m+3)\sigma_d^2 \sigma_\varepsilon^2 + \sigma_\varepsilon^4}$$
(A.26)

$$= p_2 - Hs_1 \tag{A.27}$$

where $\kappa = \frac{a_A \sigma_d^2}{a_I (\sigma_d^2 + \kappa_2^2 (1 + \kappa_1) \sigma_{\varepsilon}^2)}$. First, it is easy to show that $0 < \kappa < 1$ since, following our assumption on the coefficients of risk aversion, $a_A < a_I$.³ Second, it is also straightforward to show that 0 < H < 1. We therefore obtain the second important result of the model: There is a price response upon the release of the stale signal s_2 , i.e., $E[p_2 + d_2 - p_1|s_1 > 0] > 0$.

Trivially, $p_0 = 0$, and by plugging in the equilibrium functional form of p_2 in equation (A.27), one can show that $(1 - m)(2\kappa_2 - \kappa_1) - H > 0$. This leads to the third main result of the model: Compared to the benchmark rational model, the price over-reacts on average in the first period when the real information (s_1) is released, i.e. $E[p_1 - p_0|s_1 > 0] > \kappa_1 s_1$. Since there is an over-reaction when the actual news is released, one can interpret the continued over-reaction when the stale information is released as momentum from the first trading period to the second.

The intuition for the price behavior at t = 1 and t = 2 is the fact that the inattentive investors' aggregate demand in the second period, and hence their price impact at t = 2, is

 $^{^3\}mathrm{As}$ was stated earlier, all results follow through even without the simplifying assumption on the coefficients of risk aversion.

fully anticipated in the first period by the attentive investors. As a result, to take advantage of their informational advantage, the attentive investors let the price over-react. Additionally, the attentive arbitrageurs cannot fully remove the dividend risk and so p_2 does respond to the stale news release.

We collect the above results in the following proposition.

Proposition 1 The model's equilibrium prices can be ranked as follows if $s_1 > 0$:

$$0 < E[p_3 + d_3|s_1] < E[p_1|s_1] < E[p_2 + d_2|s_1]$$
(A.28)

which leads to an over-reaction upon the release of the initial information, continued overreaction upon the release of the stale information, and a subsequent reversal:

$$E[p_1 - p_0|s_1 > 0] > \kappa_1 s_1 \quad and \quad E[p_2 + d_2 - p_1|s_1 > 0] > 0 \quad and \quad E[p_3 + d_3 - p_2|s_1] > 0$$
(A.29)

If $s_1 < 0$, then the price ranking is reversed:

$$0 > E[p_3 + d_3|s_1] > E[p_1|s_1] > E[p_2 + d_2|s_1]$$
(A.30)

and the return dynamics are also reversed:

$$E[p_1 - p_0|s_1 > 0] < \kappa_1 s_1 \quad and \quad E[p_2 + d_2 - p_1|s_1 > 0] < 0 \quad and \quad E[p_3 + d_3 - p_2|s_1] < 0$$
(A.31)

The case with $s_1 > 0$ is illustrated as the dashed line on Figure A.2.⁴ The main empirical prediction of this model is the fact that the release of stale information will lead to return momentum, followed by a reversal. Since the attentive investors know that the release of stale information will be confounded for new information by the inattentive investors, they therefore let the price at t = 1 be higher than in the efficient market case (in the case of "good" news) and on average profit from it.

It is economically intuitive to expect that, the smaller the proportion of inattentive investors in the market, the smaller the over-reaction to the release of stale information and hence the smaller the reversal. This additional result of the inattention model 2 is summarized in the following proposition.

Proposition 2 First, the return over-reaction when the stale information is released at t = 2 decreases in the proportion of attentive investors in the market. Second, the return reversal

⁴Note that the benchmark case of the model can be recovered as $\sigma_{\varepsilon}^2 \to \infty$, which implies that $E[p_3|s_1] = E[p_1|s_1] = E[p_2|s_1] = \kappa_1 s_1$.

after the stale information is released decreases in the proportion of attentive investors.

$$\frac{\partial E\left[\frac{p_2+d_2-p_1}{p_1}|s_1>0\right]}{\partial m} < 0 \qquad and \qquad \frac{\partial E\left[\frac{p_3+d_3-p_2}{p_2}|s_1>0\right]}{\partial m} > 0 \tag{A.32}$$

where the inequalities are reversed if $s_1 < 0$.

2.5 Alternative Inattention Model 2: Confounding the Re-release with New Information

The key to the predictions of model 2 is that the inattentive do not realize that the information in the re-release has already been impounded in prices. Thus, if the re-release is good news, the inattentive believe that the fair price immediately after the re-release is higher than the fair price immediately before the re-release. Here we present an alternative version of the inattention model 2 where the inattentive are indeed inattentive to the initial release, but at t=2 they use lagged prices to update their beliefs. This results in the same implications as the original inattention model 2.

In this alternative version of model 2, we assume that the inattentive investors do not observe s_1 at t = 1. Rather, at t = 2, they infer from the first-period's price p_1 that the attentive investors have observed a (noisy) private signal about the terminal value, which they impounded into the price. We assume that this believed private signal is uncorrelated with the subsequent public signal (s_2) . Basically, in this version, the inattentive investors do not realize that the information in the re-release has already been impounded into the price by the attentive investors. They believe that the price increase is due to another source of information, a fact that the attentive investors can take advantage of.

The price at t = 3 is as in the benchmark model. At t = 2, the attentive investors, who know that $s_1 = s_2$, behave as in the main version of model 2 and their per-agent demand is given by equation (A.17). The inattentive investors do not observe s_1 but they believe that $s_2 = d_3 + \varepsilon_2$. In addition, they learn from p_1 about the believed uncorrelated private signal observed by the attentive investors. This learning requires a price conjecture, which we define as $p_1 = C \times s'_1$ where C is some constant that will be determined in equilibrium and s'_1 is the signal the inattentive investors believe the attentive investors observed at t = 1. By calculating the expectation and variance of $p_3 + d_3$ conditional on s_2 and the p_1 conjecture, one can show that the per-agent demand of the inattentive investors is given by:

$$x_{I,2} = \gamma_{I,2} \left(\kappa_2 (s_1' + s_2) - p_2 \right) \tag{A.33}$$

While the inattentive agents' price conjecture is $p_1 = C \times s'_1$, we know that the true equilibrium price is given by $p_1 = B \times s_1$ where B is some constant to be determined in equilibrium. As a result, we have $s'_1 = \frac{B}{C}s_1$. By equating total demand to the risky asset's supply and replacing s'_1 by $\frac{B}{C}s_1$, we can solve for the t = 2 equilibrium price:

$$p_2 = m\kappa_1 s_1 + (1 - m)\kappa_2 \left(\frac{B}{C}s_1 + s_2\right)$$
(A.34)

where we simplified the algebra using the same (non-restrictive) assumption on the coefficients of risk aversion as in the main version of model 2. While equation (A.34) is the equilibrium as seen by the attentive investors, the inattentive agents believe that all market participants have the same information as them, i.e., that all agents believe s_2 to be a new piece of information and that p_1 reflects an uncorrelated private signal. As a result, the inattentive investors' believed equilibrium price is:

$$p_2 = \kappa_2 (s_1' + s_2) \tag{A.35}$$

At t = 1, the attentive investors know the "true" price conjecture given by equation (A.34) and they know that $s_2 = s_1$. As a result, their per-agent demand is given by:

$$x_{A,1} = \frac{\left(m\kappa_1 + (1-m)\kappa_2\left(\frac{B}{C} + 1\right)\right)s_1 - p_1}{a_A \sigma_d^2}$$
(A.36)

At t = 1, in this alternative version of model 2, the inattentive investors do not observe s_1 and they believe that p_2 is given by equation (A.35) where s'_1 is a noisy signal of d_3 that is uncorrelated with s_2 . As always, the inattentive agents believe that $s_2 = d_3 + \varepsilon_2$. As a result, the inattentive per-agent demand is given by:

$$x_{I,1} = \frac{0 - p_1}{a_I(\sigma_d^2 + 2\kappa_2^2(\sigma_d^2 + \sigma_{\varepsilon}^2))}$$
(A.37)

By equating total demand to the risky asset's supply, we can solve for the equilibrium price as:

$$p_1 = \frac{m\left(m\kappa_1 + (1-m)\kappa_2\left(\frac{B}{C} + 1\right)\right)}{m + (1-m)\kappa} s_1$$
(A.38)

where $\kappa = \frac{a_A \sigma_d^2}{a_I(\sigma_d^2 + 2\kappa_2^2(\sigma_d^2 + \sigma_{\varepsilon}^2))}$ and $0 < \kappa < 1$. Equation (A.38) defines the constant *B* since, in equilibrium, it must be that $p_1 = B \times s_1$. We therefore have:

$$B = \frac{m^2 \kappa_1 + m(1-m)\kappa_2}{m + (1-m)\kappa - \frac{m(1-m)\kappa_2}{C}}$$
(A.39)

In order to solve for the other constant C, we must first solve for the t = 1 equilibrium under the beliefs of the inattentive investors. They believe that the attentive investors observe their private signal s'_1 , that they view s_2 as a new piece of information ($s_2 = d_3 + \varepsilon_2$), and that their t = 2 price conjecture is given by equation (A.35). As a result, the inattentive agents believe that the attentive investors' per-agent demand is in fact given by:

$$x_{A,1} = \frac{\kappa_2 s_1' - p_1}{a_A (\sigma_d^2 + \kappa_2^2 (\sigma_d^2 + \sigma_{\varepsilon}^2))}$$
(A.40)

By equating total demand and supply, we can obtain the equilibrium price as observed by the inattentive investors:

$$p_1 = \frac{m\kappa_2}{m + (1 - m)\kappa} s'_1 \equiv C \times s'_1 \tag{A.41}$$

where $\kappa = \frac{a_A(\sigma_d^2 + \kappa_2^2(\sigma_d^2 + \sigma_{\varepsilon}^2))}{a_I(\sigma_d^2 + 2\kappa_2^2(\sigma_d^2 + \sigma_{\varepsilon}^2))}$ and $0 < \kappa < 1$. Equation (A.41) allows the identification of the constant *C* in equilibrium, which we can replace in the expression for *B* and obtain:

$$B = \frac{m\kappa_1 + (1-m)\kappa_2}{m + (1-m)\kappa} \tag{A.42}$$

Based on the above, it is easy to show that C - B < 0, which implies that $s'_1 > s_1$. This means that, in equilibrium, the inattentive investors overstate the signal observed by the attentive. This leads to an over-reaction, a fact the attentive investors take advantage of. By replacing the values of B and C in the equilibrium price function (equations (A.34) and (A.38)), it is straightforward to show that all the results from the main inattention model 2 hold in this alternative version. First, there is an over-reaction to both the initial release and the stale release and second, there is a reversal following the stale release.

3 Appendix: Robustness Checks

In this section we perform a number of robustness tests for our main finding – that the release of the LEI is associated with a significant return response in the aggregate stock market, as well as the Treasury bond market. We focus the tests on the 5-minute return interval from 9:59:59 to 10:04:59 where the identification is cleanest and again only use the announcements in our sample for which there are no other macroeconomic announcements occurring at the same time.

3.1 Different Measures of Inattentive Investors' Expectations

In the analysis up to this point we have used the median of analyst forecasts as a proxy for the inattentive investors' expected value of the LEI release. As mentioned earlier, the analyst forecasts are often reported a week or two before the LEI index release, which means that even if the analysts are fully attentive, they cannot perfectly replicate the index at the time they report the expected LEI value. Further, since the inattentive investors view the re-release of the information as news, they do not think that it is already impounded in asset prices, and it is therefore reasonable to assume that they use the reported median analyst forecast (e.g., obtained from Bloomberg) as an estimate of the information that is impounded in prices. Nevertheless, in this subsection we consider alternative proxies for the inattentive agents' expectations.

In addition to median analyst forecast (called E1), we consider the expected change in the LEI index to be: zero (E2), the time t historical average of changes in the LEI index (E3), the 1-year moving average of changes in the LEI index (E4), and the last month's change in the LEI index (E5). Panel A of Table A.1 shows that all of these measures give similar announcement return responses both in terms of economic and statistical significance for the aggregate stock market, as well as for the Treasury bond returns. Thus, the results are not very sensitive to the particular measure of inattentive investors' expectations.

Next, we turn to various sub-sample regressions. The first two rows of Panel B of Table A.1 show the results of the 5-minute announcement return stock market regressions where we split the sample in two. The effect is similar across the two subsamples, which indicates that there has not been significant change in the response to stale information on the part of inattentive investors, even though the release of the LEI is a recurring event. The final two rows of Panel B of Table A.1 shows the results when focussing on "large" surprises to the LEI by first excluding announcement dates where there is no discrepancy between the expected LEI and the released LEI (i.e., announcement surprise equal to zero), and second when the surprise is larger than one standard deviation (where the standard deviation is measured on the full sample of LEI surprises). The announcement response is indeed higher for more extreme LEI surprises, which indicates that inattentive investors are sensitive to the size of the "surprise" and do not simply naively trade in the direction of the announcement. When the surprise is defined relative to the average analyst forecast, the announcement response increases from 4.5 basis points for the full sample to 9.1 basis points when the announcement surprise is larger than one standard deviation. All the measures of inattentive's expectations yield qualitatively similar results across these sub-samples.

3.2 Autocorrelation in Returns

We ensure that the announcement return cannot be explained by autocorrelation in intraday stock returns. To control for the possible autocorrelation in intraday stock market returns and also increase the power of the tests, we use pooled return regressions:

$$R_{i,t} = \alpha_i + \sum_{m=1}^{6} \beta_m R_{i-m,t} + \sum_{k=0}^{5} \gamma_k \left(\Delta LEI_{i-k,t} - E_{t-} \left[\Delta LEI_{i-k,t} \right] \right) + \varepsilon_{i,t}$$
(A.43)

where t refers to the announcement month, and $i \in \{1, 2, ..., 6\}$ refers to a particular 5-minute interval (where, by example, i = 1 refers to the return interval 9:59:59 to 10:04:59, while i - m = -5 refers to the return in the interval 9:30:00 to 9:34:59), and where $\Delta LEI_{i-k,t} = \Delta LEI_t$ if i - k = 1 and zero otherwise. Thus, in this specification we include the returns across all intervals in the 9:30–10:30 window.

Table A.2 presents the coefficient estimates and R^2 's for the five different measures of inattentive investors' expectations of the change in the LEI index, as explained earlier. The standard errors are clustered by announcement date (i.e., t) and are adjusted for heteroskedasticity and 6 lags of autocorrelation. The results are almost identical in sign and magnitude to those obtained earlier: there is a positive and statistically significant return response to the LEI announcement (γ_0). The insignificance of the coefficients on lagged returns suggests that autocorrelation is not an issue. The coefficients on the LEI surprise for the returns after the announcement interval ($\gamma_1, \gamma_2, ..., \gamma_5$) are typically negative, consistent with our earlier finding of post-announcement reversal, but they are not significant. Thus, as we have hinted at earlier, the reversal is not immediate but instead occurs over the span of almost two days, until the close of the day after the announcement.

References

- [1] DellaVigna, Stefano, and Joshua Pollet, 2009, Investor Inattention and Friday Earnings Announcements, *Journal of Finance* 64, 709-749.
- [2] McGuckin, Robert H., Ataman Ozyildirim, and Victor Zarnowitz, 2007, A More Timely and Useful Index of Leading Indicators, *Journal of Business and Economic Statistics* 25, 110-120.
- [3] Tetlock, Paul C., 2011, All the News That's Fit to Reprint: Do Investors React to Stale Information, *Review of Financial Studies*, forthcoming.

Table A.I

Alternative Measures of the Expectations of the Inattentive

on expectations measure). The full sample period is from February 1997 to February 2009. Returns are continuously compounded and expressed as percentages. Standard errors are corrected for heteroskedasticity. * denotes significant at the 10% level, ** denotes significant at the 5% level, and expectations measure), and when the announcement surprise is at least one standard deviation away from zero (from 32 to 38 observations depending the second half of the sample (58 observations), when the LEI announcement surprise is different from zero (from 74 to 115 observations depending on last announcement. Panel B reports similar regression results using S&P500 across different subsamples: the first half of the sample (58 observations), *** denotes significant at the 1% level in a two-tailed test. inattentive investors: (E1) median analyst forecast as used elsewhere in the paper, (E2) zero, (E3) historical average, (E4) moving average, and (E5) the (from 10:00am to 10:04am) on different normalized measures of the LEI surprise. We propose five different methods to calculate the expectations of the Panel A of this table reports estimates from OLS regressions of five-minute Treasury notes and bonds futures returns, as well as S&P500 futures returns

 $R_t = \gamma + \delta \left(\Delta LEI_t - E_{t-} \left[\Delta LEI_t | Ex \right] \right) + \varepsilon_t$

$\begin{array}{c} \mathrm{S\&P500} \\ (\pm \sigma_{Surprise}) \end{array}$	S&P500 (Surprise $\neq 0$)	S&P500 (Second half of sample)	S&P500 (First half of sample)	Panel B: Subsamples	S&P500	30-year Treasury bond	10-year Treasury bond	5-year Treasury bond	2-year Treasury bond	Panel A: Full sample	LEI announcement return	
$0.091^{**} \ (0.038/18.6\%)$	$0.056^{**}\ (0.023/11.7\%)$	0.049^{**} $(0.023/7.7\%)$	0.048^{**} $(0.022/8.0\%)$		0.045^{**} $(0.019/6.7\%)$	$-0.032^{***} \ (0.009/12.9\%)$	$-0.021^{***} \ (0.006/13.5\%)$	$-0.013^{***} \ (0.004/10.6\%)$	-0.005^{***} $(0.002/8.0\%)$		$\delta ({ m s.e.}/R^2)$	E1
$0.047^{*}\ (0.028/6.9\%)$	$0.030^{*}\ (0.016/2.9\%)$	$0.026\ (0.023/2.2\%)$	$0.040^{*}\ (0.023/5.6\%)$		$0.029^{*}\ (0.016/2.8\%)$	-0.022^{**} $(0.009/6.5\%)$	-0.014^{***} $(0.005/6.0\%)$	-0.008^{**} $(0.004/4.5\%)$	-0.004^{***} $(0.002/5.5\%)$		$\delta \ ({ m s.e.}/R^2)$	E2
0.041^{*} $(0.028/6.9\%)$	$0.030^{**}\ (0.015/3.1\%)$	$0.028\ (0.023/2.5\%)$	$0.039^{*}\ (0.022/3.5\%)$		$0.030^{**}\ (0.015/3.1\%)$	-0.022^{**} $(0.009/6.6\%)$	-0.013^{***} $(0.005/5.7\%)$	-0.008^{**} $(0.004/4.4\%)$	-0.004^{**} $(0.002/5.7\%)$		$\delta ({ m s.e.}/R^2)$	E3
$0.076^{***} \ (0.028/15.8\%)$	0.041^{***} $(0.015/5.5\%)$	$0.035\ (0.023/2.4\%)$	$0.049^{**}\ (0.022/8.2\%)$		$0.040^{***} \ (0.015/5.5\%)$	$\begin{array}{c} -0.024^{***} \\ (0.008/7.9\%) \end{array}$	$-0.015^{***} \ (0.005/6.7\%)$	-0.009^{**} $(0.004/5.0\%)$	$-0.005^{***} \ (0.002/6.3\%)$		$\delta ({ m s.e.}/R^2)$	E4
$0.069^{**} \ (0.030/13.6\%)$	$\begin{array}{c} 0.044^{**} \\ (0.017/6.7\%) \end{array}$	$0.044^{*}\ (0.023/6.1\%)$	$0.038^{*} \ (0.022/5.0\%)$		$\begin{array}{c} 0.041^{***} \ (0.015/5.6\%) \end{array}$	-0.025^{***} $(0.008/8.4\%)$	$-0.012^{***} \ (0.005/4.9\%)$	-0.007^{**} $(0.003/3.7\%)$	-0.004^{***} $(0.001/5.3\%)$		$\delta \ ({ m s.e.}/R^2)$	E5

Table A.II Autocorrelation in Intraday Returns

This table reports estimates from pooled OLS regressions of five-minute S&P500 futures return on six lagged returns and on the same-day LEI announcement using five different methods to calculate the expectations of inattentive investors: (E1) median consensus analyst forecast, (E2) zero, (E3) historical average, (E4) moving average, and (E5) last announcement. The sample period is from February 1997 to February 2009. The *i*'s span the six return intervals from 10:00am to 10:30am. The total number of observations in each regression is therefore 6 times 116. Returns are continuously compounded and expressed as percentages. Standard errors are corrected for heteroskedasticity and serial correlation. * denotes significant at the 10% level, ** denotes significant at the 5% level, and *** denotes significance at the 1% level in a two-tailed test.

	E1	E2	E3	E4	E5
β_1	$0.027 \\ (0.042)$	$0.031 \\ (0.041)$	$0.036 \\ (0.041)$	$0.031 \\ (0.042)$	$0.031 \\ (0.041)$
β_2	$\begin{array}{c} 0.027 \\ (0.046) \end{array}$	$\begin{array}{c} 0.026 \\ (0.045) \end{array}$	$\begin{array}{c} 0.029 \\ (0.046) \end{array}$	$\begin{array}{c} 0.029 \\ (0.046) \end{array}$	$0.028 \\ (0.046)$
eta_3	-0.035 (0.040)	-0.040 (0.040)	-0.040 (0.041)	-0.037 (0.040)	-0.033 (0.040)
β_4	-0.018 (0.055)	-0.015 (0.060)	-0.015 (0.061)	-0.015 (0.059)	-0.012 (0.058)
β_5	$0.018 \\ (0.041)$	$\begin{array}{c} 0.013 \\ (0.042) \end{array}$	$\begin{array}{c} 0.013 \\ (0.043) \end{array}$	$\begin{array}{c} 0.013 \\ (0.042) \end{array}$	-0.001 (0.041)
eta_6	$\begin{array}{c} 0.019 \\ (0.039) \end{array}$	$\begin{array}{c} 0.010 \\ (0.038) \end{array}$	$\begin{array}{c} 0.010 \\ (0.037) \end{array}$	$\begin{array}{c} 0.010 \\ (0.037) \end{array}$	$\begin{array}{c} 0.014 \\ (0.036) \end{array}$
γ_0	0.045^{***} (0.018)	0.027^{*} (0.015)	0.031^{**} (0.015)	0.040^{***} (0.014)	0.040^{***} (0.015)
γ_1	-0.008 (0.012)	-0.002 (0.011)	-0.006 (0.011)	-0.001 (0.012)	$0.012 \\ (0.011)$
γ_2	-0.018 (0.017)	-0.008 (0.015)	-0.009 (0.015)	-0.014 (0.015)	-0.017 (0.015)
γ_3	-0.013 (0.016)	-0.005 (0.013)	-0.002 (0.014)	-0.007 (0.013)	$\begin{array}{c} 0.003 \\ (0.015) \end{array}$
γ_4	-0.014 (0.017)	-0.017 (0.018)	-0.017 (0.019)	-0.009 (0.015)	-0.015 (0.020)
γ_5	-0.012 (0.012)	-0.015 (0.013)	-0.013 (0.013)	-0.015 (0.012)	-0.001 (0.015)
N	696	696	696	696	696
R^2	0.038	0.022	0.025	0.029	0.029

$$R_{i,t} = \alpha_i + \sum_{m=1}^{6} \beta_m R_{i-m,t} + \sum_{k=0}^{5} \gamma_k \left(\Delta LEI_{i-k,t} - E_{t-} \left[\Delta LEI_{i-k,t} \right] \right) + \varepsilon_{i,t}$$