Friction Loss Coefficient for Laminar flow

In different geometries

By Nhu-Chi Ha

Mentor: Professor Bruce A. Finlayson

Chemical Engineering

University of Washington

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Introduction

The purpose of the project was to determine the friction loss coefficient of the laminar flow, which is useful in microfluidic devices, to analyze the pressure drop in different shape of the models. COMSOL Multiphysics was used in 2-D axial symmetry model. Geometry of Eight models is shown below in Figure 1.

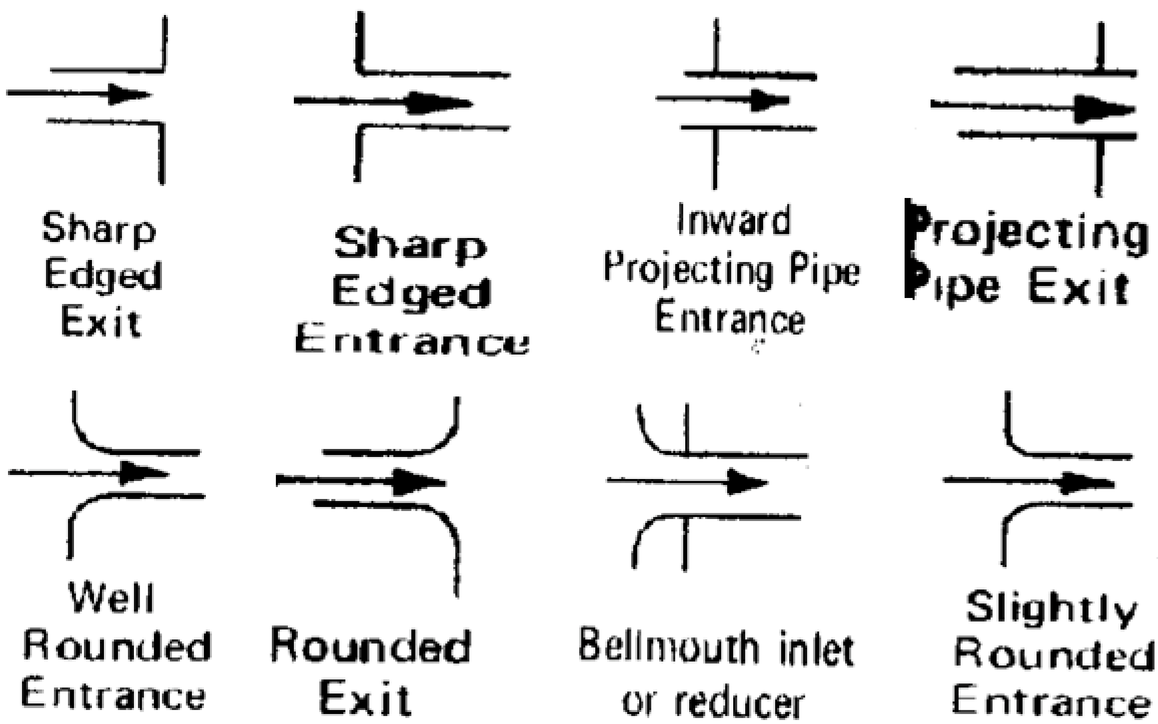

Figure 1. Eight models: Different Geometries in microfluidic devices

General dimensions, subdomain, and boundary conditions:

In Comsol Multiphysics, all of the parameters were dimensionless. General conditions for subdomain were set for all models: with the density was equal to the Reynolds number ranged from 0 to 100. The viscosity was set to 1 for every models.

From the Figure 2, general conditions for Boundary involved: boundary 1 was set axial symmetry. The boundary 2 was laminar flow with the velocity to be 1 if the flow entered the small entrance, and to be -1 if the flow entered the large entrance. The boundary 3,4,5,7 would be no slip. If the flow entered the small entrance, the boundary 8 would be set to normal pressure, if the flow entered the large entrance, that boundary would be set to neutral. Finally, the boundary 7 would be neutral (considered there was no wall on both sides).

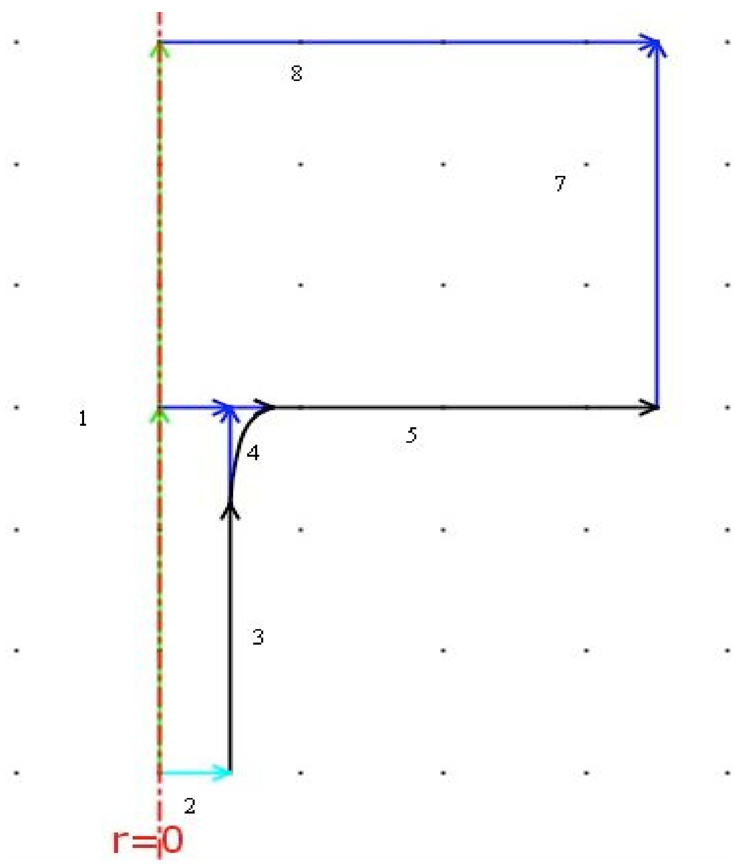

Figure 2. General shape and dimension for eight cases

More specifically, each model would have different length, width, height and shape at the fitting.

Materials and Methods

From Comsol Multiphysics, the steady state, incompressible Navier-Stokes Equation was used to find the total pressure drop in each model. The incompressible Navier-Stokes equation is given by Eq. 1:

$$\rho \mathbf{u} \bullet \nabla \mathbf{u} = -\nabla p + \eta \nabla^2 \mathbf{u}, \quad \nabla \bullet \mathbf{u} = 0 \tag{1}$$

From equation 1, the nondimentional Navier-Stokes equation is derived [2]:

$$Re\frac{\partial \mathbf{u}'}{\partial t'} + Re\mathbf{u}' \bullet \nabla' \mathbf{u}' = -\nabla' p' + \nabla'^2 \mathbf{u}'$$

where Re is the Reynolds number, **u** is the velocity, ρ is the density of the fluid, p is the pressure, η is the viscosity.

From the Boundary Integration of Comsol, the total pressure drop would be determined using equations 2 and 3:

$$\Delta p' = \frac{\int_0^{0.5} p\, r\, dr}{\int_0^{0.5} r\, dr} \tag{2}$$

$$\Delta p_{total} = \frac{\mu < v_1 >}{d_1} \Delta p' \tag{3}$$

For the fully developed flow in the small and large pipes, the pressure drop would be determined from equation 4 and 5:

$$\boxed{\Delta P_{small} = \frac{12 \times \mu \times \langle v_1 \rangle \times L_1}{d^2{}_1} = 36} \tag{4}$$

$$\boxed{\Delta P_{large} = \frac{12 \times \mu \times \langle v_2 \rangle \times L_2}{d^2{}_2} = 0} \tag{5}$$

Where <v1> is the velocity in the small pipe, <v2> is the velocity in the large pipe, d1 is the diameter of the small pipe, d2 is the diameter of the large pipe, L1 is the fully developed length of the small pipe, L2 is for the large pipe.

By continuity, the velocity in the large pipe would be calculated using equation 6:

$$\boxed{d_1^{\,2}\left\langle v_1 \right\rangle = d_2^{\,2}\left\langle v_2 \right\rangle} \qquad (6)$$

Because the diameter of the large pipe is large and can be expanded to infinity (no wall on both sides), the average velocity of the large pipe was negligible and the pressure drop would be negligible.

The excess pressure drop at the fitting would be calculated using equation 7:

$$\Delta p_{excess} = \Delta p_{total} - \Delta p_{small} - \Delta p_{large} \qquad (7)$$

Finally, the friction loss coefficient would be calculated using the equation 8:

$$\boxed{K_L = \frac{\Delta P_{excess} \times d_1}{\mu \times \left\langle v_1 \right\rangle}} \qquad (8)$$

RESULT and DISCUSSION

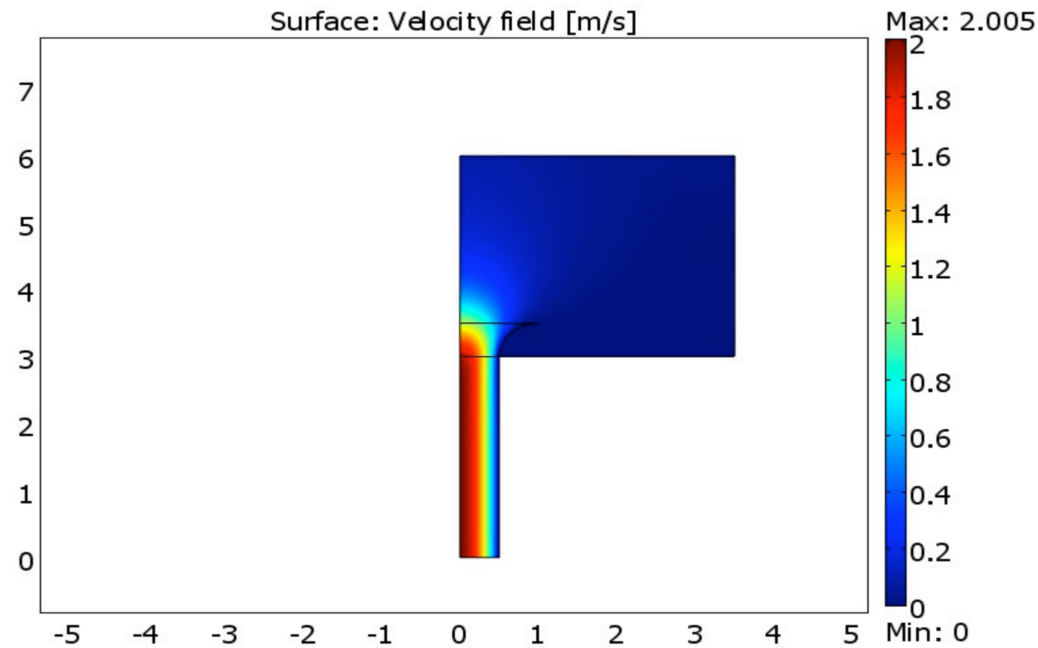

Figure 3. Bellmouth Model with velocity profile

Table 1: KL Values for all 8 models with different geometries at Re=0

GEOMETRIES	KL	Degrees of freedom	No. of Elements
Sharp edge (small entrance)	69.23	86001	14775
Sharp edge (large entrance)	69.12	65039	14275
Well round (small entrance)	60.89	85300	14507
Well round (Large entrance)	60.91	65039	14275
Slightly round	56.636	58354	15350
Bellmouth	74.51	78210	16670

Projecting (small entrance)	94.74	78250	13203
Projecting (large entrance)	94.742	62105	16520

When the Reynolds number is small, the kinetic energy change is negligible, the flow entered or exit the small pipe had the same KL value (Re=0) [1]. The KL values in the table 1 obtained in the average values of taking three different mesh elements.

From Fig. 4, three models were put into a same group to be compared by having the same flow direction into the small pipe entrance: 1 is for the well round shape, 2 is for sharp edge shape, 3 is for the projecting shape. From Fig.5, five other models were compared to each other by having the same flow direction into the large pipe entrance: 1 is for the well round shape, 2 is for the sharp edged shape, 3 is the slightly round shape, 4 is for the bellmouth shape and 5 is for the projecting shape.

From Fig. 5, the model with the well round geometry had the lowest KL value based on its shape; the excess pressure drop was small compared to others. The same trend occurred in other five models, based on their geometries, the model with slightly round geometry had the least resistance to the flow, and therefore, it had the lowest KL value.

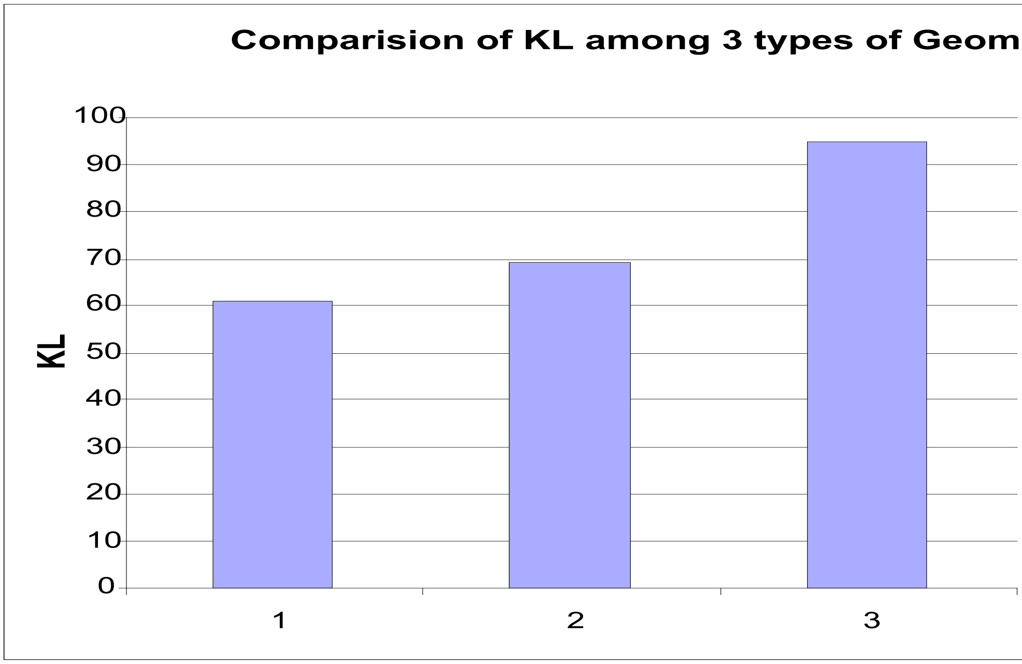

Figure 4. Comparison of KL for Well round, projecting, and sharp edge models (Re=0)

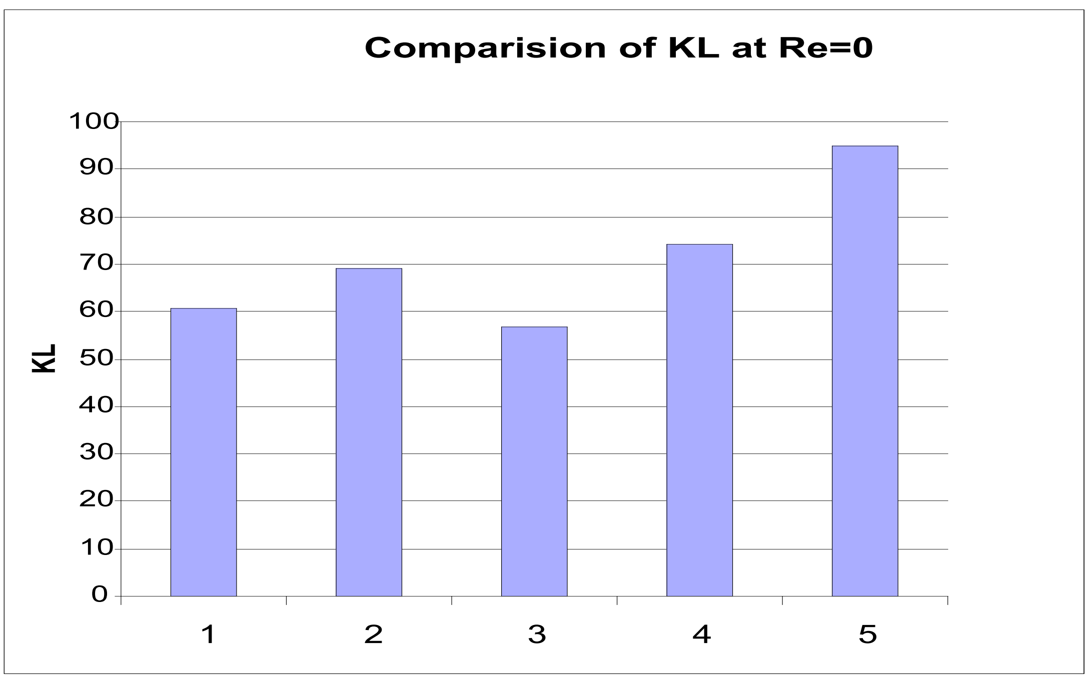

Figure 5. Comparison KL for well round, projecting, bellmouth, slightly round, sharp edged models (Re=0)

In the range of Reynolds number from 0 to 100, the group of three models also had the similar trends compared with each other, and the model with well round geometry also had the lowest KL values. As the Reynolds numbers increased, the KL values would become constant.

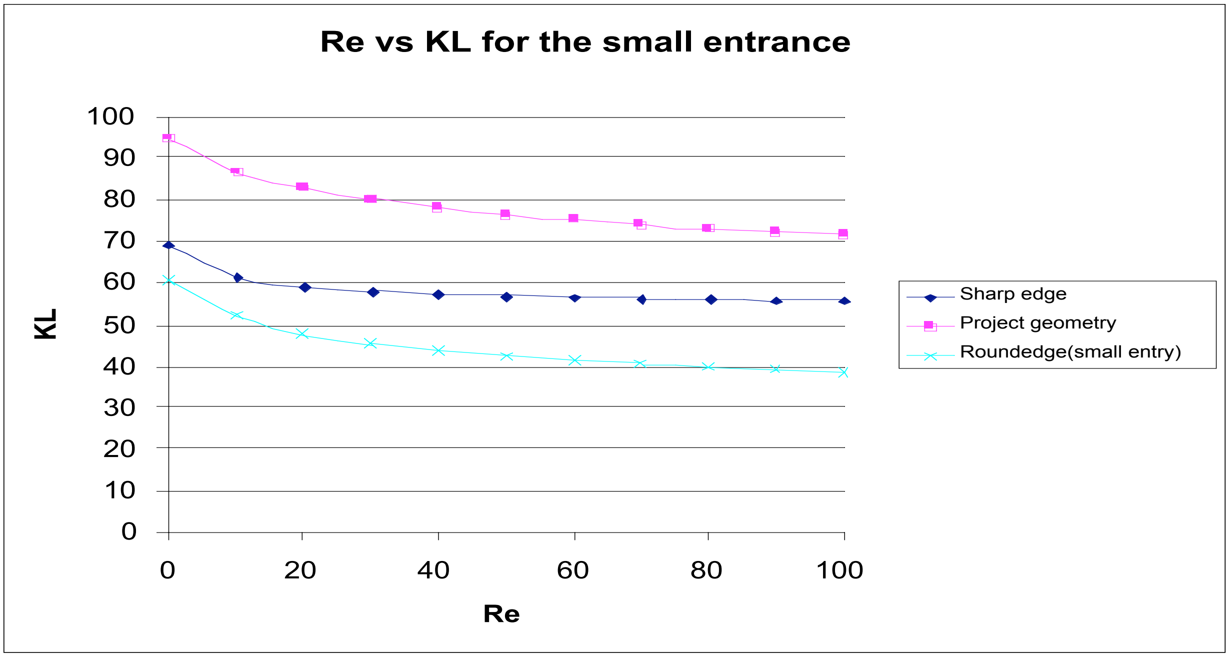

Figure 6. Comparison of KL among 3 shapes at different Reynolds numbers

From Fig. 7, the group of five models had the similar trends compared with each other and with the fig. 8.4, the laminar flow excess pressure drop due to contraction in a circular channel [1]. When the fluid flowed into the large pipe entrance, due to the geometry, the excess pressure drop increased in the order from slightly round, well round, sharp edged, bellmouth to projecting models.

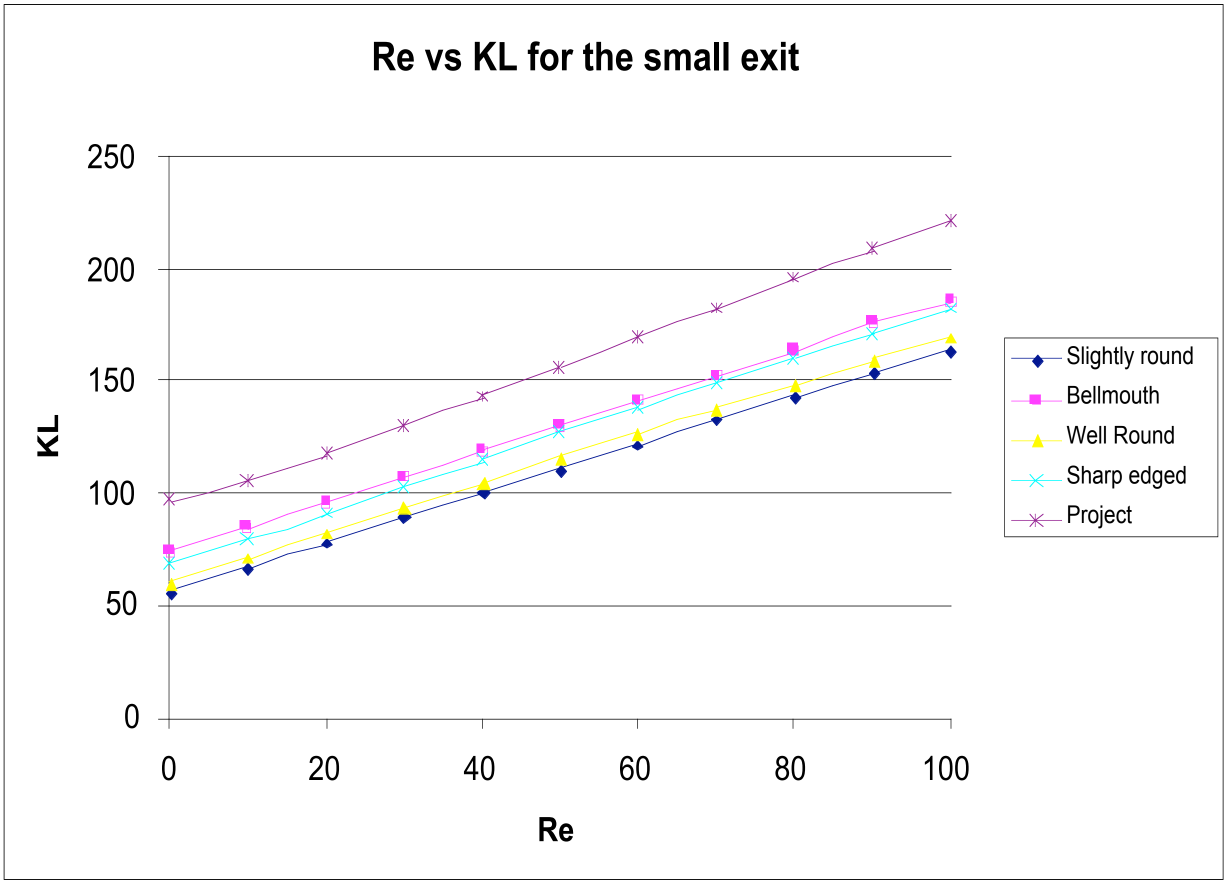

Figure 7. Comparison of KL among 5 shapes at different Reynolds numbers.

Finally, the result for each model was checked to see if the Comsol worked correctly. The streamline profile showed that the flow was fully developed. From Figure 10, it appeared to have some flow across the interior boundary of the bell mouth, which was set to be no slip. By using the cross plot, the velocity across that boundary was close to 0, based on fig. 12.

CONCLUSION

Overall, Comsol provided good results. By using Comsol, the KL can be estimated for a lot of different geometry. Therefore, without using any device, with the given geometry, the pressure drop in the microfluid devices could still be obtained by using the correlation KL, the pressure drop coefficient.

RECOMMENDATION

To obtain better data, the triangle with the red dot in the middle can be used to refine the local regions such near boundaries and separation points. Always check the boundary condition and interior boundary condition for complex geometries.

Reference

Finlayson, Pawel and others. Micro Instrumentation. Wiley-VCH Verlag GmbH & Co. KGaA. 2007. [1]

Finlayson, Bruce. Introduction to Chemical Engineering Computing. John Wiley & Sons, Inc., 2006. [2]

Appendix A

Appendix A1: Sample Calculation

By continuity:

$$d_1^{\,2}\left\langle v_1 \right\rangle = d_2^{\,2}\left\langle v_2 \right\rangle$$

where d_1 is the diameter of the small pipe, and d_2 is the diameter of the large pipe, $<v_1>$ and $<v_2>$ is the average velocities in the small and large pipes respectively. The average velocity in the large pipe was equal to 0.

The fully developed pressure drop in the small pipe was

$$\Delta P_{small} = \frac{12 \times \mu \times \left\langle v_1 \right\rangle \times L_1}{d^2{}_1} = 36$$

where L=3, <v1> =1, d1=1

The fully developed pressure drop in the larger pipe was:

$$\Delta P_{l\arg e} = \frac{12 \times \mu \times \left\langle v_2 \right\rangle \times L_2}{d^2{}_2} = 0$$

Where L=3, $<v_2>=0$, $d_2=7$

The total pressure drop was found from Comsol:

$$\Delta P' = \frac{\int_0^{0.5} p \times r dr}{\int_0^{0.5} r dr} = 105.136$$

$$\Delta P = \frac{\mu \times < v_1 >}{d_1} \times \Delta P' = 105$$

The excess pressure drop is equal to :

$$\Delta P_{excess} = \Delta P_{total} - \Delta P_{small} - \Delta P_{l\arg e} = 105 - 36 = 69$$

Finally, KL was found:

$$K_L = \frac{\Delta P_{excess} \times d_1}{\mu \times \left\langle v_1 \right\rangle} = 69$$

Appendix A2: Additional Streamline profiles for other models:

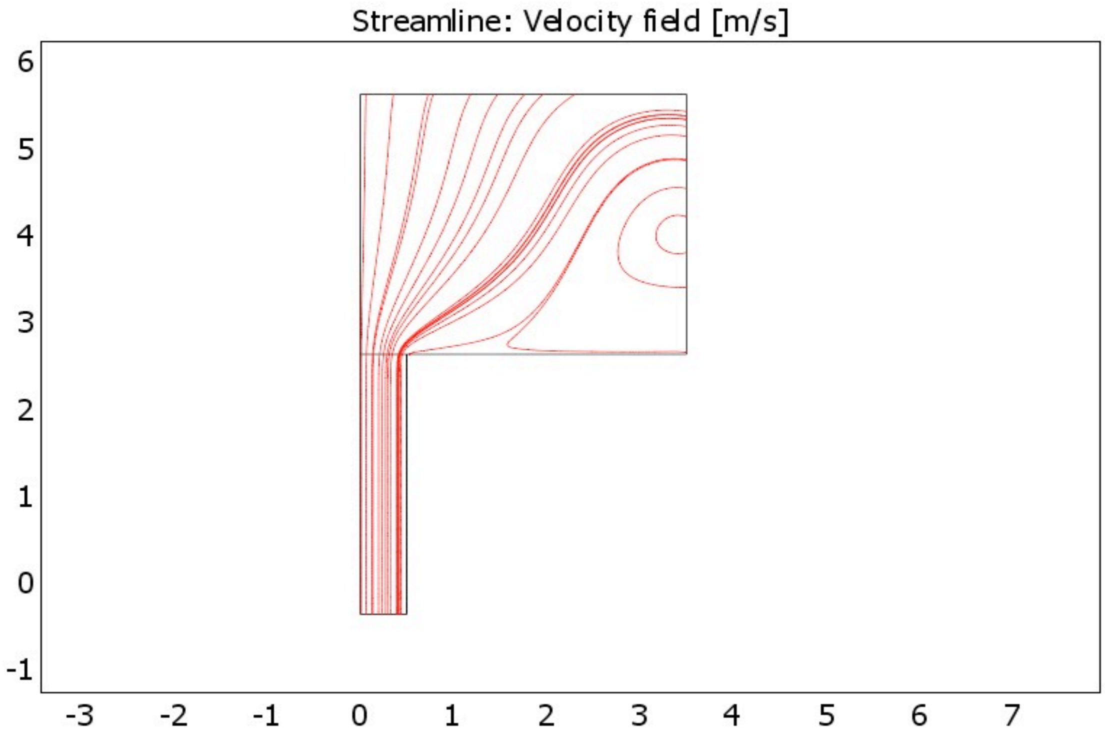

Figure 8 . Sharp edged exit (Re=0)

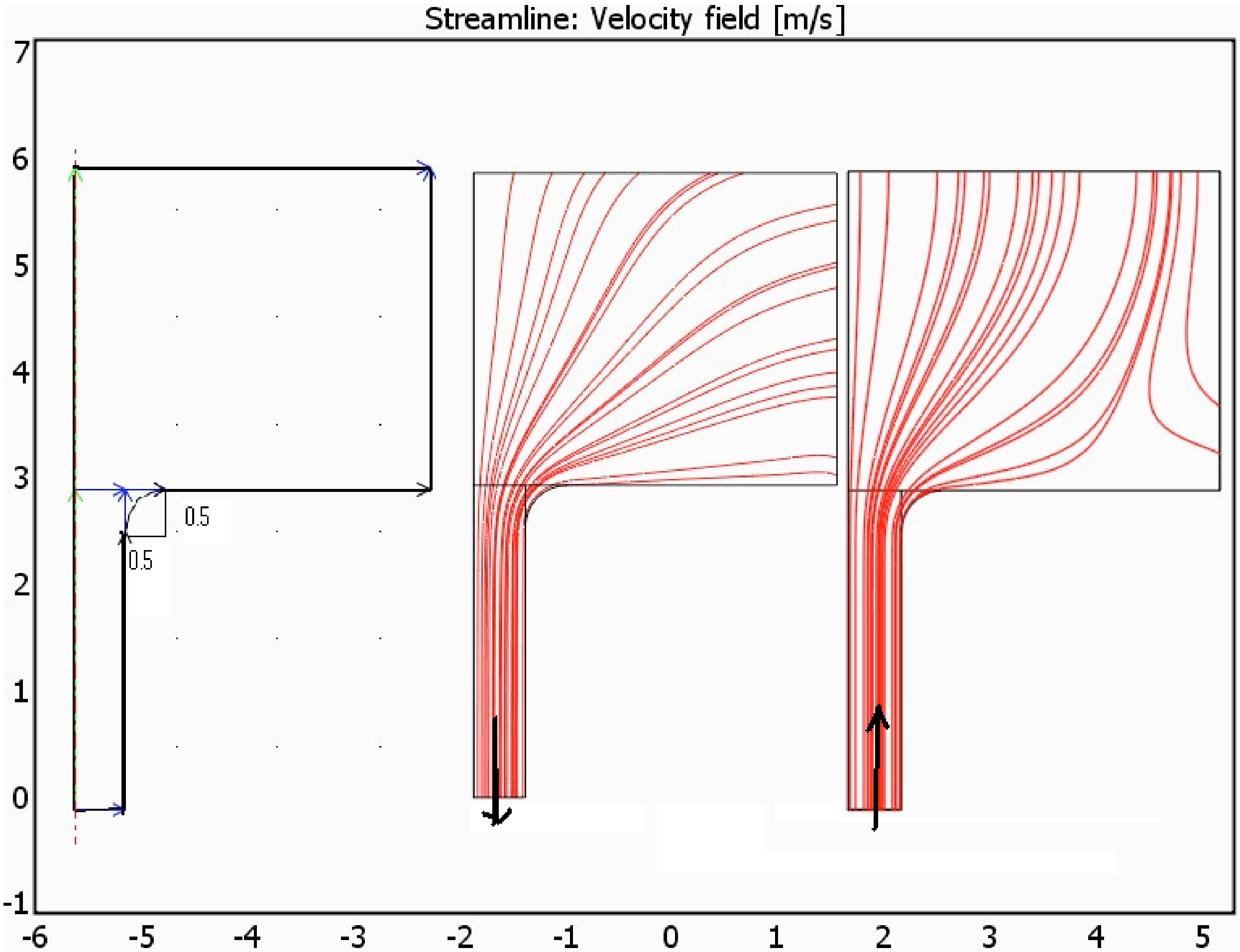

Figure 9 . Rounded exit
& well rounded entrance (Re=0)

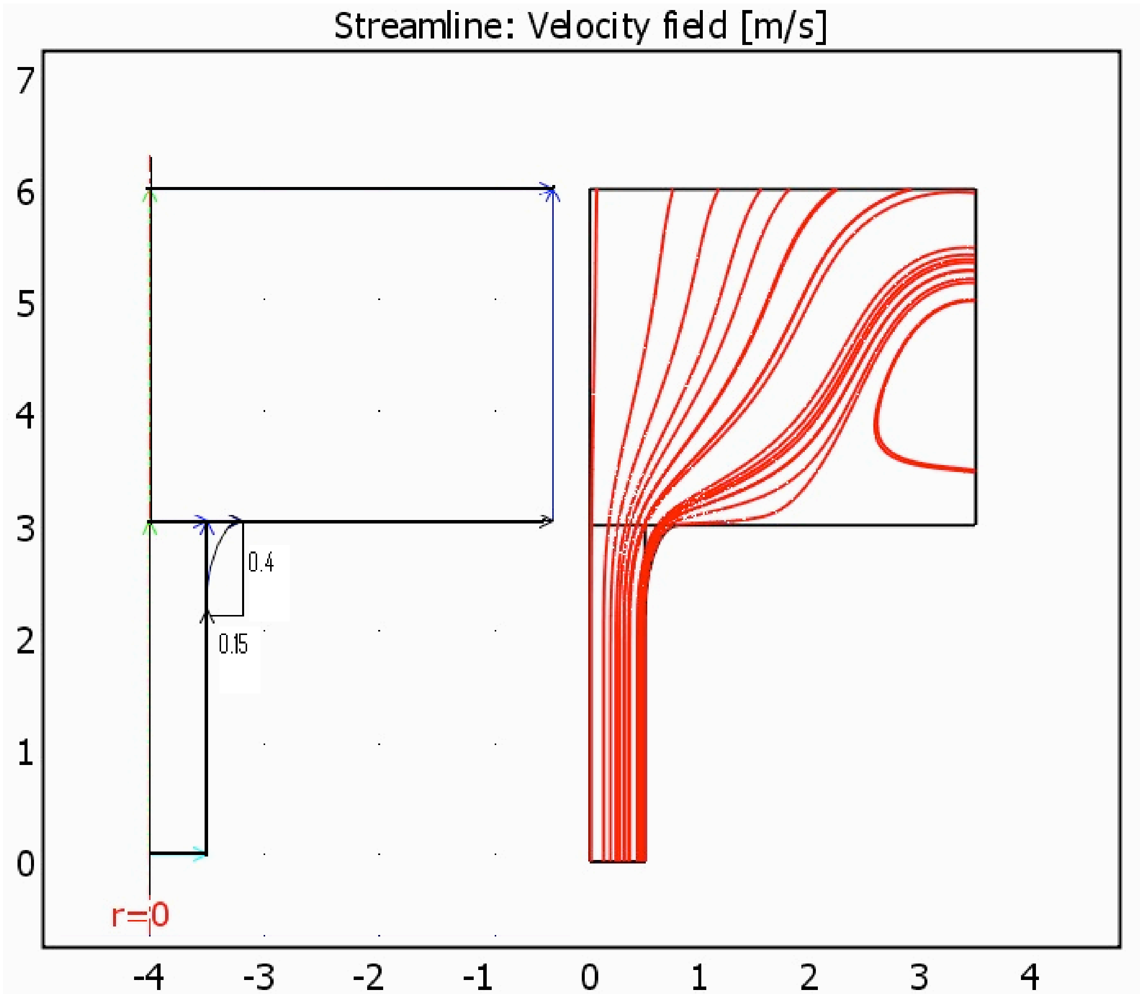

Figure 10. Slightly round entrance

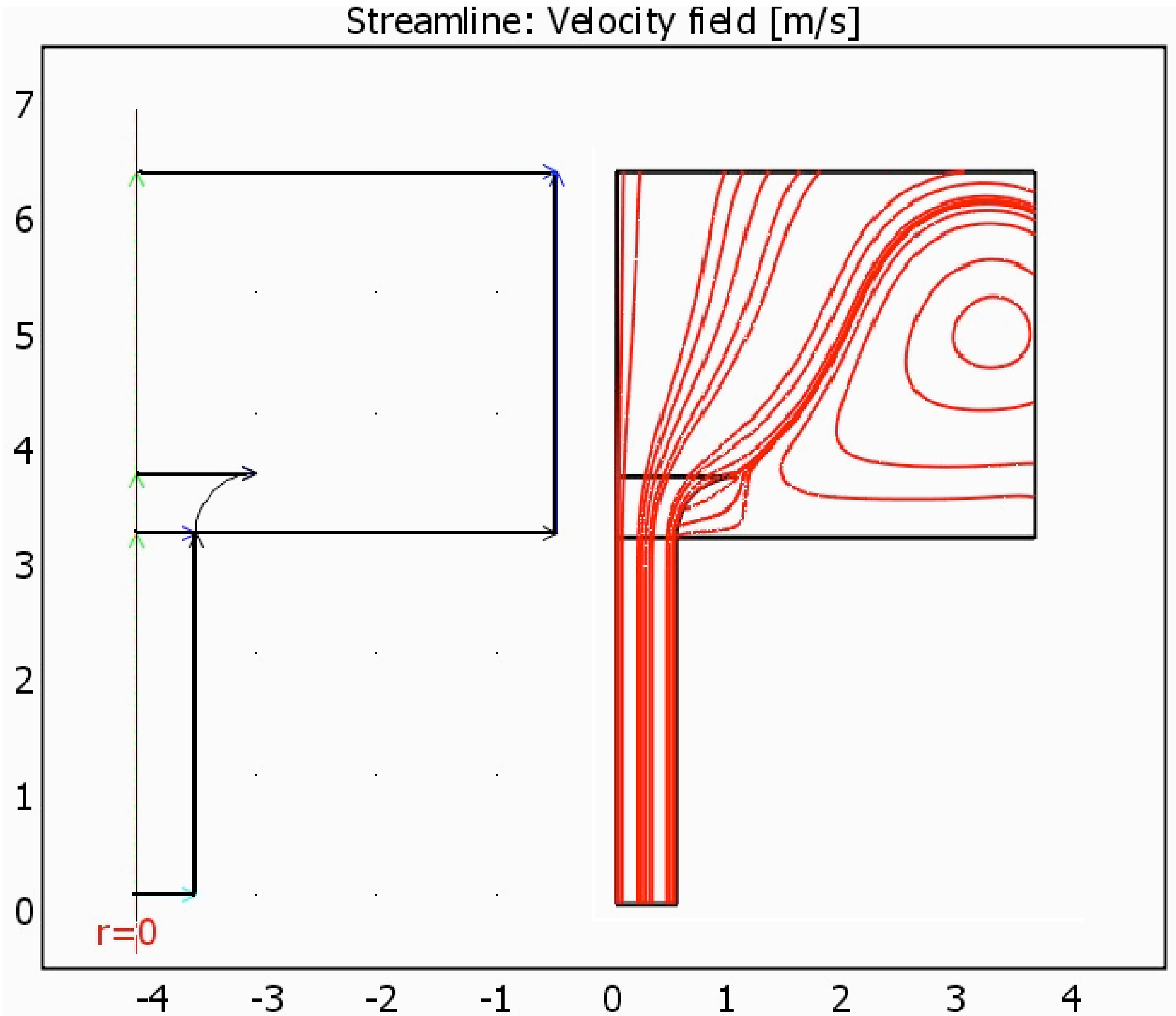

Figure 11. Bellmouth Streamline profile at Re=0

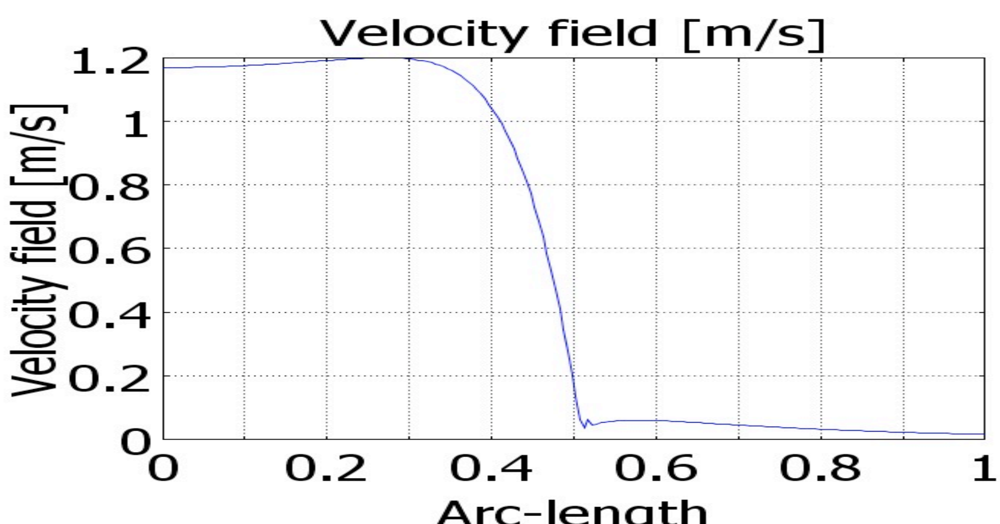

Figure 12. Velocity at the interior boundary of the curve

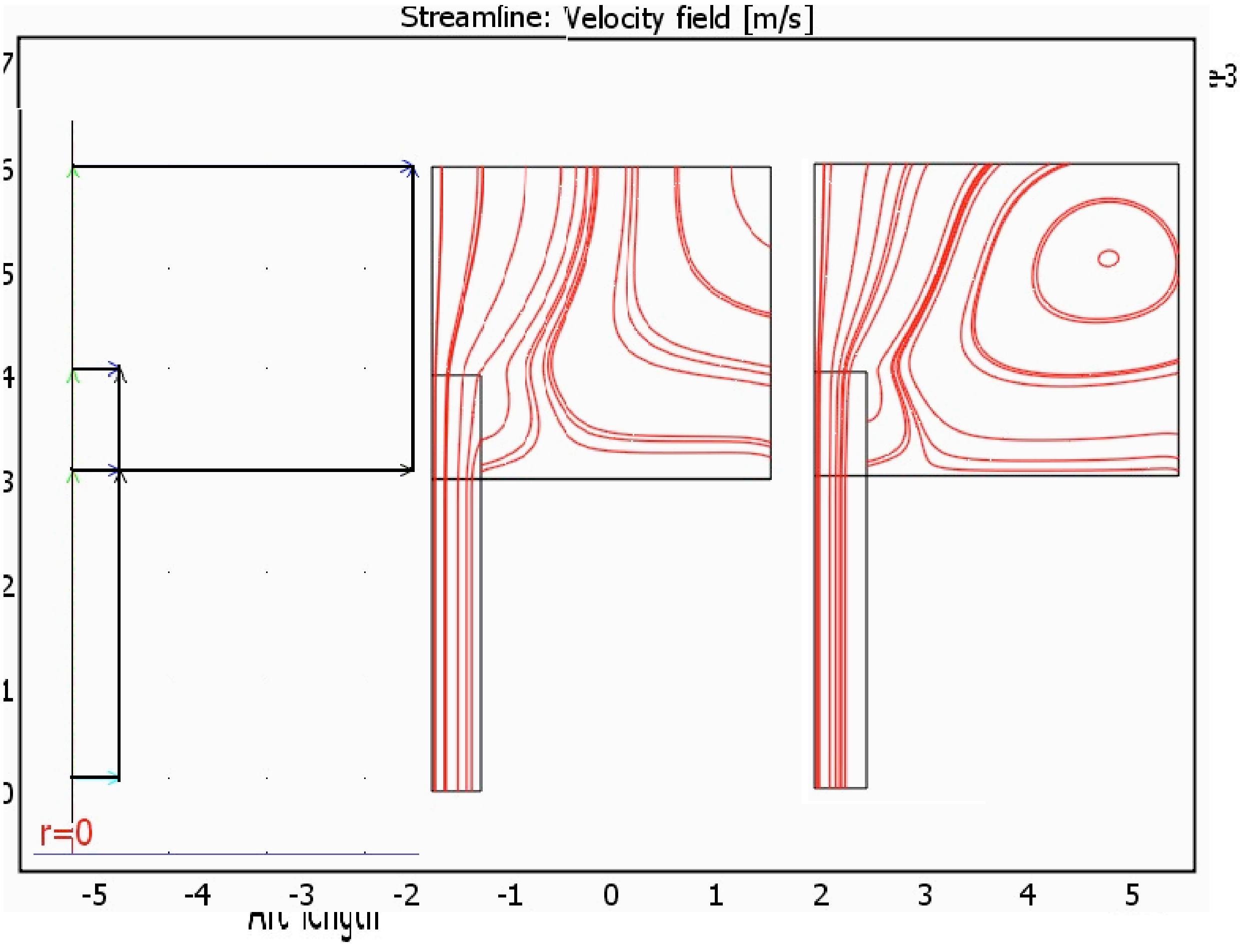

Figure 13 . Projecting pipe exit, projecting pipe entrance