Fluid Dynamics Analysis of a Modified Venturi Model

Undergraduate Independent Research

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INTRODUCTION

As the world continues to downsize, microfluidic devices have become a forefront in advancing technologies. One approach for optimizing a device is to study its flow characteristics; however, traditional methods may be difficult to perform on such small-scale models. A solution to this problem may be to simulate the device using computer software, such as FEMLAB, to determine the flow characteristics, properties and feasibility of the device. In a particular case, a company has developed a microchip for measuring the composition of a liquid flowing through a continuous system. This device would allow the bleed line to flow through the microchip for liquid composition analysis. With this in mind, the objective of this study is to determine the flow characteristics of a venturi contraction with a bleed line and investigate the potential effectiveness of the device. For this problem, two models are used to find the fluid properties through the device. To simplify the simulation model, the following assumptions are made:

- Liquid has water properties (e.g. $\mu = 0.001$ kg/m·s, $\rho = 1000$ kg/m3)
- Liquid is fully developed laminar flow at the entrance
- No-slip boundary conditions

The first model to be simulated in FEMLAB uses a two-dimensional axi-symmetric approach to determine whether the magnitude of the pressure drop through the venturi will allow for a fluid to flow through the bleed line. The two-dimensional model will also establish the velocity profile through the contraction at a given flow rate for investigation of flow abnormalities. Once the pressure drop is known, a three-dimensional model will be used to find the volumetric flow rate and velocity of the liquid through the bleed line for different viscosities.

TWO-DIMENSIONAL AXI-SYMMETRIC MODEL

For this model, the magnitude of the pressure drop through the venturi pipe is analyzed using the following dimensions and flow rate (seen in figure 1):

- Pipe diameter, D: 0.254 m
- Volumetric flow rate, Q_{in}: 1 L/min
- 3:1 contraction

The flow rate and dimensions used in this 2D analysis were estimations given of the actual device; however, in the three-dimensional model, the real dimensions are used to provide more accurate results of how the device would perform. This model was prepared to verify that the pressure drop across the contraction is significant enough to draw liquid through the bleed line.

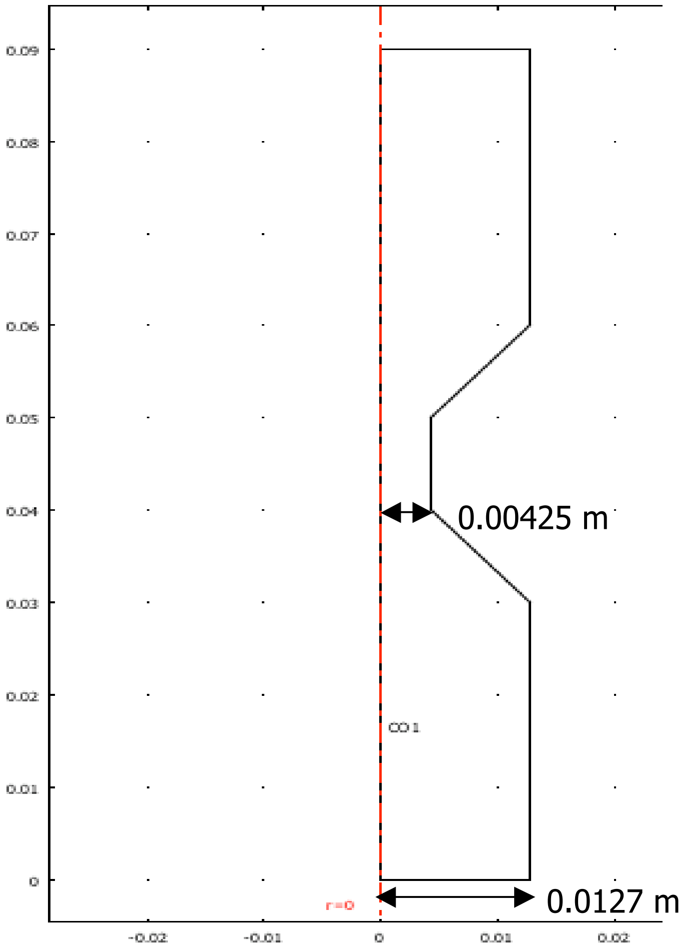

Figure 1: FEMLAB axi-symmetric representation of the venturi contraction model

Before beginning the simulation, the Reynolds number for the liquid was calculated to confirm that in fact the flow through the pipe was laminar using equation 1. The calculated Reynolds numbers are $\mathrm{Re}_{pipe} = 836$, $\mathrm{Re}_{con} = 2500$, and $\mathrm{Re}_{bl} = 148$.

$$\mathrm{Re} = \frac{\rho v D}{\mu} \qquad \text{[Eqn 1]}$$

The input values into FEMLAB were as follows:

- ○ Density, ρ: 1000 kg/m3
- ○ Viscosity, μ: 0.001 kg/m•s
- ○ Velocity, v_{in}: 0.0329 m/s

where the velocity into the pipe is calculated from the equation 2.

$$v = \frac{Q}{A}, \quad A = \frac{\pi D^2}{4} \qquad \text{[Eqn 2]}$$

The boundary condition entering the pipe was specified as Inflow/Outflow velocity, where as the boundary condition exiting the pipe was Normal Flow/Pressure set at a value of zero. All edge boundary conditions were specified as No Slip. To help FEMLAB converge to an answer, the parametric solver function was used by changing the entering velocity by factors of 10. Figure 2 illustrates the results of the pressure drop plot for 7264 elements and 33371 degrees of freedom solved and an entering velocity of 0.0329 m/s.

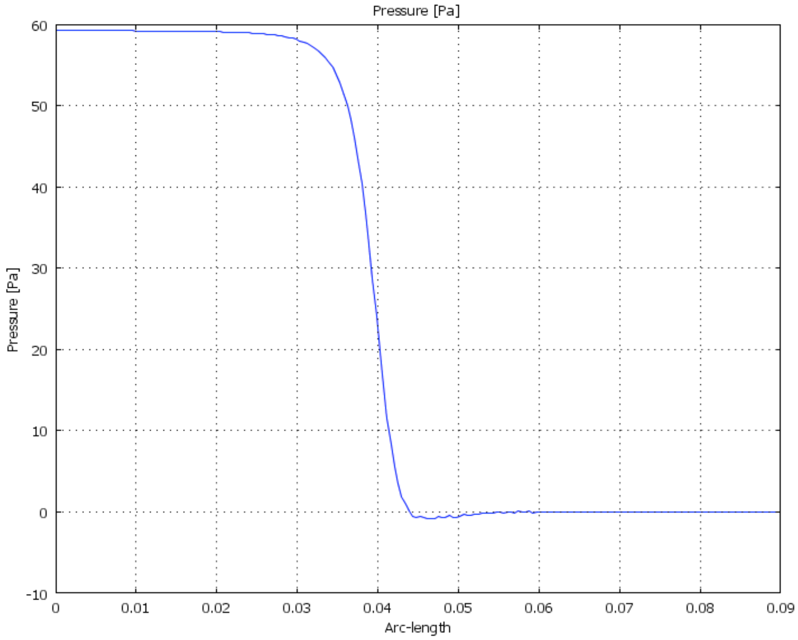

Figure 2: Pressure drop of the flowing liquid as a function of length for a velocity of 0.0329 m/s entering the Venturi. The x-axis has units in meters.

From figure 2, we can see that FEMLAB calculated the maximum pressure drop to be about 56.9 Pa for the given dimensions. Once this pressure drop was found, I calculated an approximate velocity through a channel of length 10 cm and a diameter of 2 mm using the Hagen-Poiseuille equation shown below (equation 3). This calculation is to approximate a velocity of the liquid if a channel were to be added to the Venturi model. The calculated velocity is about 0.074 m/s.

$$\Delta P = \frac{32 v \mu L}{D^2} \qquad \text{[Eqn 3]}$$

Figure 3 shows the streamline plot for the velocity profile through the device. From this plot you can see that after the contraction a vortex is created that increases in size as the entering velocity is increased.

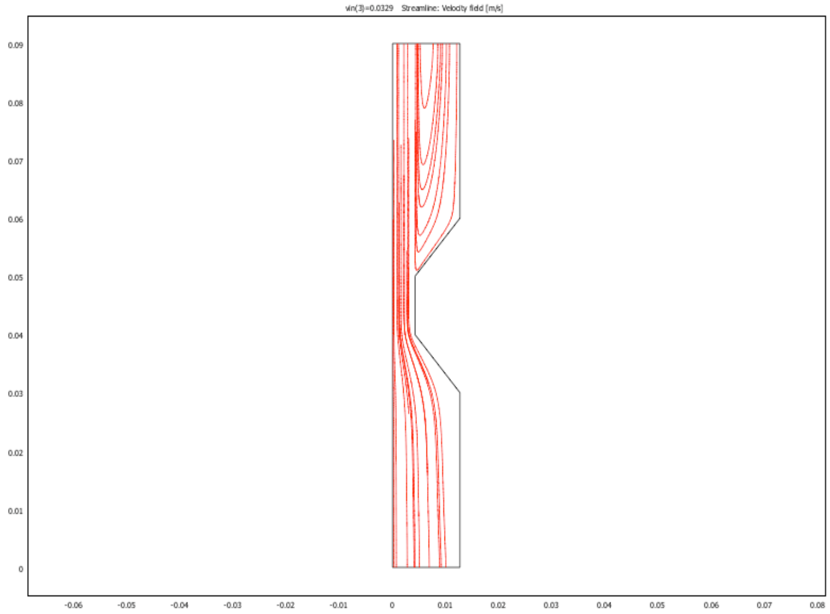

Figure 3: Streamline velocity field for a liquid velocity of 0.0329 m/s. This figure illustrates the circulation that occurs after the contraction.

THREE-DIMENSIONAL MODEL ANALYSIS

For this model, the bleed line was added into the device to find out the fluid velocity through the channel for the given flow rate and dimensions:

- Volumetric flow rate, Q_{in}: 100 mL/min
- Pipe diameter, D: 6.35 mm
- 3:1 contraction
- Channel diameter, D_{bl}: 2 mm

Figure 4 is a detailed schematic of the device that displays all the appropriate dimensions. This figure is the model represented in FEMLAB with the arrows indicating the direction of the flow through the venturi contraction device.

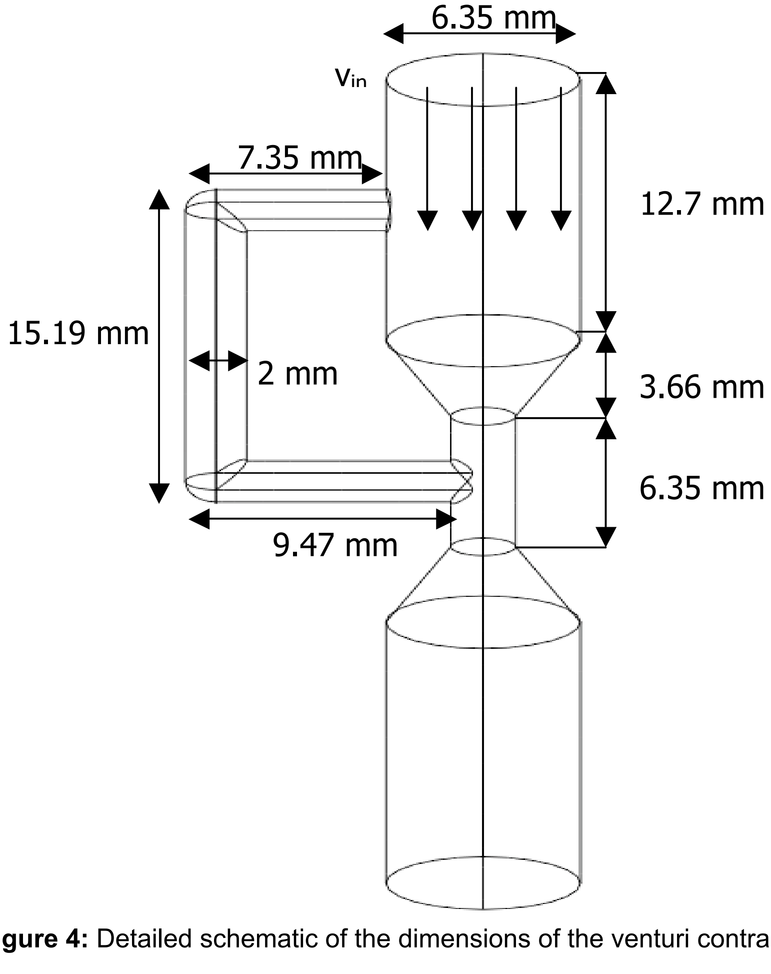

Figure 4: Detailed schematic of the dimensions of the venturi contraction device

The boundary conditions specified for the 3D model are the same as those for the 2D axi-symmetric model; however, the input values have changed due to the correction in the device dimensions. The input values for the 3D model are listed below:

- Density, ρ: 1
- Kinematic viscosity range, μ/ ρ: $10^{-5} – 10^{-6}$ m^2/s
- Velocity, v_{in}: 0.0526 m/s or 52.6 mm/s

To allow FEMLAB to converge to a solution easier, the momentum equation is divided by the density (1000 kg/m^3). The number of elements and degrees of freedom FEMLAB is able to solve for this model is 7628 and 41750, respectively. For this model, the parametric solver function is used in FEMLAB by changing the liquid viscosities to examine what values will converge to a solution. Figures 5 and 6 are velocity profile plots that view how the fluid responds through the device for a viscosity of 0.004183 kg/m•s, which corresponds to a value 4.183 times that of water.

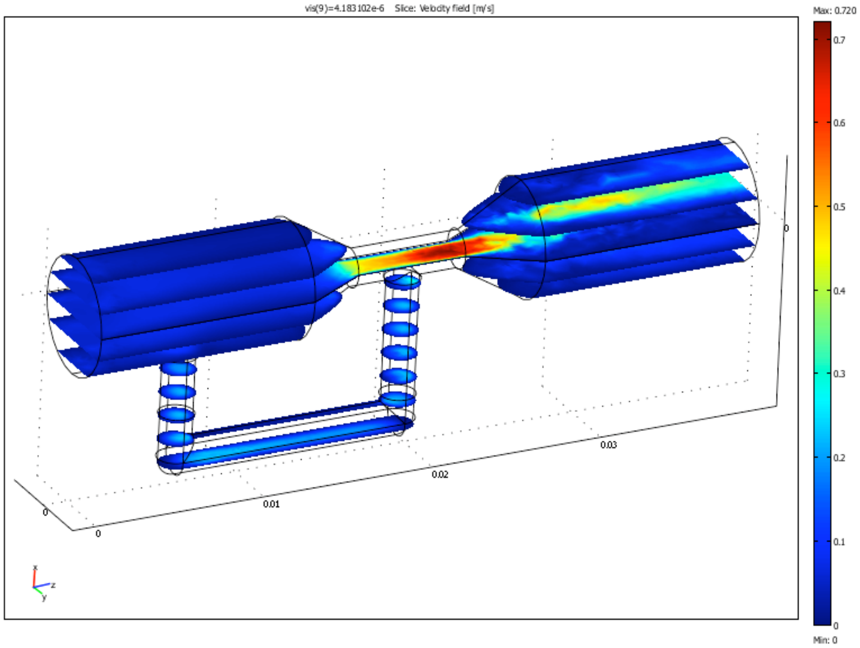

Figure 5: Slice plot of the velocity profile for an inlet velocity of 0.0526 m/s and a viscosity of 0.004183 kg/m*s.

The figure below illustrates the streamline velocity profile through the device for a flow rate of 100 mL/min. As determined in the 2D analysis, the pressure drop created by the contraction is sufficient enough to cause the liquid to flow through the bleed line.

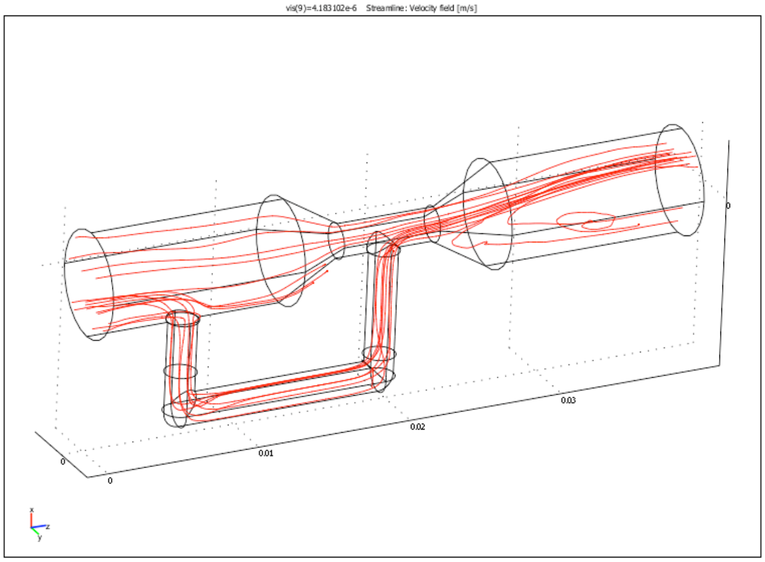

Figure 6: Streamline plot of the velocity profile for an inlet velocity of 0.0526 m/s and a viscosity of 0.004183 kg/m*s.

Since FEMLAB is unable to converge to a solution for viscosities lower than 0.004183 kg/m•s, figure 7 was prepared to correlate the liquid velocity through the bleed line with viscosity for the given dimensions. This plot allows the results from FEMLAB to be extrapolated/interpolated and applied to a wider range of fluid viscosities for industrial use.

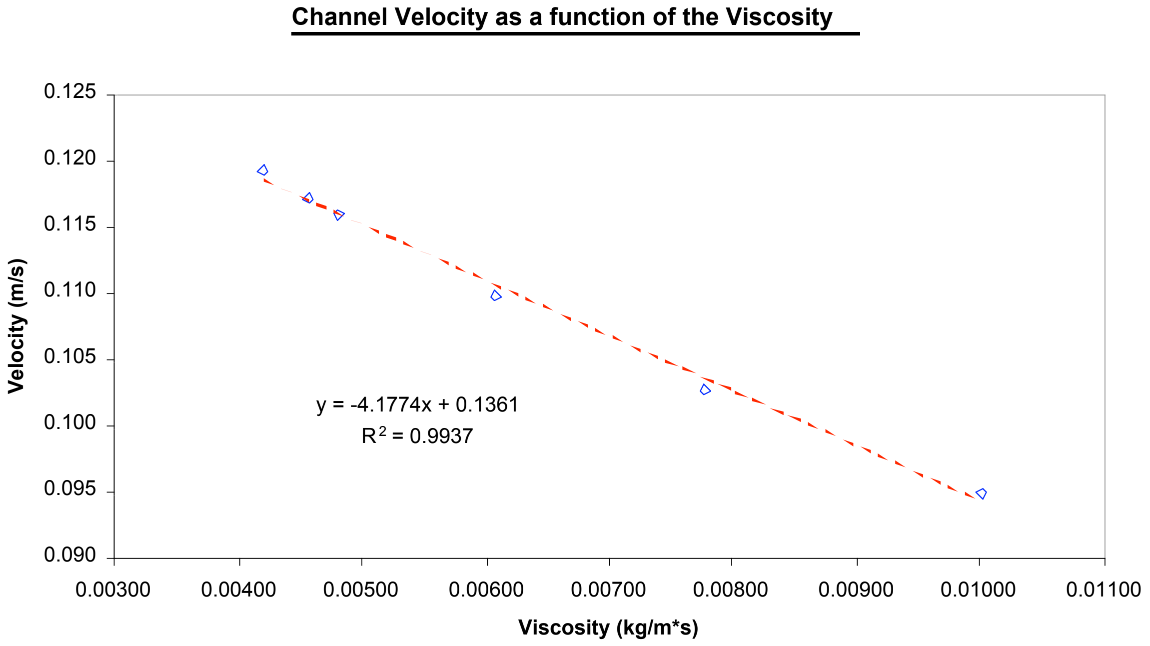

Figure 7: This plot demonstrates how a change in the viscosity affects the velocity through the channel.

To obtain the plot above, table 1 shows the bleed line flow rates for each viscosity that FEMLAB converged for. To ensure that the law of mass conservation was being withheld, the flow rates entering in the bleed line were checked with the flow rates exiting making sure that the values were equal.

Table 1: Results from FEMLAB for different viscosities

Viscosity (kg/m*s)	**$q_{bl, in}$ (m^3/s)**	**v_{bl} (m/s)**	**Re_{bl}**	**_P (Pa)**
0.00418	3.748E-07	0.119	57.04	108.98
0.00456	3.685E-07	0.117	51.46	116.78
0.00480	3.646E-07	0.116	48.37	121.63
0.00606	3.4526E-07	0.110	36.26	145.50
0.00775	3.229E-07	0.103	26.52	173.96
0.01	2.989E-07	0.095	19.03	207.78

Pressure values outputted from FEMLAB were checked with those using the Hagen-Poiseuille equation. Table 2 shows the results and the error associated with the FEMLAB's outputs. The velocity calculated using the equation 3 used the pressure drop value outputted by FEMLAB. This value is compared to the velocity associated with FEMLAB's output for the bleed line flow rate. The last column shows the % difference of the two values.

Table 2: Comparison of FEMLAB outputs

		FEMLAB Values		Hagen-Poiseuille		
Viscosity (kg/m*s)	**$q_{bl, in}$ (m^3/s)**	**v_{bl} (m/s)**	**_P (Pa)**	**v_{bl} (m/s)**	**_P (Pa)**	**Velocity Error**
0.00418	3.748E-07	0.119	125	0.137	108.98	14.70%
0.00456	3.685E-07	0.117	131	0.132	116.78	12.17%
0.00480	3.646E-07	0.116	134.3	0.128	121.63	10.42%
0.00606	3.4526E-07	0.110	151.25	0.114	145.50	3.95%
0.00775	3.229E-07	0.103	173	0.102	173.96	0.55%
0.01	2.989E-07	0.095	117.8	0.054	207.78	43.30%

[Comment by instructor: The values listed as FEMLAB values are obtained from the simulations. The values listed as Hagen-Poiseuille values are computed as follows. The velocity is computed using the Hagen-Poiseuille law and the FEMLAB pressure drop. The pressure drop is computed using the Hagen-Poiseuille law and the FEMLAB velocities. Thus, they are the inverse of each other. Of course, the Hagen-Poiseuille law ignores the contraction, bends, and expansion, and is thus a rough approximation. The total pressure drop would be greater than that indicated by the Hagen-Poiseuille law, which is verified in the table.]

CONCLUSION

From the results of the 2D axi-symmetric model analysis, it was established that the pressure drop generated by the venturi contraction is sufficient enough to cause liquid to flow through the channel. Moving on to the 3D model, the velocity values were calculated corresponding to the viscosities that FEMLAB converged for, but as shown in table 2, there is an error associated with each value that FEMLAB outputted. The objective of this study was accomplished for the specific dimensions; however, further studies may be conducted to fit specific applications. Figure 7 is provided to a correlate the fluid velocity through the bleed line for a range viscosities. Depending on the application, the venturi contraction with a bleed line device may be a feasible device for industrial use.