Phan-Thien-Tanner Modeling of a Viscoelastic Fluid in the Stick-Slip Scenario

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Introduction

Objectives

The goal of this research is to model the flow of an elastic polymer and find a correlation between the first normal stress difference and the pressure drop. The normal stress difference is very difficult to measure in practice, but very important in determining the viscoelastic properties of the fluid. However, pressure is very easy to measure, and if a correlation between the two can be determined, the pressure drop could then be used to determine these viscoelastic properties, more specifically: elasticity and Weissenberg number. Table 1 defines the terms mentioned above.

Table 1: Important Parameters

ε	Elasticity of the fluid, parameter in PTT model
$\Delta P/L$	Predicted drop in pressure for fully developed flow per unit length using the Phan-Thien-Tanner (PTT) model
$\Delta P_{excess1}$	The difference of the measured drop in pressure through the model and the drop in pressure that occurs when We = 0.
$\Delta P_{excess2}$	The difference of the measured drop in pressure through the model and the fully developed pressure drop in PTT
τ_{xx}	Shear stress at the inlet, also the normal stress at the inlet.
We	Weissenberg Number: $\lambda\langle u\rangle/y$. Where λ is the time constant that describes how fast the polymer "forgets" its shape, <u> is the average velocity, and y is half the height of the model.

Phan-Thien-Tanner Model

The PTT model models the viscoelastic flow of an elastic polymer. The defining equation of this model is Eq. 1, where $\underline{\underline{\tau}}$ is the stress tensor and $\underline{v}$ is the velocity vector.

$$\underline{\underline{\tau}}(1 + We \bullet \varepsilon(\tau_{xx} + \tau_{yy})) + We(\underline{v} \bullet \nabla\underline{\underline{\tau}} - \nabla\underline{v}^T \bullet \underline{\underline{\tau}} - \underline{\underline{\tau}} \bullet \nabla\underline{v}) = \nabla\underline{v} + \nabla\underline{v}^T \quad [1]$$

For this research, we non-dimensionalized every quantity; that with our boundary conditions (described in the next section) give us the following relations:

$$u = \frac{1}{2}\frac{\Delta P}{L}(1-y^2) + \frac{\varepsilon We^2}{2}\left(\frac{\Delta P}{L}\right)^3(1-y^4) \quad [2]$$

$$\tau_{xy} = -\frac{\Delta P}{L}y \quad [3]$$

$$\tau_{xx} = 2We\left(\frac{\Delta P}{L}y\right)^2 \qquad [4]$$

$$\tau_{yy} = \dot{\gamma}_{xx} = \dot{\gamma}_{yy} = 0 \qquad [5]$$

$$\dot{\gamma}_{xy} = \frac{\Delta P}{L}y + 2\varepsilon We^2\left(\frac{\Delta P}{L}y\right)^3 \qquad [6]$$

To use the correlations above, the term $\frac{\Delta P}{L}$ must be calculated at the given ε and We using Eq. 7.

$$1 = \frac{1}{3}\frac{\Delta P}{L} + \frac{2\varepsilon We^2}{5}\left(\frac{\Delta P}{L}\right)^3 \qquad [7]$$

The first normal stress difference, N, is calculated as $N = \tau_{xx} - \tau_{yy}$, but these stresses are measured at the inlet where $\tau_{yy} = 0$, so $N = \tau_{xx}$.

Femlab Model

The elastic polymer's fluid flow was modeled using Comsol's Femlab program. The PTT model's equations (Eqs. 2-6) and the boundary conditions depicted below in Fig. 1 were programmed into the Femlab for multiple We (0-5) at a constant ε = 0.02.

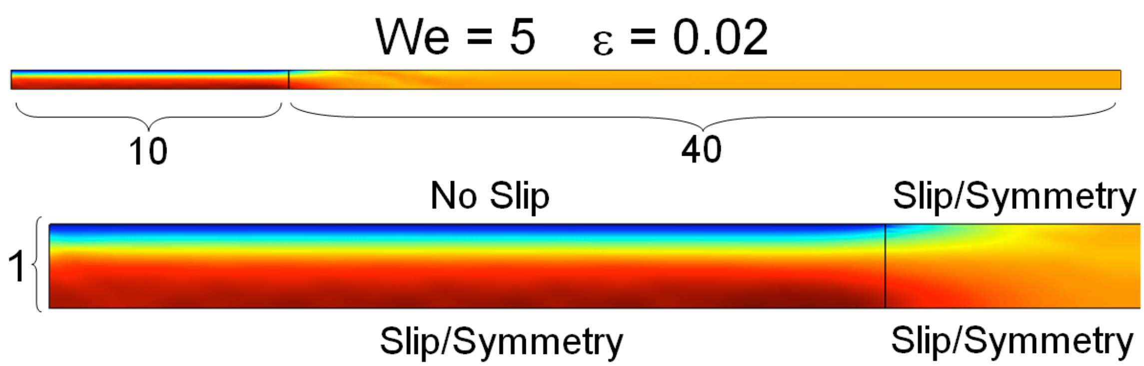

Figure 1: Velocity profile for a fluid with a We = 5 and an ε = 0.02. This is a portrayal of the stick-slip scenario, where after 10 units of length, the upper boundary becomes frictionless; this closely resembles the end of a pipe.

Other important conditions that were input into Femlab were an average velocity, <u>, at the inlet of 1, a kinematic viscosity, η, of 1, and a density, ρ, of 0. Two different mesh sizes were used; the high mesh had 5480 elements and 43,331 degrees of freedom, and the medium mesh had 2836 elements and 22,829 degrees of freedom.

Results

Table 2 summarizes the results for each different We; ΔP/L was calculated using Eq. 7, while τ_{xx} and ΔP were calculated by Femlab.

Table 2: Numerical results for the stick-slip scenario at different We.

Mesh	We	?	ΔP/L	τ_{xx}	ΔP	$\Delta P_{excess1}$	$\Delta P_{excess2}$
High	0	0.0200	3.00	0.00	30.86	0.00	-0.86
High	0.1	0.0200	2.99	1.79	31.31	0.44	-1.37
High	0.5	0.0200	2.86	8.18	31.97	1.10	-3.37
Medium	1	0.0200	2.59	13.37	31.02	0.15	-5.16
High	1	0.0200	2.59	13.37	30.90	0.04	-5.05
Medium	2	0.0200	2.10	17.72	27.70	-3.16	-6.65
Medium	3	0.0200	1.78	19.02	24.97	-5.90	-7.16
Medium	4	0.0200	1.56	19.35	22.91	-7.95	-7.36
Medium	5	0.0200	1.39	19.33	21.33	-9.54	-7.43

Three graphs were generated from this data. All three have two types of pressure drops. $\Delta P_{excess1}$ is the measured pressure drop at a given We minus the pressure drop at for the same scenario but with We = 0. $\Delta P_{excess2}$ is the measured pressure drop at a given We minus the pressure drop predicted by PTT (L times ΔP/L).

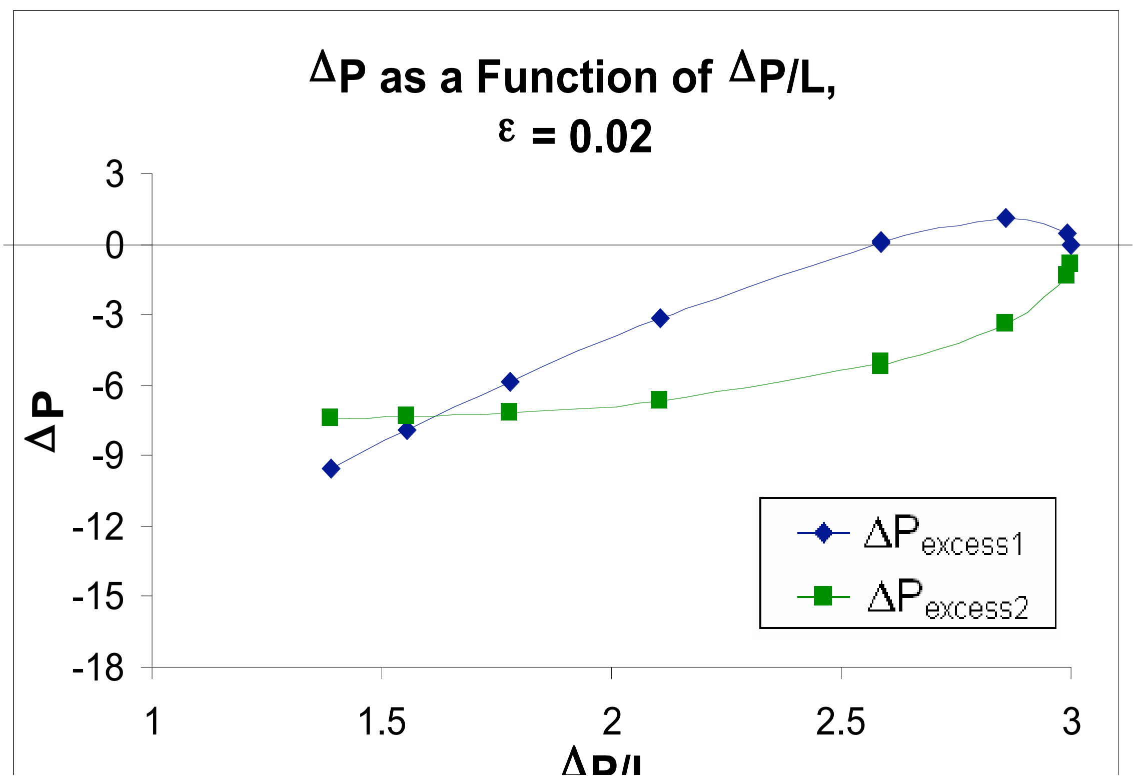

Figure 2: A plot of the pressure drop against the calculated $\frac{\Delta P}{L}$.

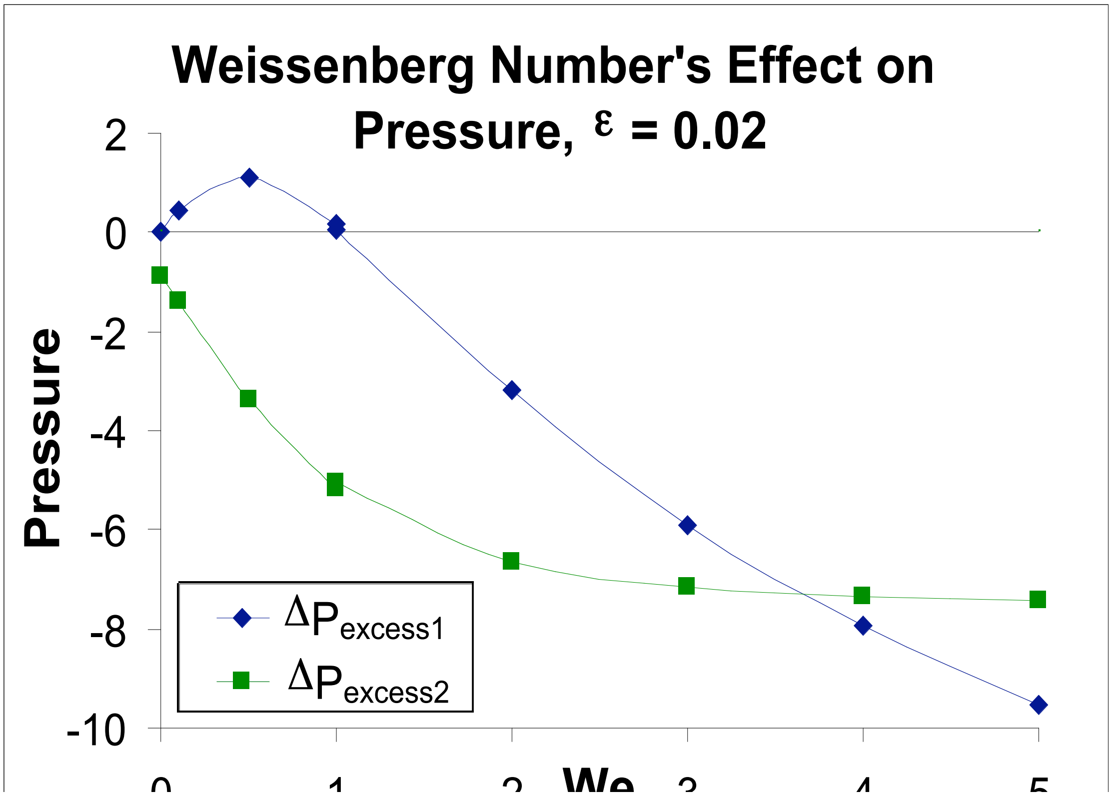

Figure 3: A plot of the pressure drop against We.

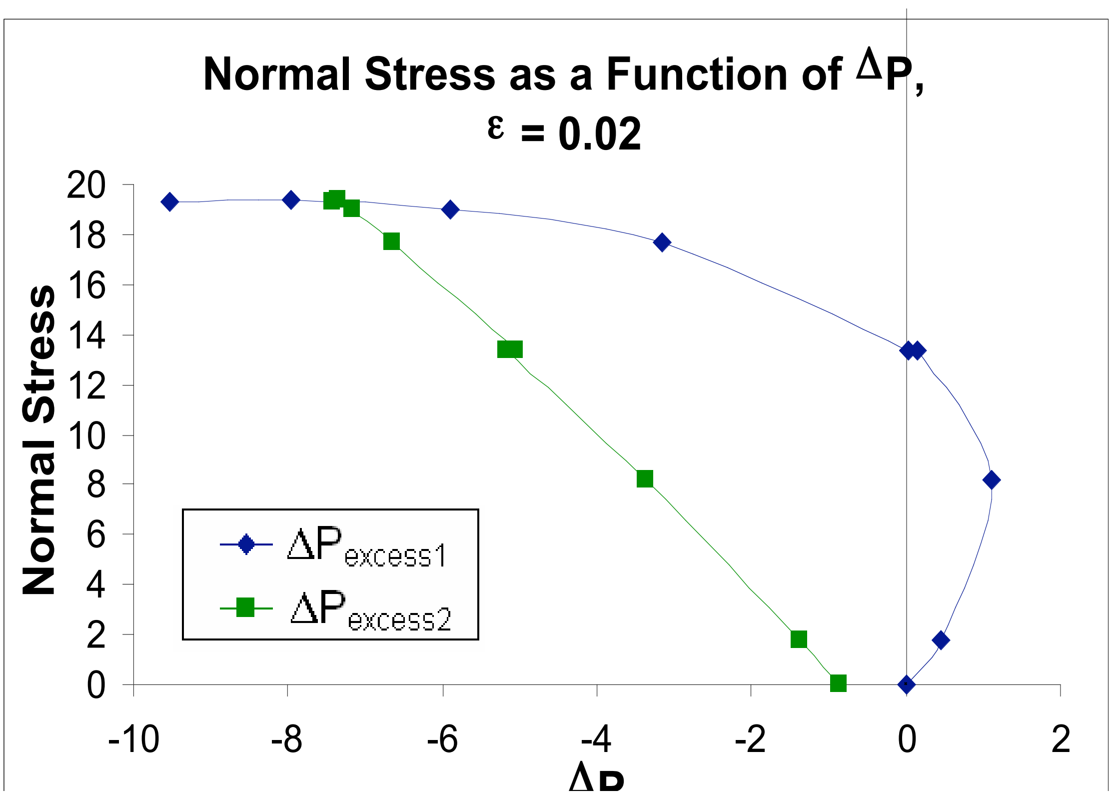

Figure 4: A plot of the normal stress (τ_{xx}) against the pressure drop.

Discussion

When the results in Table 2 were found, a couple things were unexpected. The most obvious being that both $\Delta P_{excess1}$ and $\Delta P_{excess2}$ are negative as the We goes up. As We gets larger a swelling as the fluid exits the outlet of a pipe can be seen to increase as well. This would imply positive values for $\Delta P_{excess1}$ and $\Delta P_{excess2}$. However, we have non-demonsionalized the pressure and in doing so have given it an inverse relationship to <u>. As We increases, the <u> also increases ($We = \lambda\langle u\rangle/y$), but as the <u> increases, the pressure decreases, and this decrease is strong enough to overcome any increase in pressure that may have occurred due to swelling.

The other odd result was for $\Delta P_{excess1}$. The $\Delta P_{excess1}$ for We=1 was positive. Further tests were done for We < 1 with a higher mesh to flush out this anomaly, but it only showed that we have found a ‘hump’ as seen in Figs. 2-4. This ‘hump’ is an unknown anomaly. It could be a problem with Felmlab, it could be a problem with the PTT model, or it could be the way it is; there is no clear answer at this time.

Despite these inconsistencies, Figs. 2-4 can be used to determine the normal stress, We, and ε, assuming that these three graphs are made many more times for many different ε, and that a physical model that relates well to the stick-slip scenario must be used in the experimentation.

Steps to Determine the Properties of a Viscoelastic Fluid Using Figures 2-4

1. The experiment run with a Newtonian liquid (We = 0) and the pressure drop recorded.
2. The experiment run with the fluid of interest and the pressure drop recorded.
3. Subtract the pressure drop in step 1 from the pressure drop in step 2., this is $\Delta P_{excess1.}$
4. Use Fig. 2 to check the elasticity.
    a. Find the for ΔP/L for the for $\Delta P_{excess1}$
    b. Multiply the for ΔP/L by the length of the pipe and subtract that from the pressure drop in step 2., this is $\Delta P_{excess2.}$
    c. The $\Delta P_{excess2}$ found in step 4b. should have the same ΔP/L as found in step 4a.
    d. If they match move on to step 5., if not, try a different ε and repeat this step.
5. Once the elasticity is found, use Fig. 3 to find the We. Check the result using Eq. 7.
6. Use Fig. 4 to determine the normal stress.

Conclusions and Recommendations

This research did not produce a perfect result; however, it did prove that this method could be used in industry to determine the properties of a viscoelastic fluid including the first normal stress. The ‘hump’ should be further researched to determine its origin.

Many things need to be done before this method can be used. First and foremost, the results must be checked using a fluid with known properties. Before that can really be done, a scenario that can be better produced physically must be used, such as a 4:1:4 contraction. Then this model must be tested extensively for many different elasticities; the more elasticities it is tested for, the wider the range that this model can predict. Figures 2-4 must be reproduced for each elasticity tested, and more figures like Fig. 4 could be produced for quantities like the shear rate, the time constant, or the shear stress. It is easy to foresee the process for determining these properties being done by a computer with a database full of the correlative data. This would make the iterative process for determining the elasticity mush faster.