Mixing and Pressure Drop in Serpentine Mixers

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Introduction

The purpose of this research is to determine the affects of two entering fluids mixing in the serpentine mixer at different conditions. Three cases were considered: 1) varying Reynold number 2) varying Peclet number and 3) changing mixer length. By using a serpentine mixer model, one can estimate the mixing effect depending on its condition.

Background

A good way to predict the concentration behavior is by looking at the Reynold and Peclet number.

$$Re = \frac{\rho D u_s}{\mu} \quad \text{(Eq. 1)}$$

Reynold number is the ratio of initial and viscous forces where ρ is density, D is diameter, u_s is average velocity, and μ is viscosity.

$$Pe = \mathrm{Re} \times Sc \quad \text{(Eq. 2)}$$

Peclet number is the multiple value of Reynold and Schmidt number, where

$$Sc = \frac{\nu}{D} \quad \text{(Eq. 3)}$$

defines the ratio of momentum and diffusivity, ν is the kinematic velocity, and D is the mass diffusivity.

To determine how well the concentrations were mixed, the mixing cup and variance equations were used, respectively:

$$c_{mixingcup} = \frac{\int_A c(x,y,z)v(x,y)dxdy}{\int_A v(x,y)dxdy} \quad \text{(Eq. 4)}$$

$$c_{\mathrm{var}iance} = \frac{\int_A \left[c(x,y,z) - c_{mixingcup}\right] v(x,y)dxdy}{\int_A v(x,y)dxdy} \quad \text{(Eq. 5)}$$

Where the mixing cup is defined as the concentration of fluid if the flow emptied to a cup that was well stirred, and variance is the overall change of concentration.

Methods

- **Calculating Variance Values**

The serpentine mixer model used to research this topic was made previously by another research student of Professor Finlayson. To apply for this research, minor modifications were made. In order to become familiarized with this previously made model, values were calculated to agree with literature values[1]. These values are shown in Table 1.

Table 1. Literature value for serpentine mixer

Re	Pe	Variance	Equivalent length
0.25	250	5.31e-8	
0.5	500	1.56e-5	225
1.0	1000	2.60e-4	250
2	2000	1.12e-3	400
4	4000	2.1e-3	700
8	8000	3.03e-2	600

Where equivalent length = length of straight channel of same dimension that gives same variance

In COMSOL, after solving the model for Re = 1, Pe = 1000, 14,912 no. degrees of freedon, and 8418 elements, the Boundary Integration under Postprocessing was used. The outlet boundary was chosen and entered “v” under Expression to get the value of velocity in y-direction. See Figure 1.

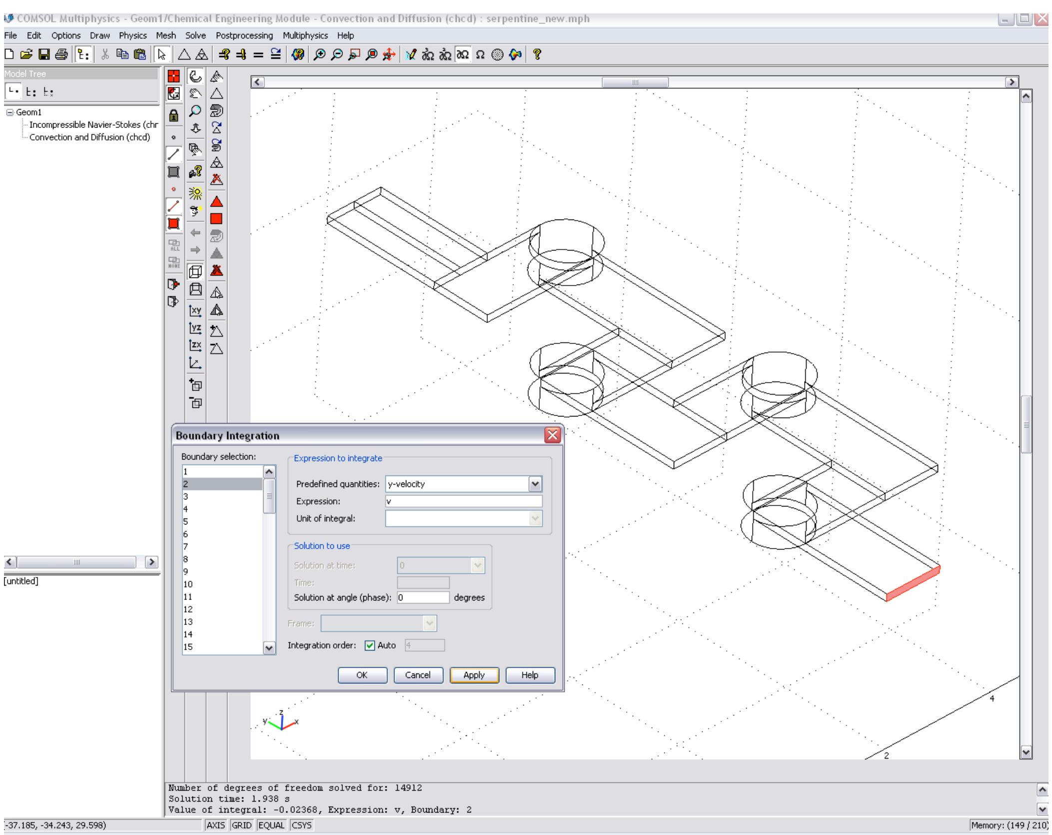

Figure 1. Example simulation of problem with Boundary Integration windows opened

Using 0.02368 as velocity value, enter “c*v/0.02368” under Expressions, where the value will be 0.489585. This value is the mixing cup concentration. To find the concentration variance, under Expression enter: (c-0.489585)^2*v/0.02368, where the final value will be 1.840207e-4. Comparing this final value to the Table 1 value, we conclude that they are not equal. This difference may be caused by the mesh in the model. Once the mesh statistic was increased, the concentration variance was calculated to equal 2.062023e-4, very close to the literature value.

- **Changing Variables**

To determine final values, model was used with mesh statistic of no. DOF solved = 42558, and elements = 25917.

Three relationships are in interest to determine the effect of the parameters: 1) Re vs. ΔP’, 2) Re vs. concentration variance, and 3) vs. concentration variance.

By definition, inlet pressure is

$$P_{avg} = \frac{\int p(x,y)dxdy}{\int dxdy} \text{ (Eq. 6),}$$

outlet pressure is

$$P_{avg} = 0 \text{ (Eq. 7),}$$

and ΔP’ is the non-dimensionalized pressure difference. Pressure drop is:

$$p_s = \frac{\eta u_s}{x_s} \text{ (Eq. 8)}$$

where η = viscosity, = average velocity, = total length of path. residence time, or

$$t_s = \frac{x_s}{u_s} \text{ (Eq. 9)}$$

Average velocity and path length are defined as

$$u_s = \frac{\eta}{\rho x_s} Re \text{ (Eq. 10)}$$

$$x_s = l_s \times w_s \text{ (Eq. 11)}$$

where l_s= dimensionless path length and w_s = 800 μ = dimensionless path width.

Because the medium concentration will be water, η = 0.001 and ρ = 1000. Therefore the average velocity becomes

$$u_s = \frac{1}{800}\text{Re} \quad \text{(Eq. 12)}$$

Four cases were created with different length of the serpentine mixer. The following four figures represent these changes:

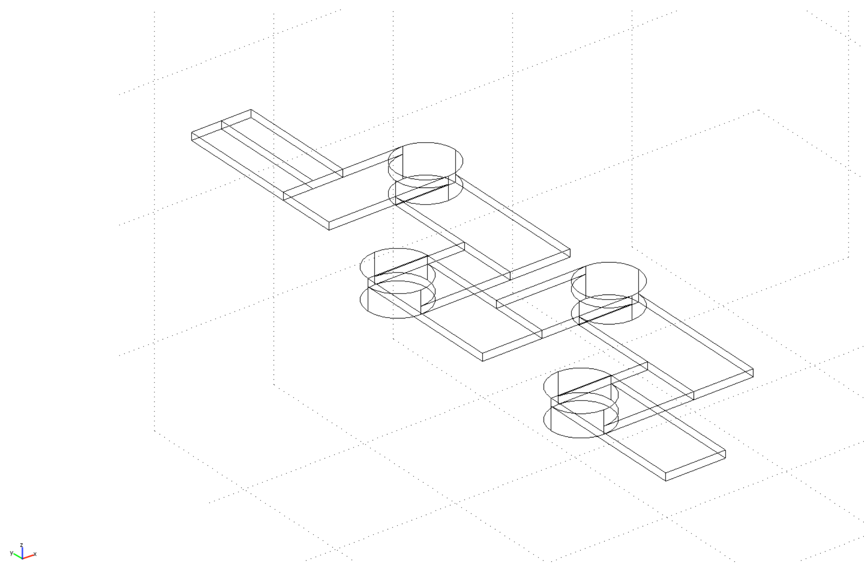

Figure 2. Case 1 with path length l_s = 20.95

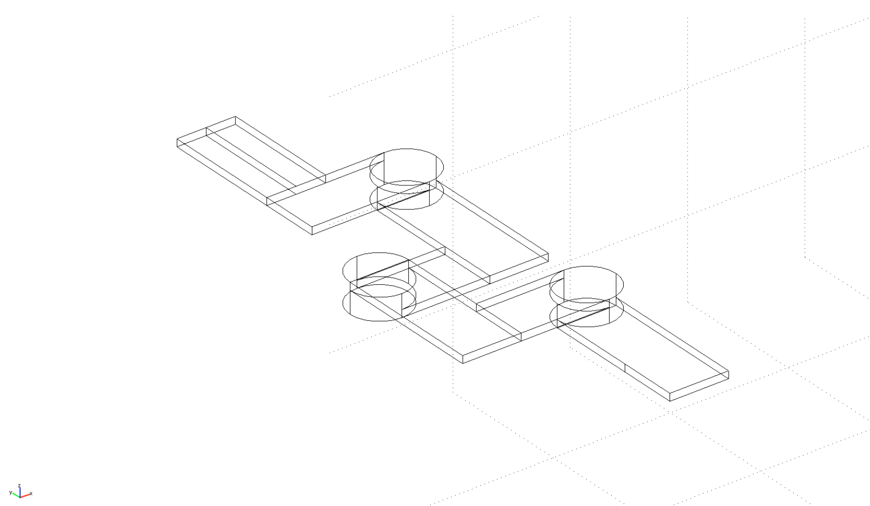

Figure 3. Case 2 with path length l_s = 13.125

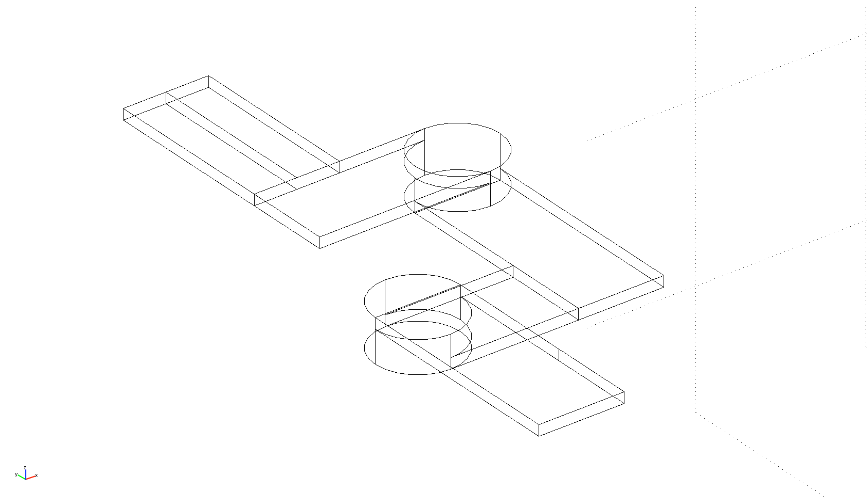

Figure 4. Case 3 with path length $l_s = 9.75$

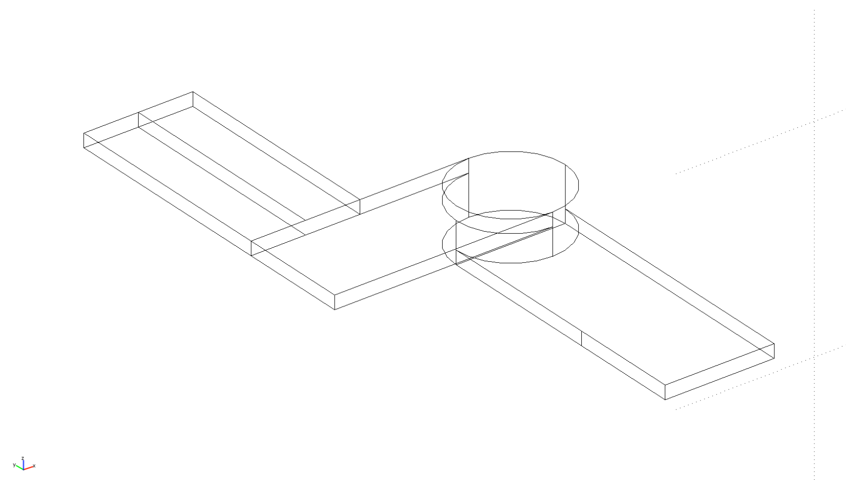

Figure 5. Case 4 with path length $l_s = 5.875$

See Sample Calculations section for path length calculations.

Results and Discussions

To find the relationships of 1) Re vs. ΔP', 2) Re vs. concentration variance, and 3) vs. concentration variance, four tables were created with respect to each case.

Table 2. Values for Case 1

Re	Pe	Variance	u_s	t_s	Ptot
0.25	250	1.65E-08	0.000313	53.632	1.86456E-05
0.5	500	7.75E-06	0.000625	26.816	3.72912E-05
1	1000	2.06E-04	0.00125	13.408	7.45823E-05
2	2000	0.001146	0.0025	6.704	0.000149165
4	4000	0.003155	0.005	3.352	0.000298329
8	8000	0.003042	0.01	1.676	0.000596659

Table 3. Values for Case 2

Re	Pe	Variance	u_s	t_s	Ptot
0.25	250	2.96E-05	0.000313	33.6	2.97619E-05
0.5	500	3.19E-04	0.000625	16.8	5.95238E-05
1	1000	8.58E-04	0.00125	8.4	0.000119048
2	2000	0.001041	0.0025	4.2	0.000238095
4	4000	0.001266	0.005	2.1	0.00047619
8	8000	0.004472	0.01	1.05	0.000952381

Table 4. Values for Case 3

Re	Pe	Variance	u_s	t_s	Ptot
0.25	250	3.22E-04	0.000313	24.96	4.00641E-05
0.5	500	0.001905	0.000625	12.48	8.01282E-05
1	1000	0.004655	0.00125	6.24	0.000160256
2	2000	0.007596	0.0025	3.12	0.000320513
4	4000	0.012324	0.005	1.56	0.000641026
8	8000	0.021642	0.01	0.78	0.001282051

Table 5. Values for Case 4

Re	Pe	Variance	u_s	t_s	Ptot
0.25	250	6.01E-05	0.000313	15.04	6.64894E-05
0.5	500	0.009071	0.000625	7.52	0.000132979
1	1000	0.020415	0.00125	3.76	0.000265957
2	2000	0.033661	0.0025	1.88	0.000531915
4	4000	0.047293	0.005	0.94	0.00106383
8	8000	0.058609	0.01	0.47	0.00212766

The following three figures will show the trend of the serpentine mixer when changing one variable with respect to another variable.

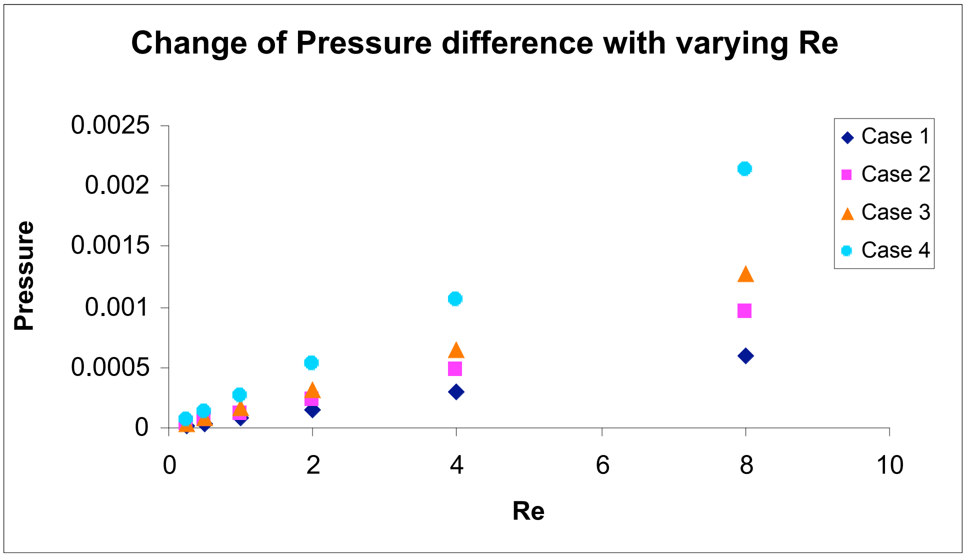

Figure 6. Change of pressure with varying Re number

Figure 6 represents the difference of pressure as the length of the mixer changes. The pressure increases when increasing Re values for all four cases. This is because when the Re number is increased, the velocity of the fluid also is affected. As the velocity is increased, the pressure becomes greater due to the addition of force induced. Pressure is more affected by the shorter serpentine mixer because to achieve the same average velocity as the longest serpentine mixer additional pressure must be forced.

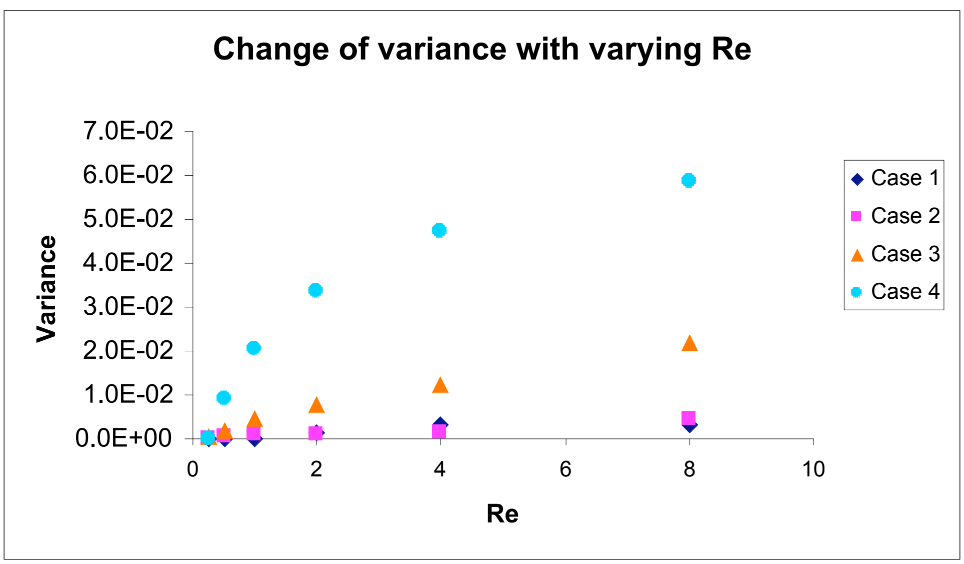

Figure 7. Affect of concentration variance for different mixer length

The next figure represents the change of concentration variance as Re number is increased for each case. All four cases follow the same trend of increasing variance as Re increases. This is also due to the increased fluid velocity, affecting the residence time in the mixer. This trend can also be seen in Figure 8. This variance is most greatly affected in Case 4 because it has a disadvantage of having a shorter length of mixer with constant average velocity as seen in Re = 8.

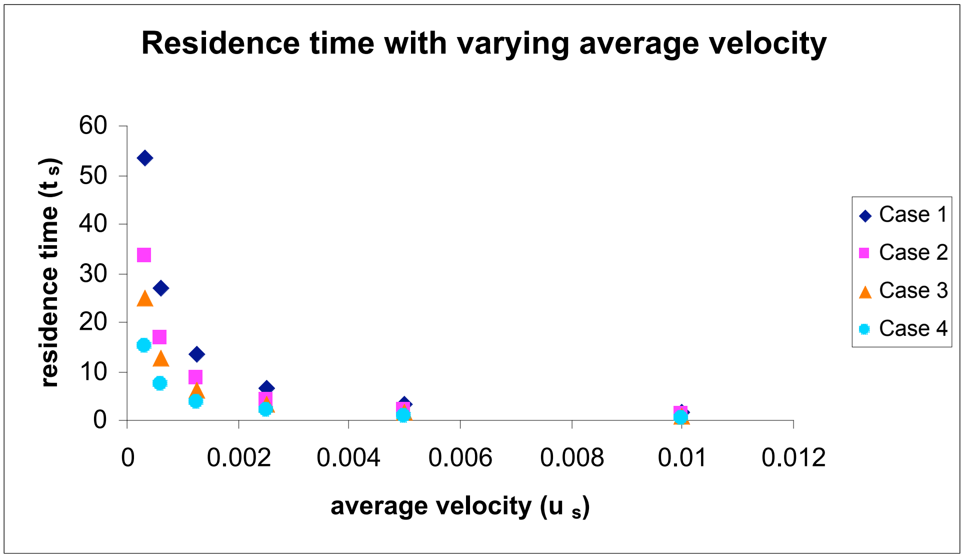

Figure 9. Residence time of all four cases

As seen in Figure 9, the residence time of all four cases decrease because the average velocity is increased. At same average velocity, Case 1 has a longer residence time than Case 4 due to the length of the mixer.

One way to check for the accuracy of COMSOL model, concentration mixing was tracked for Case 2 with different Re and Pe numbers. The following figures show these changes.

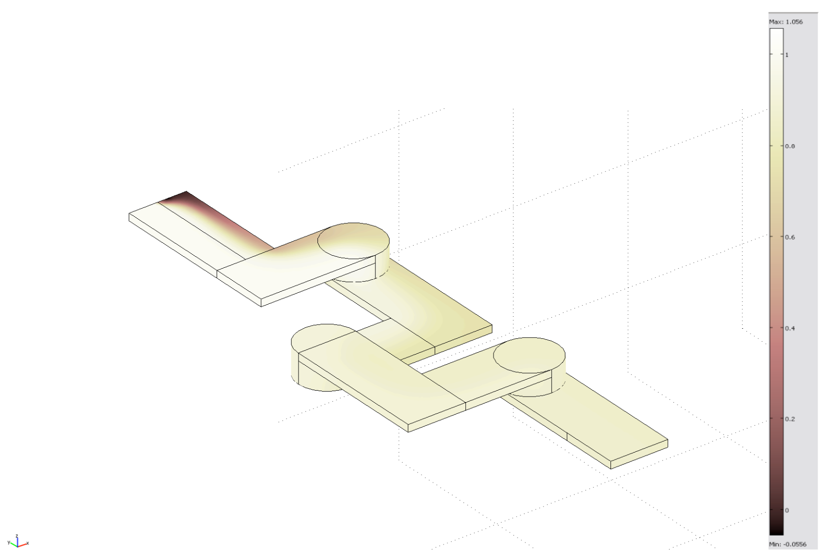

Figure 10. Re = 0.25, Pe = 250

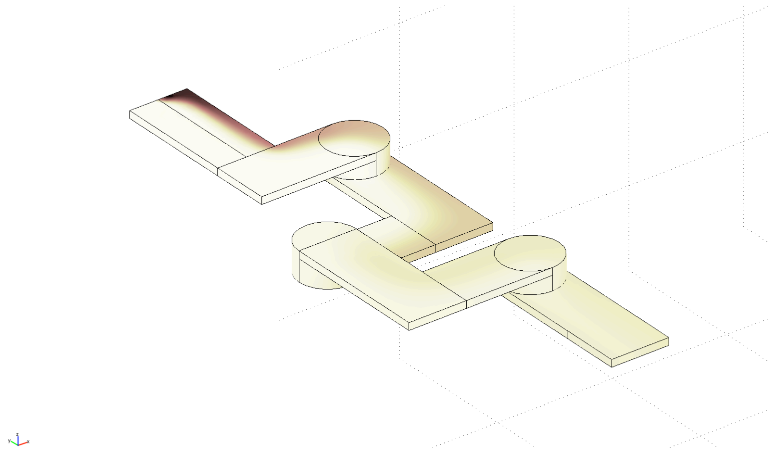

Figure 11. Re = 0.5, Pe = 500

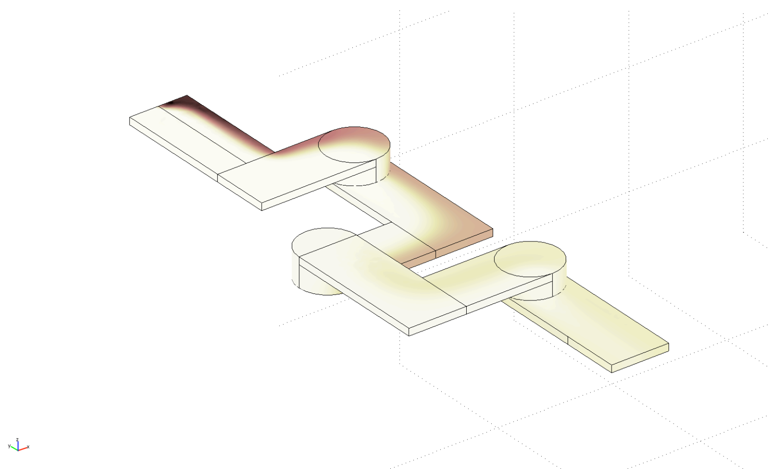

Figure 12. Re = 1, Pe = 1000

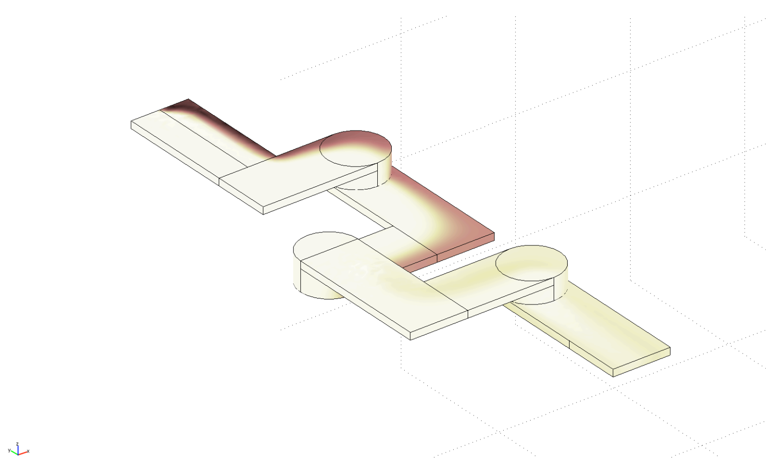

Figure 13. Re =2, Pe = 2000

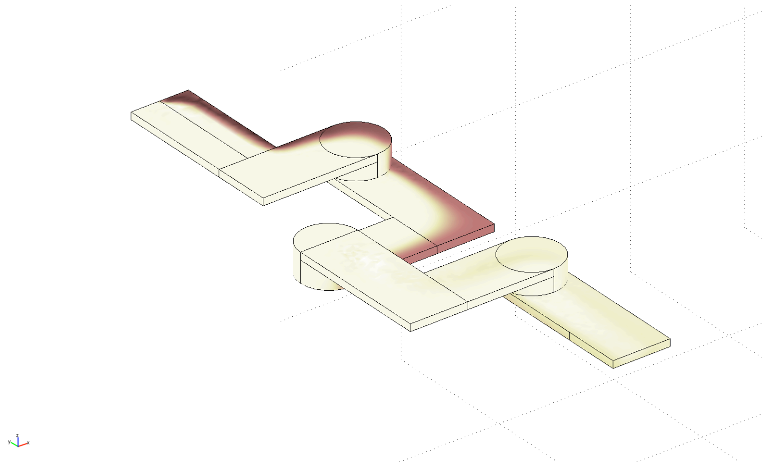

Figure 14. Re = 4, Pe = 4000

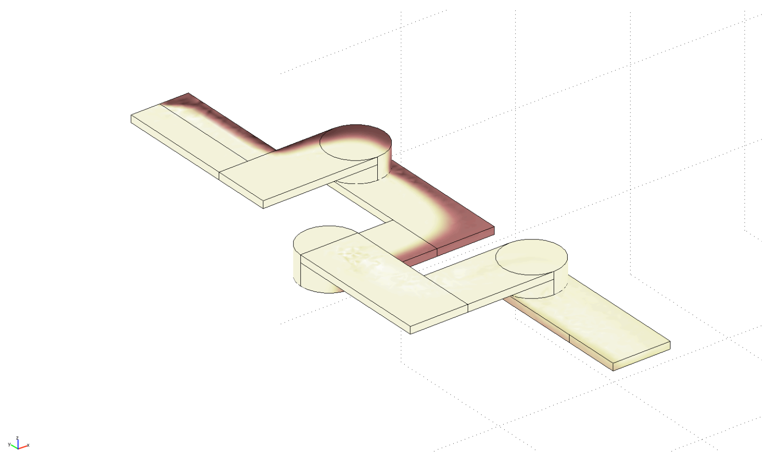

Figure 15. Re = 8, Pe = 8000

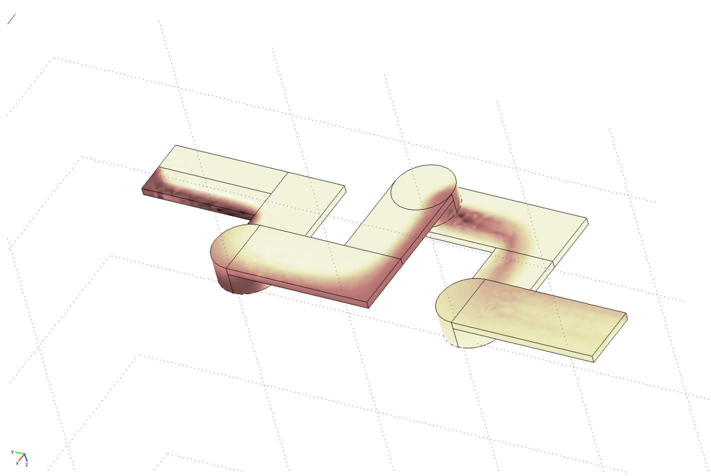

Figure 16. Re = 8, Pe = 8000, back side

At the entrance of the mixer the brown location represents water, or concentration = 0. In Figure 10 one notices the even blend of color achieved earlier in the mixer. This is due to the low velocity of fluid and increased residence time to allow well mixing. As the figures progress the brown location increases throughout the mixer, resulting in greater concentration variance and poor mixing.

Conclusion

All figures and calculated table values agree with expected results. To find an optimal Re, Pe, or length of mixer one must look at the concentration one wants to mix. Depending on its property, choose a case where the residence time will be long enough to complete the mixing and concentration variance small.

Recommendations

Another check for this model is to calculate the cross sectional velocity and concentration at exit point of Case 4 and same location of Case 1. Compare these values to see if they agree. If this is the case, then the model is working correctly.

References

[1] Koch, M. V., VandenBussche, K. M., & Chrisman, R. W. Micro Instrumentation. Weinheim: WILEY-VCH. 2007

Sample Calculations

- **Determining Mesh Statistics:**

For Re = 1, Pe = 1000

1) Mesh Statistics Used:
No. DOF = 14912, Elements = 8418, solution time: 2.062 s
Boundary Integration
y-velocity (v)
0.02368
c*v/0.02368
0.489585
(c-0.489585)^2*v/0.02368
1.840207e-4

1.840207e-4 ≠ 2.60e-4. Try again with smaller mesh

2) New Mesh Statistics:
No. DOF solved = 42558, Elements = 25917, solution time: 7.5 s
Boundary Integration
y-velocity (v)
0.02368
c*v/0.02368
0.490789
(c-0.490789)^2*v/0.02368
2.062023e-4

Continue to refine mesh to agree with literature value

3) New Mesh Statistics:
No. DOF solved = 123746, Elements = 80267, solution time = 70.141s
Boundary Integration
y-velocity (v)
0.02368
c*v/0.02368
0.491505
(c-0.491505)^2*v/0.02368
1.996474e-4

Variance decreases, use mesh statistics #2 because most closest to literature value

- **Calculating Path Length of Four Cases**

Values were obtained by looking at model and Draw Option in COMSOL

Case 1
Length = 10
Width = 2+2+2+1.75 = 7.75
Height = .8*4 = 3.2
l_s = 20.95

Case 2
Mesh Statistics:
No. DOF = 135212, Elements = 19710
Length + width = 2+2+2+2+2+2=12
Height = 0.375*3 = 1.125
l_s = 13.125

Case 3
Mesh Statistics:
No. DOF = 268476, Elements = 41846
Length + width = 9
Height = 0.375*2 = 0.75
l_s = 9.75

Case 4
Mesh Statistics:
No. DOF = 62265, Elements = 9067
Length + width = 5.5
Height = 0.375*1 = 0.375
l_s = 5.875

- **Results for Table 2**

$$x_s = l_s \bullet 800 \times 10^{-6}$$

$$x_s = 20.95 \bullet 800 \times 10^{-6} = 0.01676$$

$$u_s = \frac{1}{800}\text{Re} = \frac{1}{800}(0.25) = 3.125 \times 10^{-4}$$

$$t_s = \frac{x_s}{u_s} = \frac{0.01676}{3.125 \times 10^{-4}} = 53.632$$

$$p_s = \frac{\eta u_s}{x_s} = \frac{0.001 \times (3.125 \times 10^{-4})}{0.01676} = 1.865 \times 10^{-5}$$