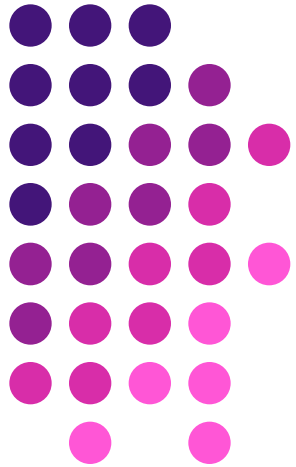
Pressure Drop of Gases in Contractions and Expansions in Microchannels

Kaarin Dawes

ChemE 499

Spring 2003



Objectives

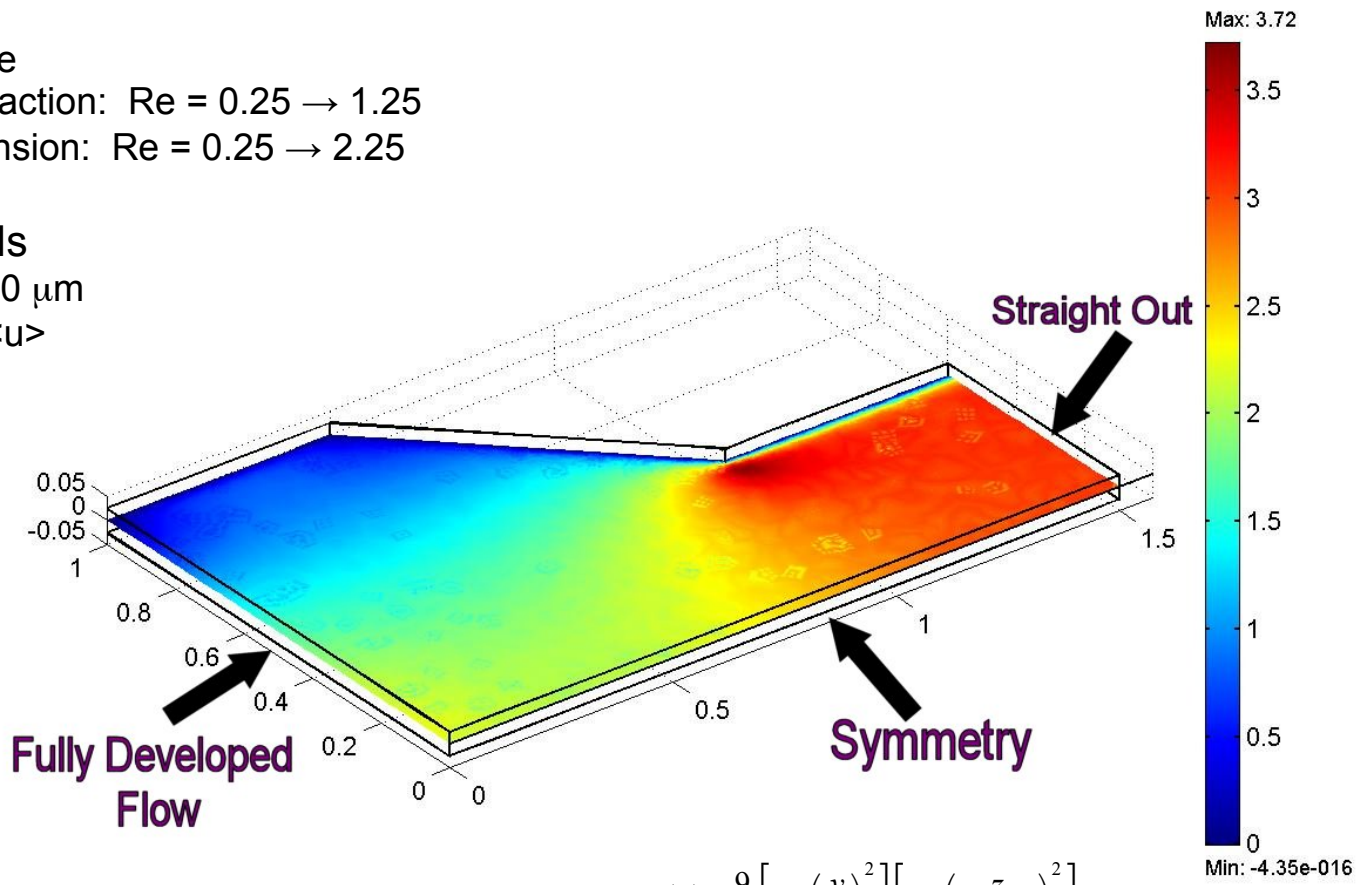
- Determine pressure drop due to an expansion or contraction
- Determine equivalent length
- Compare results to LWZ paper

Background

- Solve Navier-Stokes equations in 3-D
  - $\rho u \bullet \nabla u = -\nabla p + \mu \nabla^2 u$
- Check Perry's equation for incompressible flow
  - $Q = \dfrac{WH^3 \Delta P}{K \mu L}$
- Compute results using LWZ equation
  - $\dot{m} = \dfrac{H^3 W P_o^2}{24 RTL\mu} \left[\left(\dfrac{P_i}{P_o} \right)^2 - 1 + 12 Kn_o \left(\dfrac{P_i}{P_o} - 1 \right) \right]$

Geometry (45°)

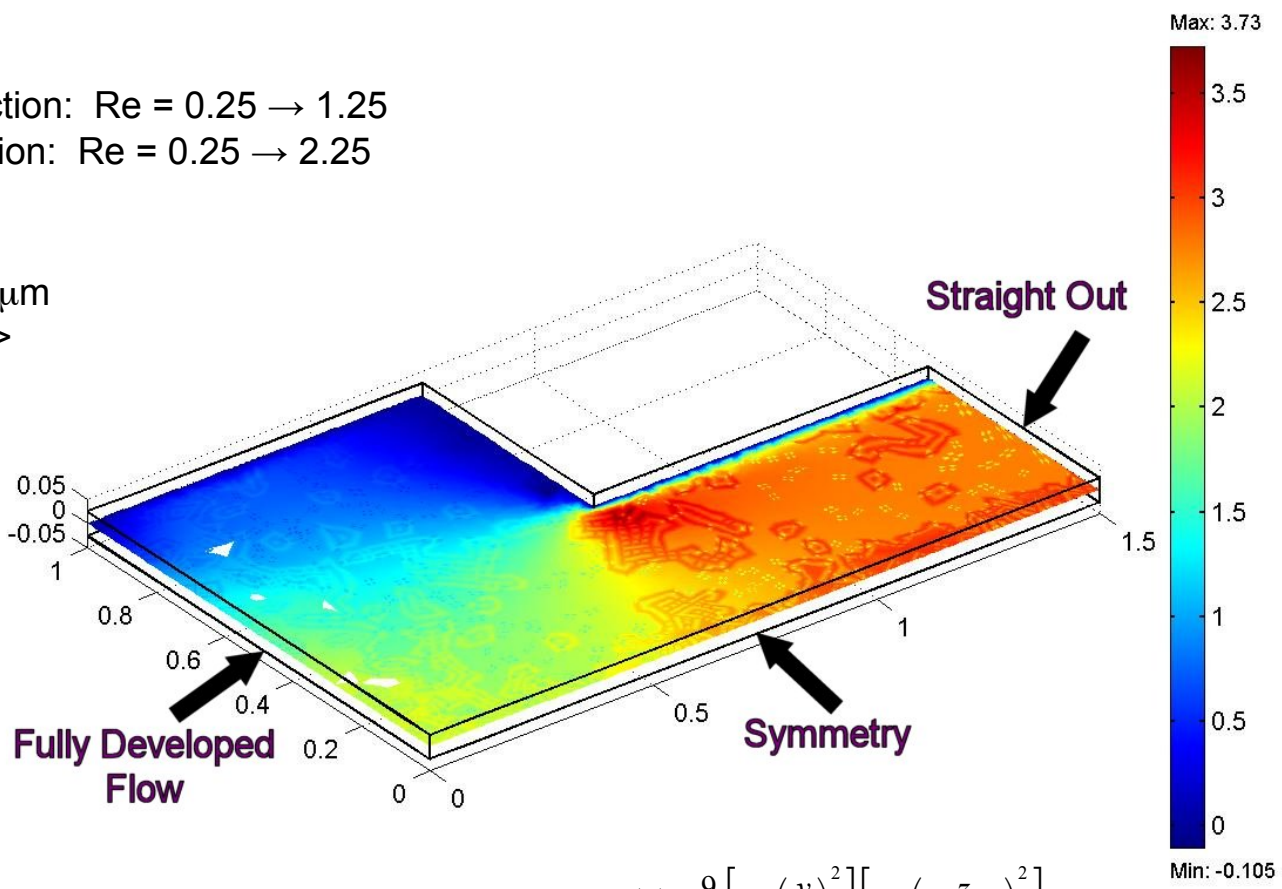
- Subdomain
  - $\mu = 1$
  - $\rho = Re$
  - Contraction: Re = 0.25 → 1.25
  - Expansion: Re = 0.25 → 2.25
- Standards
  - $x_s = 20\ \mu m$
  - $u_s = \langle u \rangle$



Inlet Velocity Profile: $u(x) = \frac{9}{4}\left[1-\left(\frac{y}{1}\right)^2\right]\left[1-\left(\frac{z}{0.025}\right)^2\right]$

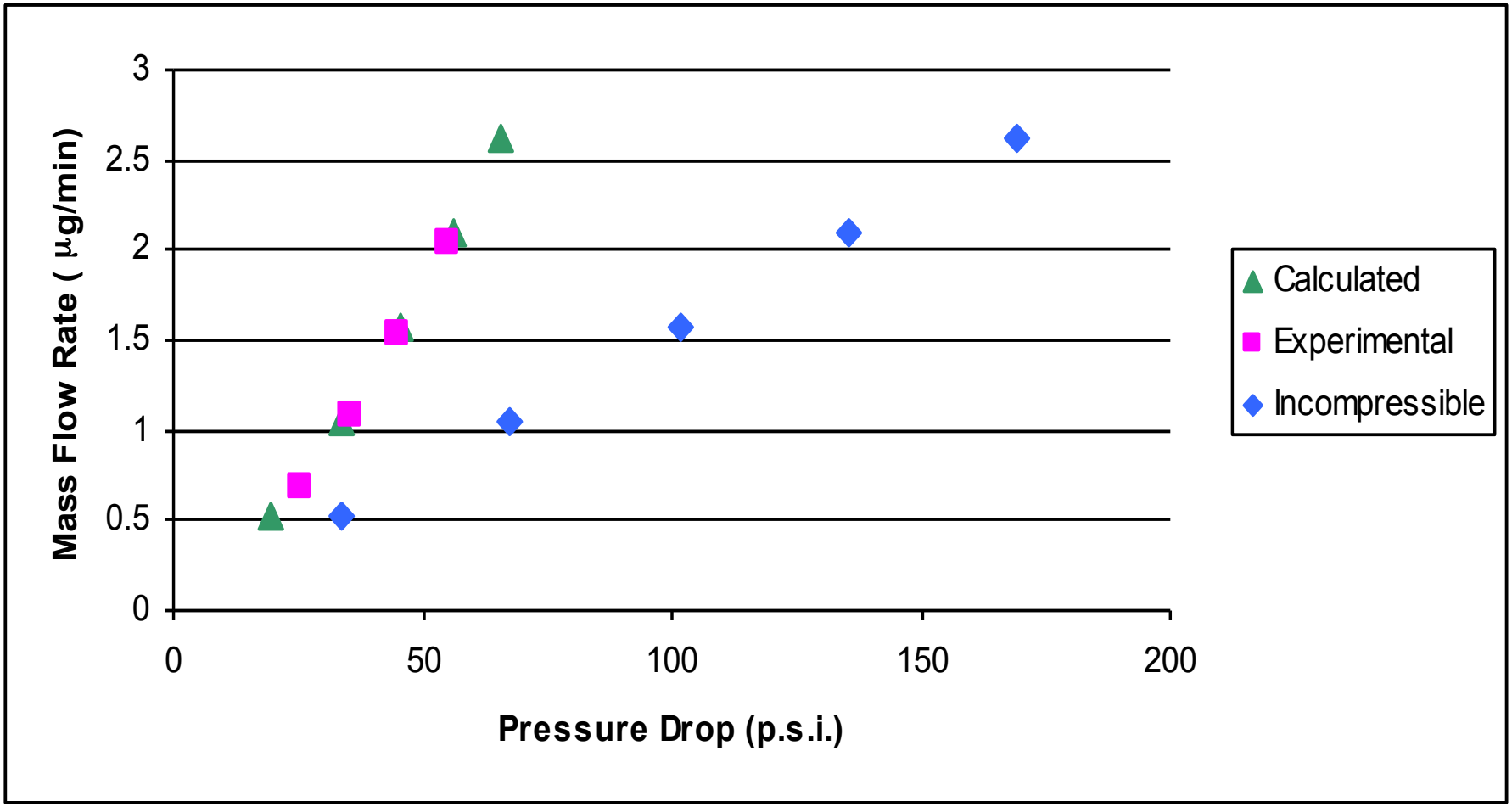
Geometry (90°)

- Subdomain
  - $\mu = 1$
  - $\rho = Re$
  - Contraction: Re = 0.25 → 1.25
  - Expansion: Re = 0.25 → 2.25
- Standards
  - $x_s = 20\ \mu m$
  - $u_s = \langle u \rangle$

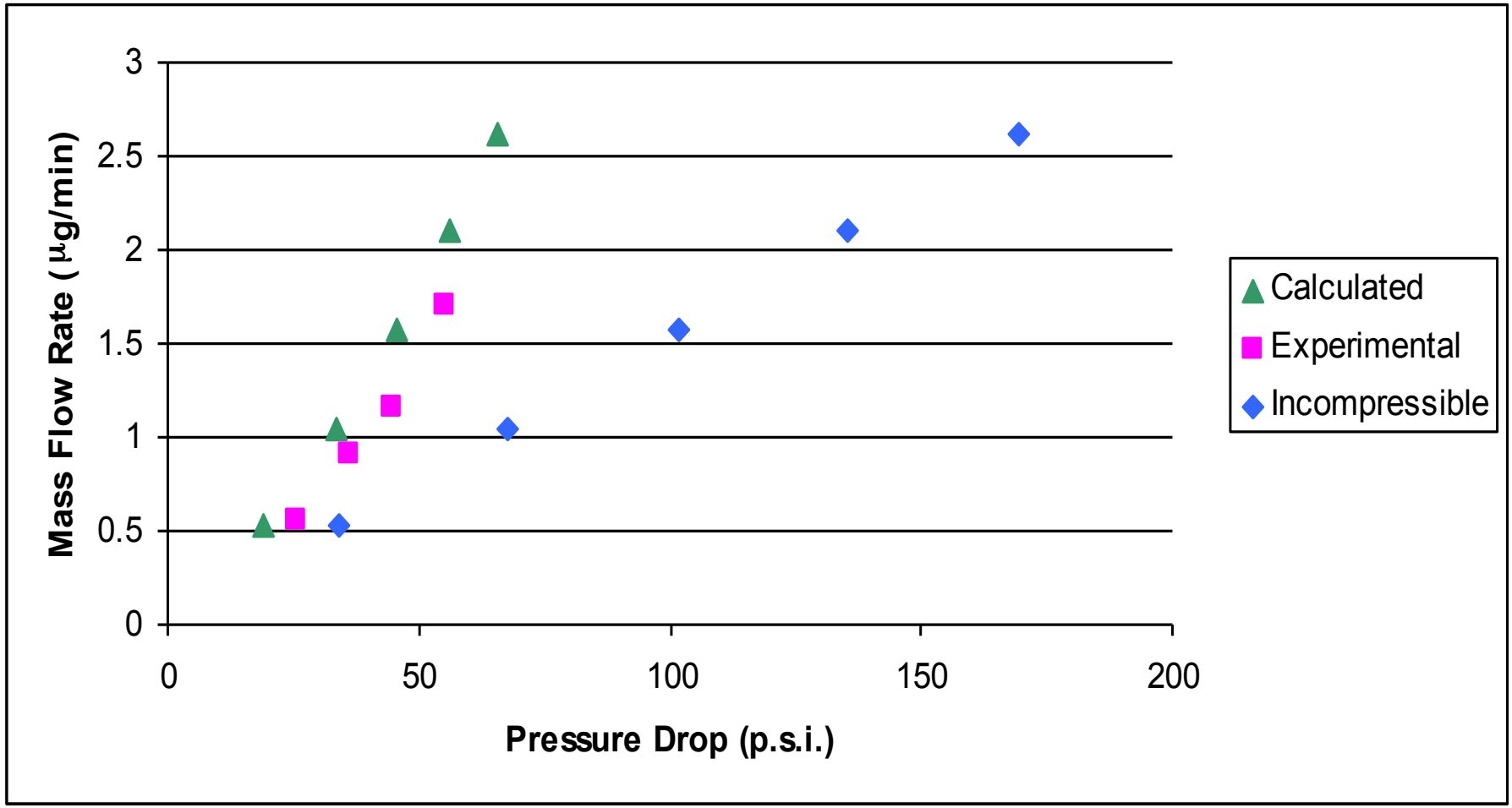


Inlet Velocity Profile: $u(x) = \frac{9}{4}\left[1-\left(\frac{y}{1}\right)^2\right]\left[1-\left(\frac{z}{0.025}\right)^2\right]$

45° Contraction



90° Contraction



Contribution to Total Pressure Drop

Flow Rate (μg/ min)	0.525	1.05	1.575	2.10	2.625
ΔP_{tr} (psi)	0.2857	0.5714	0.8571	1.1428	1.4286
Percentage of Total Pressure Drop	1.48%	1.70%	1.88%	2.03%	2.17%

Data from 45° contraction, slightly larger for 90° contraction

Equivalent Length

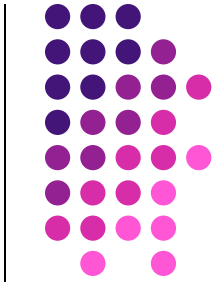
Angle	45° Contraction	45° Expansion	90° Contraction	90° Expansion
Calc'd Length (μm)	54.3	55.5	58.0	59.7
LWZ Length (μm)	500	650	1100	1200

Mesh Comparison

Mesh	Extremely Coarse	Coarser	Normal	Finer
ΔP_{tr} (psi)	0.2857	0.2838	0.2837	0.2836
% difference	--	-0.65	-0.70	-0.73
ΔP_{t} (psi)	19.267	19.265	19.265	19.265
% difference	--	-0.01	-0.01	-0.01
L_e (μm)	53.5	53.2	53.1	53.1
% difference	--	-0.65	-0.70	-0.73

Data from 45° contraction

Conclusions



- Flow cannot be modeled as incompressible
- Pressure drop due to expansion/contraction is very small part of total pressure drop
- LWZ equivalent lengths seem exaggerated