Pressure Drop Across Orifices in Microchannels

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The purpose of doing this paper is to show that Navier-Stokes equation is still able to predict the effect of the thickness of the orifice with orifice diameter as small as 8 microns.

Introduction:

A controversy was found in the literature between Hasegawa[1] and Dagan[2] papers. In Hasegawa paper, measurements of the pressure drop across orifices for laminar flow were done in an aperture diameter as small as 8 microns. They report that the Navier-Stokes equation cannot predict their results, which show an effect of the thickness of the orifice plate. However, Dagan paper mentioned that there is an analytical solution of the Navier-Stokes equation that shows the thickness of the orifice plate is important at very small Reynolds number.

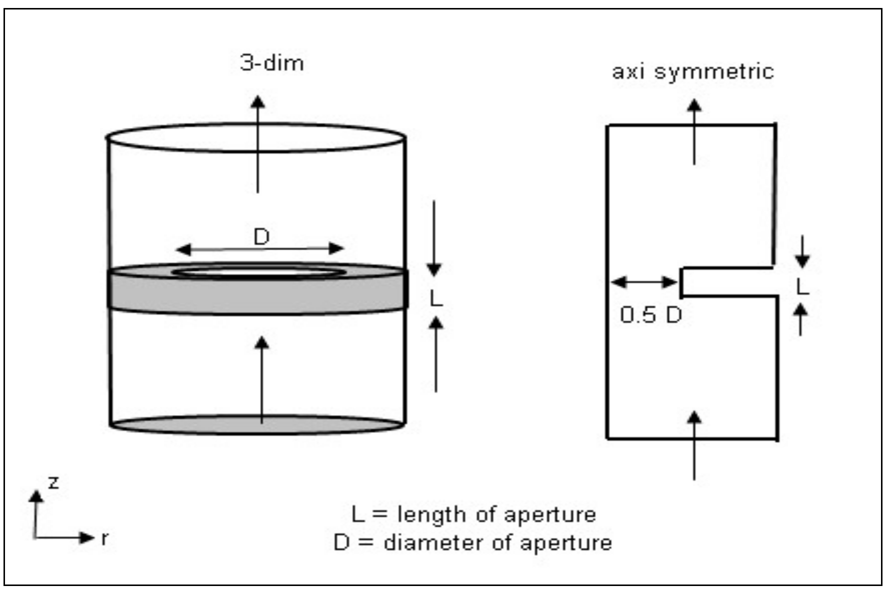

Figure 1. The three dimensional orifice and the approximation using two dimensional, axi-symmetric.

When the flow is laminar, the excess pressure drop (Δp) across orifices can be obtained by taking the pressure difference between the total pressure drop necessary to eject the fluid through the aperture and the pressure drop that would exist if only losses from fully developed flow was present[1]. The formula is represented as following:

$$\frac{2\Delta p}{\rho U^2} = K_2 + \frac{K_1}{\mathrm{Re}} \tag{1}$$

where U = mean velocity

ρ = density

K_1 and K_2 = constants

$\mathrm{Re} = \frac{\rho U D}{\mu}$ = Reynolds number

μ = viscosity

D = aperture diameter

Experiments from Hasegawa, et al.:

In Hasegawa[1] experiment, the excess pressure drops were measured for flow through very small orifices whose diameter ranges from 1 mm to 10μm using water, silicon oils, and solutions of glycerin in water. The velocities were measured along the centerline of the orifice. They stated that for larger orifices, their experimental excess pressure drops were the same as the numerical analysis of Newtonian flow, but for smaller orifices, the experimental results were higher. In addition, they calculated both theoretical and numerical values of K' as 37.7 and 27 respectively for an infinitely thin orifice in the creeping flow; these results were experimentally confirmed. Therefore, they concluded that Navier-Stokes could not be used to predict the effect of orifice thickness in microchannels.

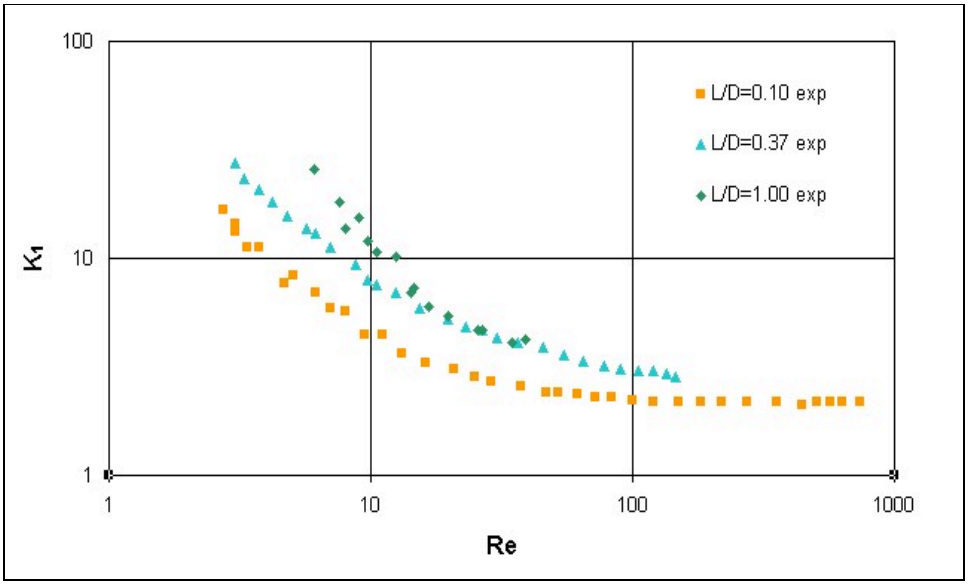

Figure 2. The experimental results taken from Hasegawa[1] for different L/D ratio. The experiment was done using water, silicon oils, and solutions of glycerin in water. This plot was reproduced from Hasegawa[1].

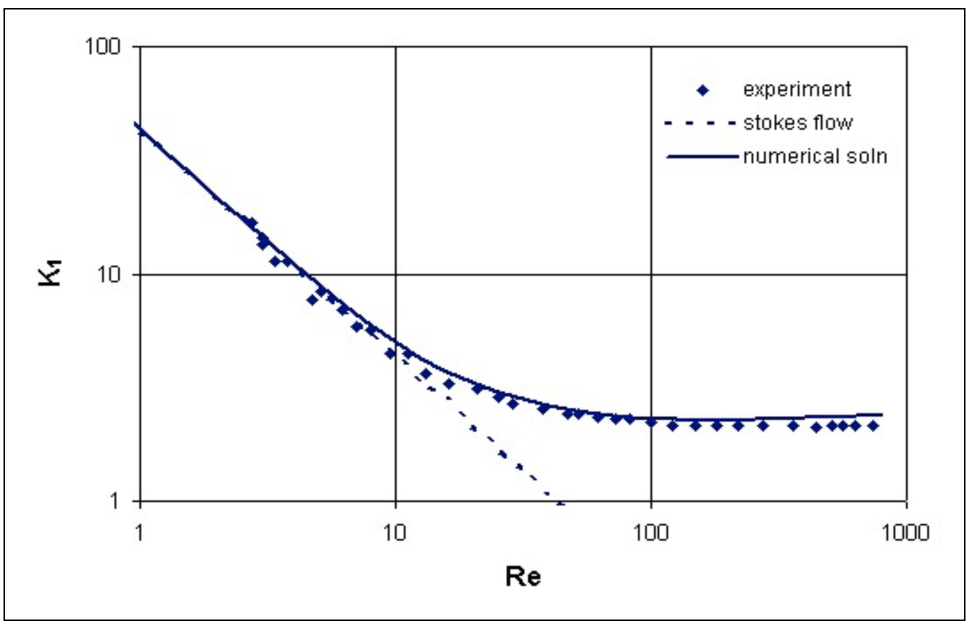

Figure 3. Numerical solution done by Hasegawa. Note here that they only have one numerical solution for different L/D ration. This plot was reproduced from Hasegawa[1].

First Evidence from Dagan, et al.:

The analytical solution from Dagan[2] was performed using Stokes flow (i.e. exact solution slow flow), of which valid only for flow at low Reynolds number.

$$\frac{2\Delta \mathrm{p}}{\rho \mathrm{U}^2} = \frac{64}{\mathrm{Re}}\frac{L}{D} + \frac{12\pi}{\mathrm{Re}} \qquad (2)$$

where ρ = density
U = average velocity
L = length of orifice
D = diameter of orifice

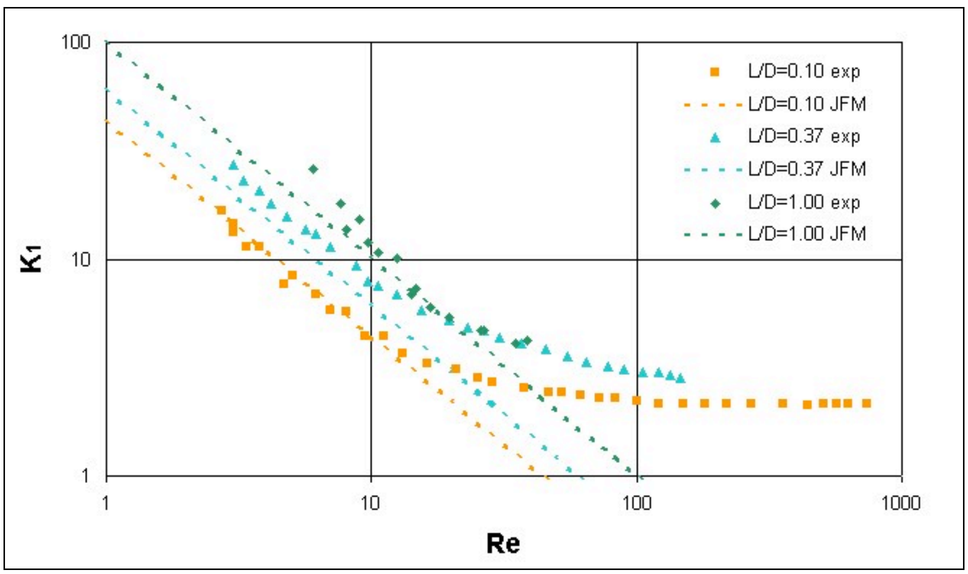

Figure 4. The first evidence from analytical solution using exact solution slow flow. The equation used to get the analytical solution, called JFM, was taken from Dagan[2].

Figure 4 above shows that pressure drop calculated using the Stokes flow follows the experiment data at low Re. It means that Navier Stokes is able to predict the effect of the orifice thickness at microchannels.

Numerical Solution from FEMLAB:

Simulation was performed in FEMLAB to get numerical solutions of the excess pressure drop. Numerical solution of K1 and Stokes flow K1 were calculated using FEMLAB, which has a built-in dimensionless Navier Stokes equation:

$$\rho_c U_c . \nabla_c U_c = -\nabla_c p + \mu \nabla_c^2 U_c \quad (3)$$

where ρ_c = density
U_c = average velocity

The geometry and dimension used in FEMLAB are described in the following:

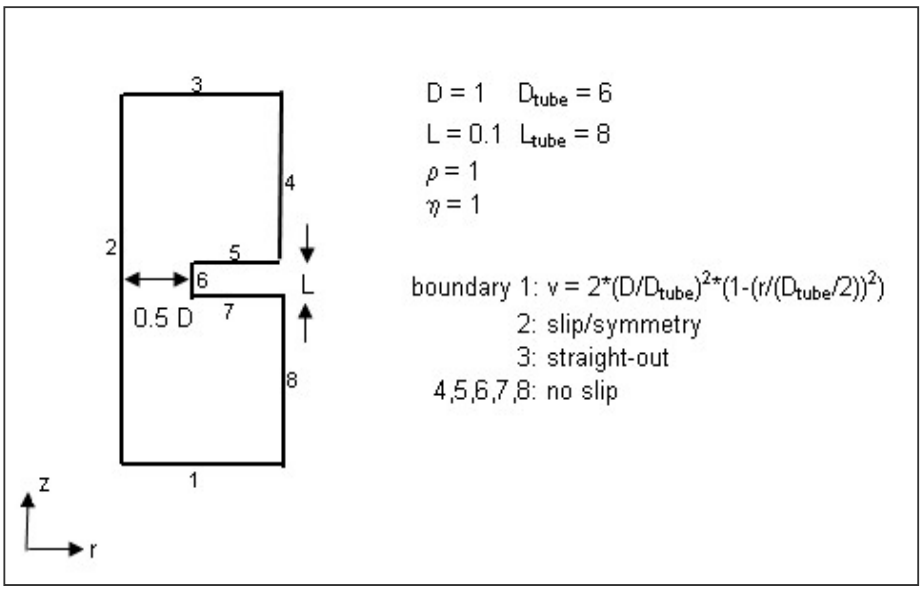

Figure 5. The model and dimension used in FEMLAB simulation. To get various result at different L/D ratio, the orifice diameter (D) was kept constant while changing the orifice thickness (L). A fully developed flow was introduced at the entrance of the channel.

The length and diameter of the domain were not given in Hasegawa[1]'s letter. Thus, for this calculation the length and diameter of the domain was defined as D_{tube}=3 and L_{tube}=8. In addition, to get more precise results, the calculations were also done using mesh and refined mesh. The effect of domain's diameter and length as well as mesh and refined mesh would be discussed more detailed later.

As a result, a plot of K1 vs. Re was obtained at different L/D ratio and compared with the experimental as well as JFM results:

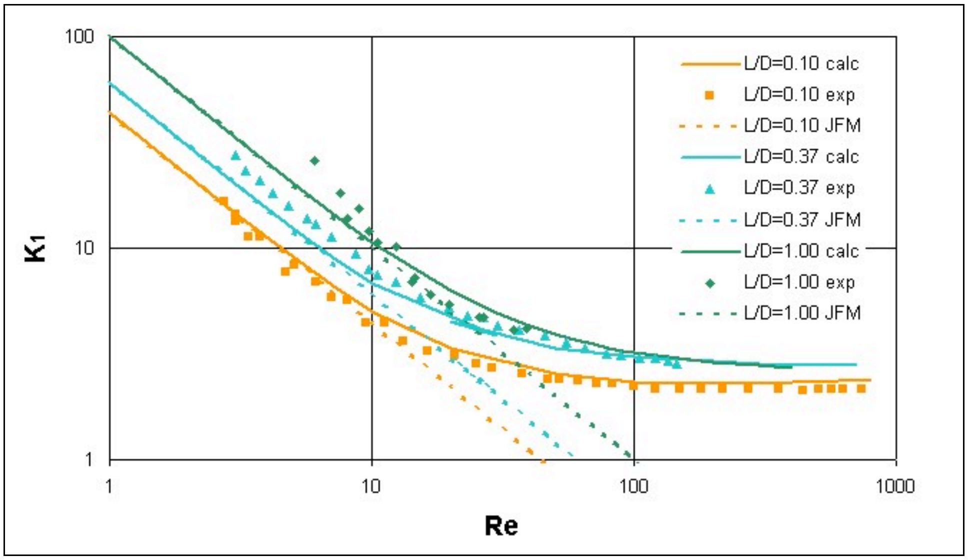

Figure 6. The dimensionless Δp (K_1) from FEMLAB simulation was plotted against Re for different L/D ratio. At low Re, K_1 decreased as the increasing of Re and starts to level off at high Re.

According to the plot above, the calculation follows both experiment and JFM results. At low Re, the pressure drop decreases with the increasing of Re and starts to level off at higher Re.

Mesh refinement was also done in order to get more precise results. The following table shows some calculations done using mesh and refined mesh at a certain L/D value.

Table 1. Comparison of Δp from mesh and refined mesh simulation at L/D=1. The difference between the two are really small and thus negligible.

L/D 1

Mesh Refine

Re	Δp_{mesh}	Δp_{refine}	% difference
1	49.81	50.24	0.864
5	50.59	50.96	0.738
10	53.20	53.38	0.340
30	74.32	74.60	0.385
50	98.44	93.78	4.73
100	160.34		

Since the difference of the calculated Δp between mesh and refine mesh approximations is less than 1% for Reynolds number less than 50 and around 5% for Reynolds number bigger than 50. Since the interest of the calculation is only at low Reynolds number, so the effect of mesh refinement can be negligible.

Another assumption was that all of those calculations were done using D_{tube} =3 and L_{tube}=8. To account the effect of tube's diameter, other calculations were done at different tube. The calculations are shown in the following:

Table 2. The Δp for different tube/domain diameter. The difference among different diameter was small and thus negligible. The calculation was performed at Re = 10 and L/D = 0.37.

Re	10
L/D	0.37

d_{tube}	Δp	%difference
2	33.8768	0.000
3	34.0688	0.567
4	34.0774	0.592

Based on the table above, the difference of the calculation using different tube's diameter values is less than 1%. Thus, the effect of tube's diameter is negligible. Analog to the tube's diameter, the effect of tube's length is also negligible.

The Effect of Domain's Length on the Orifice Pressure Drops:

At higher L/D ratio, as Re gets bigger, the domain should be set to be long enough to get a fully developed flow. To check for that, the exit velocity profile and pressure at the centerline should be observed. A hyperbolic velocity profile should be appeared at the channel exit and there should not be a negative pressure at the centerline if not the domain's length should be extended. For L/D = 1.14, the domain's length should be extended starting at Re = 200 to get a valid solution of K_1.

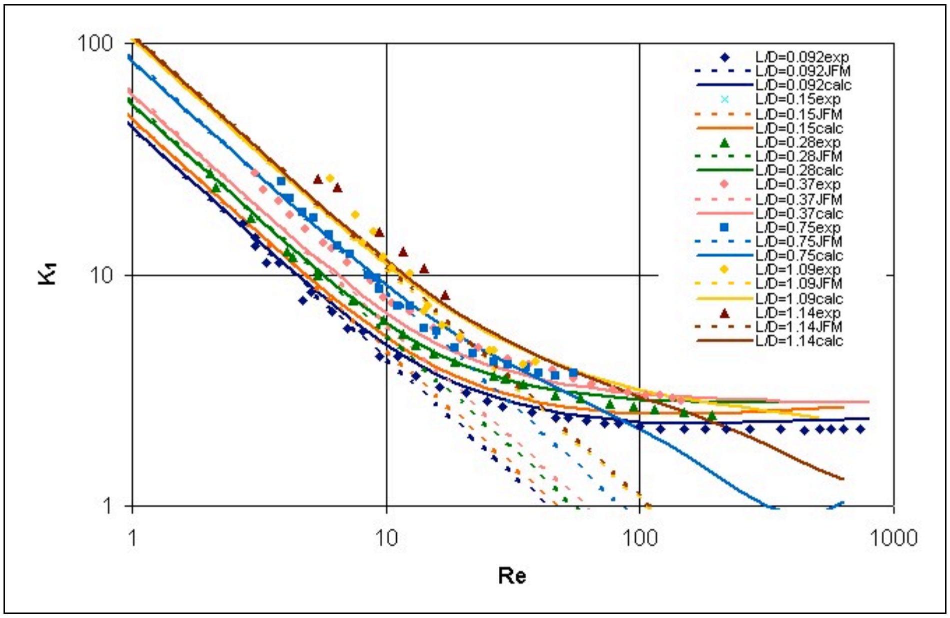

Figure 7. All results at different L/D ratio each for Hasegawa experiment result (exp, points), Dagan analytical solution (JFM, dotted line) , and numerical solution using FEMLAB (calc, solid line). Note that at high Re, for high L/D ratio, the FEMLAB solution starts to deviate from what they supposed to be.

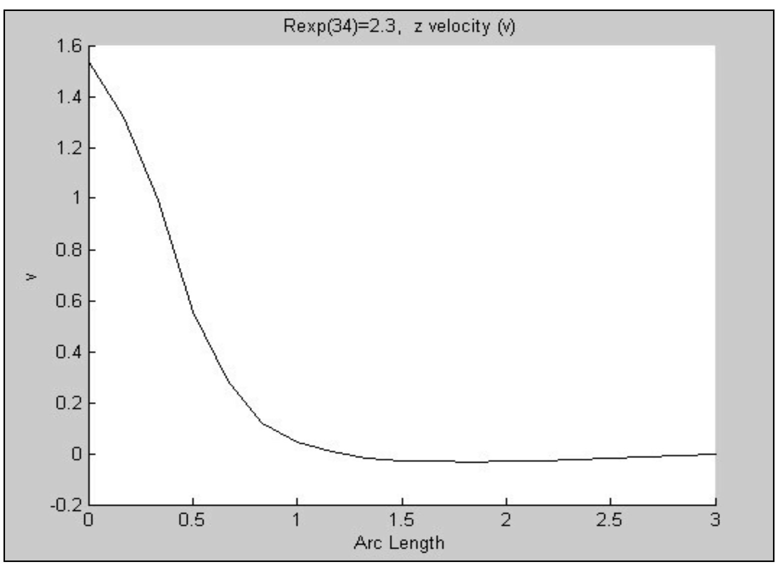

Figure 8. The exit velocity for L/D = 1.14 at Re = 200. It shows that the profile was not hyperbolic, means the velocity at the exit of the channel was not fully developed.

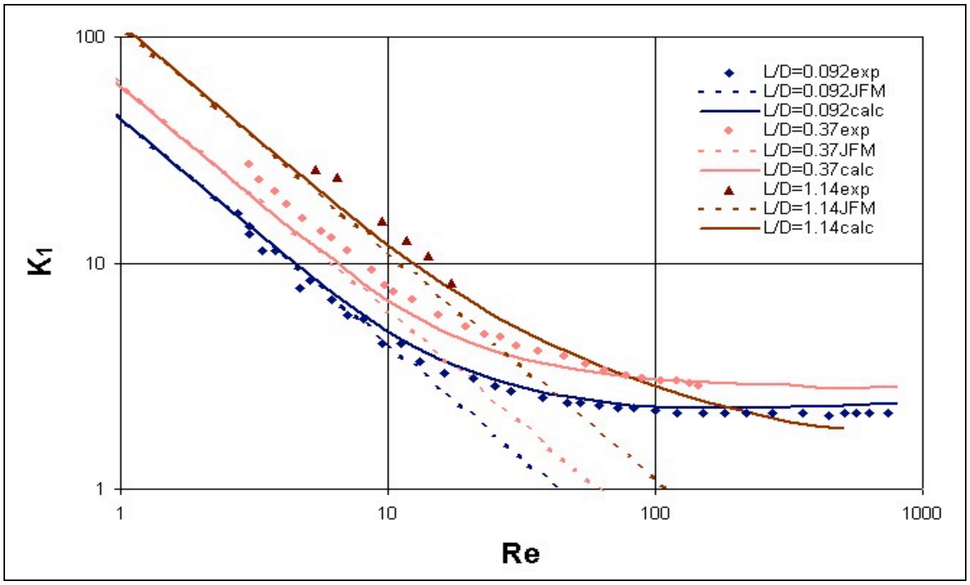

Figure 9. The results after the domain length was made longer to get fully developed velocity along the domain. Notice that at Re > 200, the domain should be made longer to get fully developed velocity along the domain.

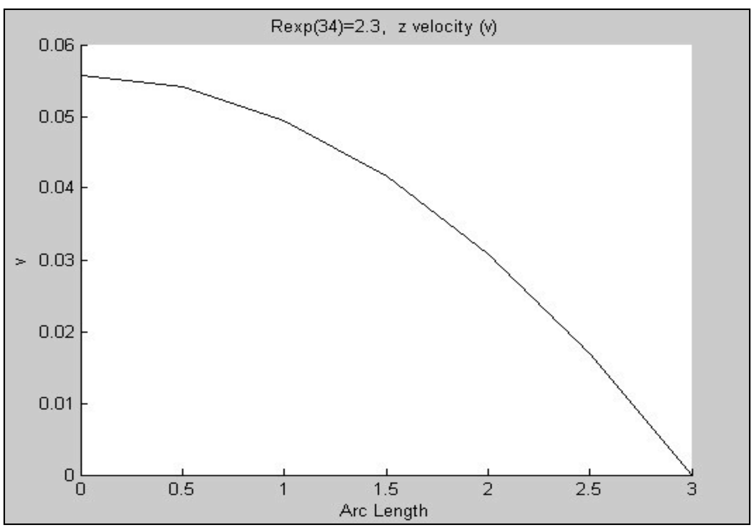

Figure 10. The exit velocity profile for L/D = 1.14 and Re = 200 after the domain was made longer. Here, the fully developed velocity was obtained along the domain.

Conclusions:

As conclusions, Navier-Stokes can still be used to predict the effect of the orifice thickness to the pressure drop across micro orifices. Hasegawa[1] misinterpreted the analytical solution of Dagan[2] and got incorrect numerical solution of the pressure drop at different L/D ratio.

From the simulation and theory from Dagan[2], we can get a conclusion that Navier Stokes is still able to predict the effect of the orifice thickness to the pressure drop across micro orifices with diameter as small as 8 microns.

At higher Re, especially for higher Re, the domain should be long enough to get fully developed velocity.

References:

[1] Hasegawa, T., M. Suganuma, and H. Watanabe, "Anomaly of excess pressure drops of the flow through very small orifices" Phys. Fluids 9, 1-3, (1997).

[2] Dagan, Z., S. Weinbaum, R. Pfeffer, "An infinite-series solution for the creeping motion though an orifice of finite length," J. Fluid Mech. 115 505-523 (1982).

Appendix:

Model.mat → a FEMLAB model for shorter domain (L=8). This model was set at L/D = 1.14 and solving for parametric solution Rexp = 0.1:0.1:3

Longer.mat → a FEMLAB model for shorter domain (L=80). This model was set at L/D = 1.14 and solving for parametric solution Rexp = 0.1:0.1:3