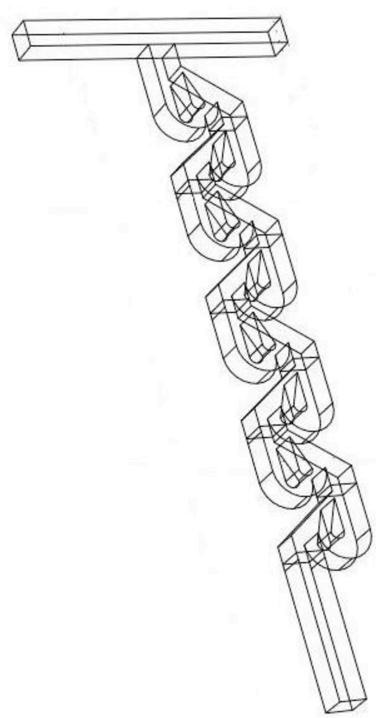
Microfluidic Research: Mixing Effectiveness of Modified Tesla Structures



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Introduction

The intent of this research report is to investigate and describe the effectiveness of a particular geometry in mixing two fluids in order to obtain an optimum exiting concentration. This is to be done by comparing how effectively the mixer operates at both different Peclet numbers and Reynolds numbers. All work will be completed in a virtual environment using Comsol Multiphysics to evaluate each defined situation.

Description

A modified Tesla mixer works by utilizing the Coanda effect to split an approaching stream and then redirect one of the diverged flows so that it will meet the second tributary. Shown in figure 1, the Coanda effect may best be described using the analogy of a spoon.

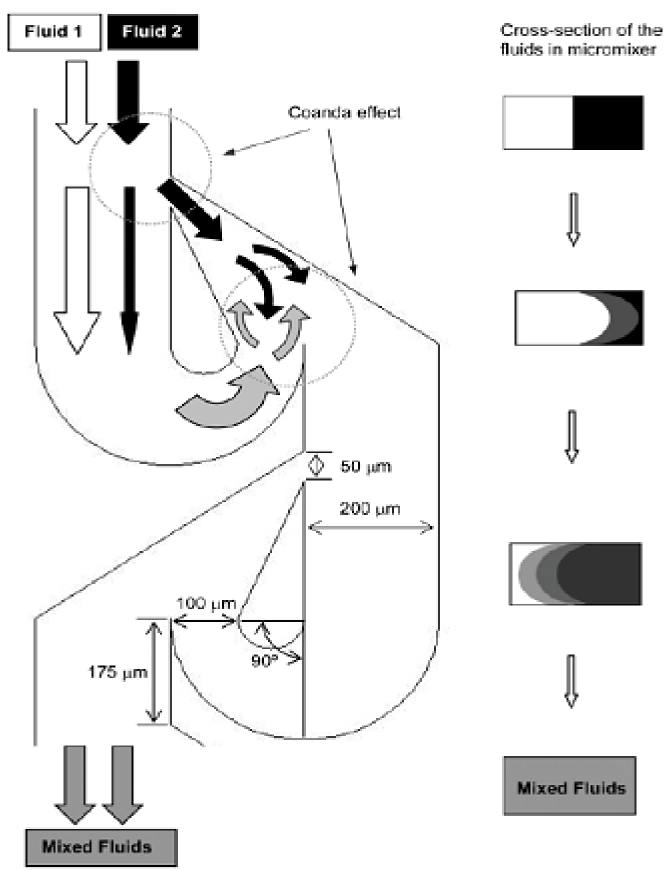

Figure 1. Modified Tesla Mixer and Coanda Effect

Consider water flowing along the convex side of a spoon. As the water flows along the curve, it follows the shape of the spoon, adhering to its overall shape, and even as it leaves the spoon the water continues to flow in the direction in which it was on the spoon. This effect is a principal consideration in describing the manner by which this mixer works. When two fluids of differing composition enter the mixer, one diverts into a separate channel by the Coanda effect as shown above. The other un-diverted fluid continues and is redirected through a 90 degree curve where it meets the smaller tributary. At this point the diverted flow again experiences the Coanda effect as the partition used to separate it and the original flow has a curved surface near where the two flows recombine. As a result, the two streams meet essentially head on and mix forcefully as a result of this collision. After this interaction, the recombined stream exits the first mixer where the process is repeated.

In order to evaluate the mixing results, Comsol numerically solves both the non-dimensional Navier-Stokes equation and the Convective-Diffusion equation. The Navier-Stokes equation describes the fluid motion and is known in the dimensional form as

$$\rho\frac{\partial u}{\partial t}+\rho u\bullet\nabla u=-\nabla p+\eta\nabla^2 u$$

and is transformed to a non-dimensional form by introducing the following parameters.

$$u^{'}=\frac{u}{u_s}\qquad p^{'}=\frac{p}{p_s}\qquad x^{'}=\frac{x}{x_s}$$

Under these definitions the Navier-Stokes equation may now be defined as follows.

$$\frac{\partial u^{'}}{\partial t^{'}}+\mathrm{Re}\,u^{'}\bullet\nabla^{'}u^{'}=-\nabla^{'}p^{'}+\nabla^{'2}u^{'}$$

$$\mathrm{Re}=\frac{\rho u_s x_s}{\eta}$$

The design of this mixer may present the Reynolds number as a key variable because of the interaction of the two diverged streams. If the velocity at which they meet is changed, the mixing effect may either improve or worsen as a result. Out of the Navier-Stokes equation, the pressure at the inlets and anywhere within the mixing unit may also be determined. Comsol evaluates the pressure at a boundary as

$$P' = \frac{\int p(x,y)\,dxdy}{\int dxdy}$$

The value obtained in Comsol is non-dimensional as to be expected from the non-dimensional organization of the Navier-Stokes equation and is converted to a dimensional form by

$$P = P_s P'$$

$$P_s = \rho u_s^2$$

Further mixing is described by the Convective-Diffusion equation, which is shown in its dimensional form here.

$$\frac{\partial c}{\partial t} + v \bullet \nabla c = D \nabla^2 c$$

Like the Navier-Stokes equation, the Convective-Diffusion equation is also made non-dimensional by introducing several new variables.

$$c' = \frac{c}{c_s} \qquad v' = \frac{v}{v_s} \qquad \nabla' = x_s \nabla \qquad t' = \frac{t v_s}{x_s}$$

$$Pe \frac{\partial c'}{\partial t'} + Pe v' \bullet \nabla' c' = \nabla'^2 c'$$

The Peclet number is a result of this non-dimensional form and describes the ratio of the mixing due to the fluid flow to that of the mixing caused by diffusion. This will also be varied throughout evaluation of the geometry in order to determine the influence of different values of the diffusivity of the material in the stream.

$$Pe = \frac{u_s x_s}{D}$$

In order to evaluate the effectiveness of the mixing process, both the concentration at several locations within the mixer and the variance of this concentration will be calculated, and these values are defined as described below.

$$c_{mixingcup} = \frac{\int_A c(x,y,z)\, v(x,y)\, dxdy}{\int_A v(x,y)\, dxdy}$$

The mixing cup concentration eliminates the possibly different rates at which fluids are flowing at a particular point and describes what the concentration of the fluid would be if it was emptied into a vessel and well mixed. Also of possible value is the optical concentration, which is simply the integral of the concentration, evaluated over a particular length, in this case the width of the mixer channel.

$$c_{optical} = \frac{\int_0^L c(x,y,z)\, dy}{\int_0^L dy}$$

The variance for each of these values is determined to observe how well mixed the fluid actually is.

$$c_{variance} = \frac{\int_A \left[c(x,y,z) - c_{mixingcup} \right]^2 v(x,y)\, dxdy}{\int_A v(x,y)\, dxdy}$$

Success in design would be noted by a mixing cup concentration of .5 with the variance minimized to whatever is conceivable.

Once a geometry has been designed, the boundary conditions upon which the above equations will be solved must be defined. For the Navier-Stokes equation, the two inlets of the mixer are defined as having prescribed flows, while the outlet is set to have zero pressure. The remaining wall surfaces are all considered to be defined by the "No-Slip" condition. The Convective-Diffusion equation is bounded at the inlets by defined concentrations of one and zero, while the outlet is set as having convective flux. The walls of the mixer are not permeable and so are all defined as insulated.

Two-Dimensional Results

The results of the calculations are shown completely in the appendices, one such result shown visually in Figure 21, that being that for the concentration throughout the mixer. All situations were calculated with a mesh composed of 15,368 elements with 103,553 degrees of freedom. The results are best checked purely on the basis of what would ideally occur. In ideal mixing, the mixing cup concentration would be 0.5 and the variance very small or ideally zero. If there are large deviations from the optimum concentration, especially if it is considerably above 0.5, then concern must be exercised in presenting the results as valid.

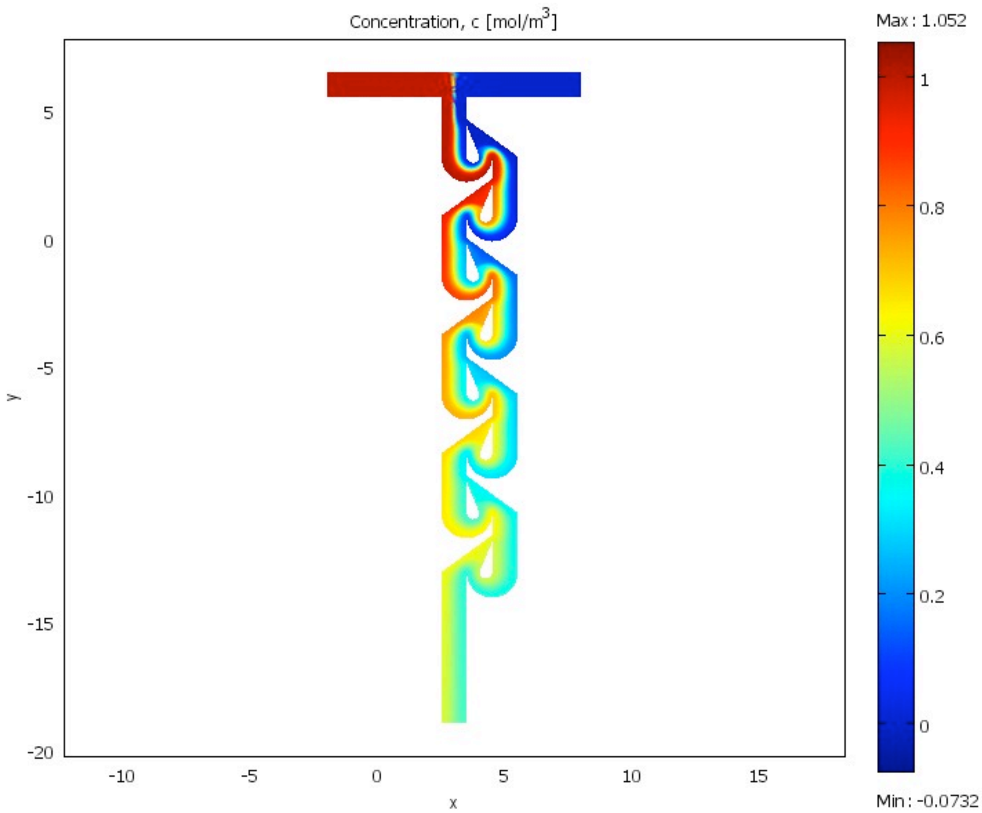

Figure 2. Solution of Convection-Diffusion equation from Comsol.

Another potential source of error in computer-based numerical evaluations is the density of the mesh used to evaluate the geometry. To illustrate the impact of the mesh, the situation with the Reynolds number equal to one and the Peclet number equal to 300 will be evaluated at three different mesh densities. The first table illustrates a very coarse mesh of 3,618 elements with 25,218 degrees of freedom. The second table has a refined mesh of 14,472 elements with 97,498 degrees of freedom. The final table presents the finest mesh used with 15,368 elements and 103,533 degrees of freedom. While that mesh does not have many more total elements than the second case, it is refined in specific locations which were initially very coarse. The most obvious impact of the refined mesh is in the values of the pressure differentials obtained in various locations within the mixer, with the refined mesh providing a value that is approximately 15% greater than the coarse mesh.

Number of Mixers	Peclet Number	$C_{mixingcup}$	$Var_{mixingcup}$	$C_{optical}$	$Var_{optical}$	Pressure'	Pressure
None	None	mol/m^3	mol/m^3	mol/m^3	mol/m^3	None	Pa
1	300	4.975E-01	1.146E-01	4.898E-01	1.147E-01	8.410E+02	2.103E+01
2	300	4.944E-01	7.102E-02	5.057E-01	7.114E-02	7.279E+02	1.820E+01
3	300	4.951E-01	4.333E-02	4.902E-01	4.335E-02	6.171E+02	1.543E+01
4	300	4.936E-01	2.703E-02	4.995E-01	2.706E-02	5.037E+02	1.259E+01
5	300	4.937E-01	1.655E-02	4.901E-01	1.656E-02	3.912E+02	9.780E+00
6	300	4.926E-01	1.026E-02	4.968E-01	1.028E-02	2.780E+02	6.950E+00
7	300	4.921E-01	6.278E-03	4.903E-01	6.280E-03	1.656E+02	4.139E+00
8	300	4.923E-01	3.841E-03	4.933E-01	3.842E-03	5.300E+01	1.325E+00
					Overall P	9.884E+02	2.471E+01

Figure 3. Coarse Mesh at Re=1 and Pe=300

Number of Mixers	Peclet Number	$C_{mixingcup}$	$Var_{mixingcup}$	$C_{optical}$	$Var_{optical}$	Pressure’	Pressure
None	None	mol/m^3	mol/m^3	mol/m^3	mol/m^3	None	Pa
1	300	4.986E-01	1.106E-01	4.928E-01	1.106E-01	9.506E+02	2.377E+01
2	300	4.988E-01	6.851E-02	5.070E-01	6.858E-02	8.225E+02	2.056E+01
3	300	4.986E-01	4.343E-02	4.939E-01	4.345E-02	6.965E+02	1.741E+01
4	300	4.987E-01	2.703E-02	5.029E-01	2.704E-02	5.682E+02	1.421E+01
5	300	4.985E-01	1.696E-02	4.955E-01	1.697E-02	4.410E+02	1.102E+01
6	300	4.986E-01	1.055E-02	5.018E-01	1.056E-02	3.128E+02	7.820E+00
7	300	4.985E-01	6.616E-03	4.963E-01	6.620E-03	1.855E+02	4.639E+00
8	300	4.985E-01	4.160E-03	4.989E-01	4.160E-03	5.829E+01	1.457E+00
					Overall P	1.117E+03	2.792E+01

Figure 4. Refined Mesh at Re=1 and Pe=300

Number of Mixers	Peclet Number	$C_{mixingcup}$	$Var_{mixingcup}$	$C_{optical}$	$Var_{optical}$	Pressure'	Pressure

Units	None	None	mol/m^3	mol/m^3	mol/m^3	mol/m^3	None	Pa
	1	300	4.998E-01	1.129E-01	4.942E-01	1.129E-01	9.939E+02	2.485E+01
	2	300	5.001E-01	7.130E-02	5.081E-01	7.136E-02	8.599E+02	2.150E+01
	3	300	4.999E-01	4.607E-02	4.952E-01	4.610E-02	7.282E+02	1.820E+01
	4	300	5.000E-01	2.925E-02	5.042E-01	2.927E-02	5.941E+02	1.485E+01
	5	300	4.998E-01	1.873E-02	4.967E-01	1.874E-02	4.610E+02	1.153E+01
	6	300	4.999E-01	1.188E-02	5.032E-01	1.189E-02	3.270E+02	8.175E+00
	7	300	4.998E-01	7.600E-03	4.975E-01	7.606E-03	1.940E+02	4.850E+00
	8	300	4.998E-01	4.874E-03	5.002E-01	4.874E-03	6.094E+01	1.523E+00
						Overall P	1.169E+03	2.923E+01

Figure 5. Locally Refined Mesh at Re=1 and Pe=300

Three-Dimensional Results

It was not expected that the creation of the three-dimensional geometry would have any impact on the mixing, as the shape of the mixer has no unique qualities in the third dimension that would have a great impact on mixing. The results mirror this, as shown in the appendices, with the variances at similar Peclet numbers being well within an order of magnitude when compared to the two-dimensional case. A sample solution for the Navier-Stokes equation can be seen in Figure 6. Although only tested at a broad range of 100, 500, and 1000 for Peclet number values, I believe the results are sufficient from these three tests to conclude that no unique results will be obtained from completing a more comprehensive analysis of the three-dimensional model. The only relevant

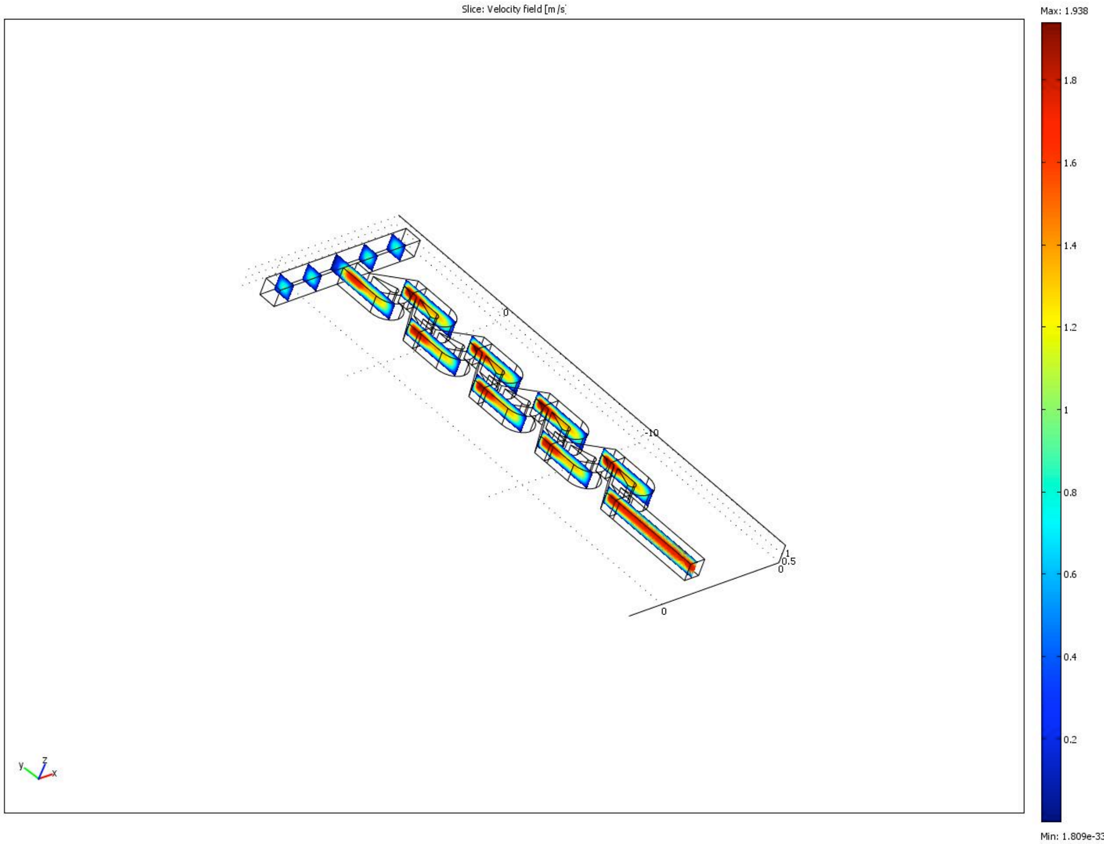

Figure 6. Solution for the Navier-Stokes equation; the colors representing velocities.

difference in values was seen in the pressures, which can most probably be accounted for in the greater friction of the additional walls that did not exist in the more simple two-dimensional case. While for the two-dimensional case the pressure reaches near 29 Pa, for the 3D case it approaches 45 Pa, so nearly a 50% increase occurs. Shown in figure 7, you can see that the two and three dimensional results still collapse onto the same curve however.

Literature Comparison

It was my intent to present a comparison of the results published by the original creators of this mixing device and show them here to either bolster or disclaim their results. However, the bizarre manner by which the group presented their results made it difficult to perform any such comparison.

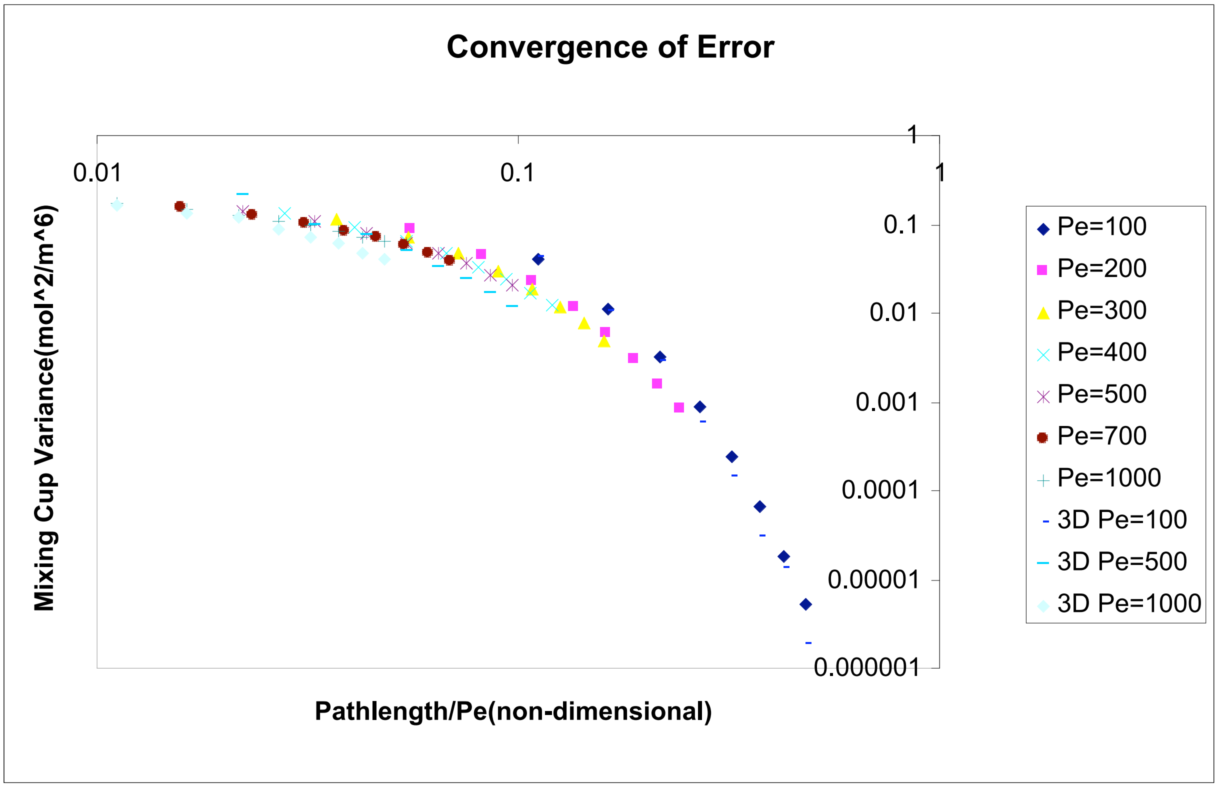

Figure 7. 2D and 3D results.

The values presented were both dissimilar to those presented here in units, and there were no tabulated values shown, so any attempt to compare results would likely have been brief and made with a number of assumptions based on their graphical results. A simple comparison may be made in that the authors of the article do declare their mixer to be a considerable improvement to a T-shaped mixer, while my results do not merit such a declaration.

T-Sensor Comparison

Comparing the tested geometry to that of a simple T-shaped mixer produced some results that I necessarily did not expect, but nonetheless are what was found, and ultimately lead to the most critical conclusion of this research. Graphing all of the data collected for both the two-dimensional and three-dimensional cases combined with the T-sensor, it appears as though one could best claim that the Tesla mixer is at best equal to the T-sensor, although one could potentially claim that the Tesla mixer is actually worse than the T-sensor. Shown in Figure 8 on the following page, these possible results are quite clear as the two mixers do not necessarily collapse perfectly onto a similar curve.

Suggestions

The modified Tesla mixer appears to be an effective microfluidic mixer however none better than a simple T-shaped mixer. Certain effects may be enhanced through some possible changes in the design. The two streams which meet in the mixer are integral to the effectiveness of the device, but it also appears that the mixing may not be quite as effective as it could be. One possible reason for this is that the opening into the alternate channel in the mixer is too small, and therefore the contraction increases the velocity too greatly. If the inlet to the other portion of the mixer is wider, the velocity change in the diverged section would be lower and so the stream which it meets may further move into the diverged section. By allowing the stream to further penetrate the diverged section, the mixing that occurs as a result of the two streams meeting may be enhanced. At this stage it appears as though mixing is confined almost, if not completely, to diffusion, and so all considerations I can make would revolve around a way to enhance the intended effect of the design of this mixer.

Conclusions

Two primary conclusions may be made as a result of the calculations performed, the results of which are shown in detail in Appendix B. First, is that as the Peclet number increases, the mixing worsens. This result is obvious, as an increasing Peclet number in this instance implies a smaller diffusivity, thus decreasing the mixing due to diffusion. The other result is more curious, but nonetheless apparent. As the Reynolds number increases, the mixing becomes worse. Because the mixing within the Tesla structure is meant to be heavily dependent on the recirculation and meeting of the two streams, it would be a logical conclusion from my position to assume that the increased Reynolds number would cause the meeting of the two streams to cause a greater disturbance of the flow, and therefore increase the mixing, however this is not so. What does appear to happen is that as a result of the higher velocity of the greater Reynolds numbers tested, the fluid simply moves faster without any mixing caused by this motion.

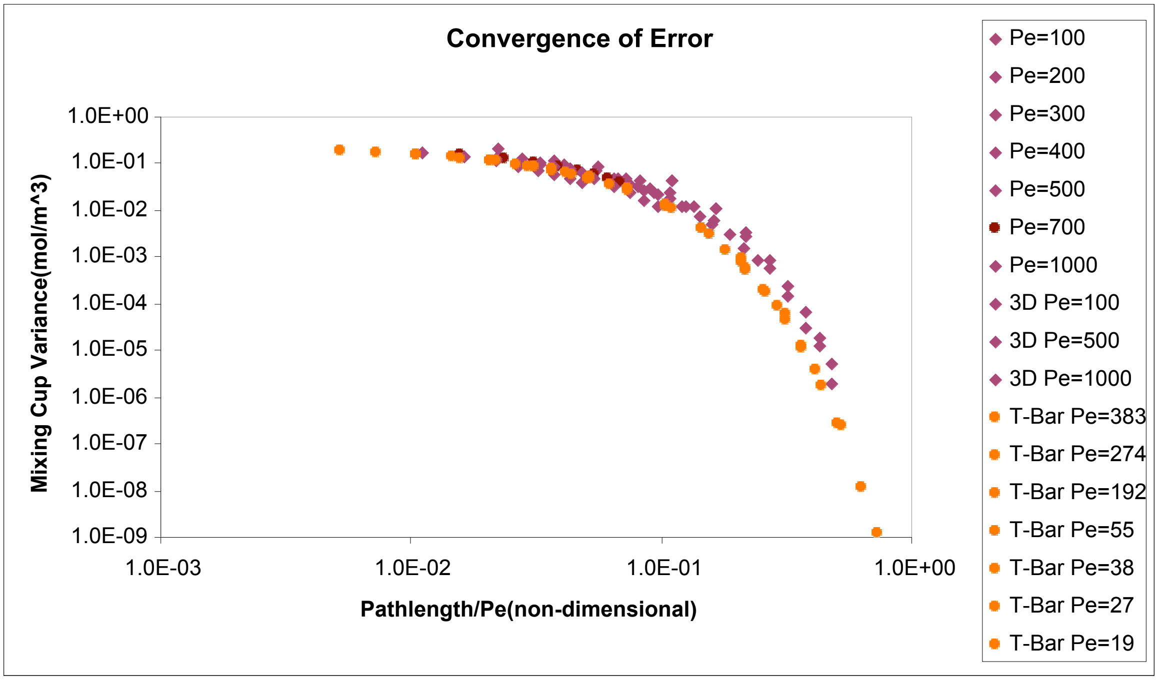

Figure 8. Tesla Mixer and T-Sensor mixing comparison.

The only result is that, instead, the fluid remains in the mixer for a lesser amount of time, thus allowing less mixing due to diffusion to occur.

The mixing cup and optical concentration variances are reasonably similar, however it appears that with increasing Reynolds numbers the two values begin to diverge from each other. Also of note, is that when the variances of different situations are compared to the path-length divided by the Peclet or Reynolds number, the resulting curves converge effectively to a single arc. Ultimately, the most encompassing conclusion that I can make about this mixer design is that at under the limited conditions in which it was tested, it offers no improvement over simpler designs.

Bibliography

Koch, M.V, Vanden Bussche, K. M, and Chrisman, R. W. *Micro Instrumentation for High Throughput Experimentation and Process Intensification-a Tool for PAT.* Weinheim: WILEY-VCH, 2007.

Finlayson, Bruce A. *Introduction to Chemical Engineering Computing.* Hoboken: John Wiley & Sons, Inc., 2006.

Hong, C.-C., J.-W. Choi, and C. H. Ahn. 2004. "A novel in-plane passive microfluidic mixer with modified Tesla structures". *LAB ON A CHIP*. 4: 109-113.

Appendix A: Sample Calculations

$$\mathrm{Re} = \frac{\rho u_s x_s}{\eta}$$

$$1\left[\text{dimensionless}\right] = \frac{\left(1000\frac{kg}{m^3}\right)\left(.005\frac{m}{s}\right)\left(200x10^{-6}m\right)}{\left(.001\frac{kg}{m \bullet s}\right)}$$

$$Pe = \frac{u_s x_s}{D}$$

$$1000\left[\text{dimensionless}\right] = \frac{\left(.005\frac{m}{s}\right)\left(200x10^{-6}m\right)}{\left(10^{-9}\frac{m^2}{s}\right)}$$

$$P_s = \rho u_s^2$$

$$.025\left[Pa\right] = \left(1000\frac{kg}{m^3}\right)\left(.005\frac{m}{s}\right)^2$$

$$P = P_s P'$$

$$29.23\left[Pa\right] = \left(.025\frac{kg}{m \bullet s^2}\right)\left(1169\right)$$

$$c_{mixingcup} = \frac{\int_A c(x,y,z)\,v(x,y)\,dxdy}{\int_A v(x,y)\,dxdy}$$

$$.4999\left[\frac{mol}{m^3}\right] = \frac{.9582\,\frac{mol}{m\bullet s}}{1.916\,\frac{m^2}{s}}$$

$$c_{optical} = \frac{\int_0^L c(x,y,z)\,dy}{\int_0^L dy}$$

$$.4989\,\frac{mol}{m^3} = \frac{.4989\,\frac{mol}{m^2}}{1m}$$

$$c_{\mathrm{var}\,iance} = \frac{\int_A \left[c(x,y,z) - c_{mixingcup}\right]^2 v(x,y)\,dxdy}{\int_A v(x,y)\,dxdy}$$

$$.1507\left[\frac{mol^2}{m^6}\right] = \frac{.2889\,\frac{mol^2}{m^4\bullet s}}{1.9168\,\frac{m^2}{s}}$$

Appendix B

Detailed Results

The first seven tables present the data which concerns varying Peclet numbers, with a Reynolds number held constant at one. The remaining tables present data in which the Reynolds number is varied, with a constant Peclet number of 300. Following that are the three-dimensional results for different Peclet numbers.

Two-Dimensional

	Number of Mixers	Peclet Number	$C_{mixingcup}$	$Var_{mixingcup}$	$C_{optical}$	$Var_{optical}$	Pressure'	Pressure
Units	None	None	mol/m^3	mol/m^3	mol/m^3	mol/m^3	None	Pa
	1	100	4.995E-01	4.117E-02	4.916E-01	4.124E-02	9.939E+02	2.485E+01
	2	100	5.000E-01	1.117E-02	5.045E-01	1.119E-02	8.599E+02	2.150E+01
	3	100	4.997E-01	3.214E-03	4.976E-01	3.218E-03	7.282E+02	1.820E+01
	4	100	4.998E-01	8.734E-04	5.010E-01	8.748E-04	5.941E+02	1.485E+01
	5	100	4.998E-01	2.437E-04	4.992E-01	2.441E-04	4.610E+02	1.153E+01
	6	100	4.998E-01	6.629E-05	5.001E-01	6.642E-05	3.270E+02	8.175E+00
	7	100	4.998E-01	1.849E-05	4.996E-01	1.853E-05	1.940E+02	4.850E+00
	8	100	4.998E-01	5.173E-06	4.998E-01	5.175E-06	6.094E+01	1.523E+00
						Overall P	1.169E+03	2.923E+01

	Number of Mixers	Peclet Number	$C_{mixingcup}$	$Var_{mixingcup}$	$C_{optical}$	$Var_{optical}$	Pressure'	Pressure
Units	None	None	mol/m^3	mol/m^3	mol/m^3	mol/m^3	None	Pa
	1	200	4.997E-01	8.657E-02	4.925E-01	8.662E-02	9.939E+02	2.485E+01
	2	200	5.001E-01	4.404E-02	5.074E-01	4.409E-02	8.599E+02	2.150E+01
	3	200	4.998E-01	2.306E-02	4.957E-01	2.308E-02	7.282E+02	1.820E+01
	4	200	4.999E-01	1.175E-02	5.031E-01	1.176E-02	5.941E+02	1.485E+01
	5	200	4.998E-01	6.061E-03	4.976E-01	6.066E-03	4.610E+02	1.153E+01
	6	200	4.999E-01	3.086E-03	5.018E-01	3.089E-03	3.270E+02	8.175E+00
	7	200	4.998E-01	1.591E-03	4.985E-01	1.592E-03	1.940E+02	4.850E+00
	8	200	4.998E-01	8.225E-04	5.001E-01	8.225E-04	6.094E+01	1.523E+00
						Overall P	1.169E+03	2.923E+01

	Number of Mixers	Peclet Number	$C_{mixingcup}$	$Var_{mixingcup}$	$C_{optical}$	$Var_{optical}$	Pressure'	Pressure
Units	None	None	mol/m^3	mol/m^3	mol/m^3	mol/m^3	None	Pa
	1	300	4.998E-01	1.129E-01	4.942E-01	1.129E-01	9.939E+02	2.485E+01
	2	300	5.001E-01	7.130E-02	5.081E-01	7.136E-02	8.599E+02	2.150E+01
	3	300	4.999E-01	4.607E-02	4.952E-01	4.610E-02	7.282E+02	1.820E+01
	4	300	5.000E-01	2.925E-02	5.042E-01	2.927E-02	5.941E+02	1.485E+01
	5	300	4.998E-01	1.873E-02	4.967E-01	1.874E-02	4.610E+02	1.153E+01
	6	300	4.999E-01	1.188E-02	5.032E-01	1.189E-02	3.270E+02	8.175E+00
	7	300	4.998E-01	7.600E-03	4.975E-01	7.606E-03	1.940E+02	4.850E+00
	8	300	4.998E-01	4.874E-03	5.002E-01	4.874E-03	6.094E+01	1.523E+00
						Overall P	1.169E+03	2.923E+01

	Number of Mixers	Peclet Number	$C_{mixingcup}$	$Var_{mixingcup}$	$C_{optical}$	$Var_{optical}$	Pressure'	Pressure
Units	None	None	mol/m^3	mol/m^3	mol/m^3	mol/m^3	None	Pa
	1	400	4.999E-01	1.299E-01	4.951E-01	1.299E-01	9.939E+02	2.485E+01
	2	400	5.002E-01	9.122E-02	5.080E-01	9.129E-02	8.599E+02	2.150E+01
	3	400	4.999E-01	6.556E-02	4.953E-01	6.558E-02	7.282E+02	1.820E+01
	4	400	5.000E-01	4.655E-02	5.047E-01	4.657E-02	5.941E+02	1.485E+01
	5	400	4.999E-01	3.324E-02	4.963E-01	3.326E-02	4.610E+02	1.153E+01
	6	400	4.999E-01	2.358E-02	5.041E-01	2.722E-02	3.270E+02	8.175E+00
	7	400	4.998E-01	1.683E-02	4.968E-01	1.684E-02	1.940E+02	4.850E+00
	8	400	4.998E-01	1.204E-02	5.001E-01	1.204E-02	6.094E+01	1.523E+00
						Overall P	1.169E+03	2.923E+01

	Number of Mixers	Peclet Number	$C_{mixingcup}$	$Var_{mixingcup}$	$C_{optical}$	$Var_{optical}$	Pressure'	Pressure
Units	None	None	mol/m^3	mol/m^3	mol/m^3	mol/m^3	None	Pa
	1	500	4.999E-01	1.419E-01	4.956E-01	1.419E-01	9.939E+02	2.485E+01
	2	500	5.002E-01	1.061E-01	5.078E-01	1.061E-01	8.599E+02	2.150E+01
	3	500	5.000E-01	8.122E-02	4.954E-01	8.124E-02	7.282E+02	1.820E+01
	4	500	5.001E-01	6.168E-02	5.049E-01	6.170E-02	5.941E+02	1.485E+01
	5	500	4.999E-01	4.707E-02	4.961E-01	4.708E-02	4.610E+02	1.153E+01
	6	500	4.999E-01	3.571E-02	5.047E-01	3.573E-02	3.270E+02	8.175E+00
	7	500	4.998E-01	2.724E-02	4.963E-01	2.725E-02	1.940E+02	4.850E+00
	8	500	4.998E-01	2.083E-02	4.999E-01	2.083E-02	6.094E+01	1.523E+00
						Overall P	1.169E+03	2.923E+01

	Number of Mixers	Peclet Number	$C_{mixingcup}$	$Var_{mixingcup}$	$C_{optical}$	$Var_{optical}$	Pressure'	Pressure
Units	None	None	mol/m^3	mol/m^3	mol/m^3	mol/m^3	None	Pa
	1	700	4.999E-01	1.580E-01	4.958E-01	1.580E-01	9.939E+02	2.485E+01
	2	700	5.002E-01	1.267E-01	5.074E-01	1.267E-01	8.599E+02	2.150E+01
	3	700	5.000E-01	1.041E-01	4.957E-01	1.041E-01	7.282E+02	1.820E+01
	4	700	5.001E-01	8.537E-02	5.050E-01	8.539E-02	5.941E+02	1.485E+01
	5	700	5.000E-01	7.026E-02	4.960E-01	7.028E-02	4.610E+02	1.153E+01
	6	700	5.000E-01	5.760E-02	5.054E-01	5.763E-02	3.270E+02	8.175E+00
	7	700	4.999E-01	4.743E-02	4.957E-01	4.745E-02	1.940E+02	4.850E+00
	8	700	4.998E-01	3.914E-02	4.994E-01	3.914E-02	6.094E+01	1.523E+00
						Overall P	1.169E+03	2.923E+01

	Number of Mixers	Peclet Number	$C_{mixingcup}$	$Var_{mixingcup}$	$C_{optical}$	$Var_{optical}$	Pressure'	Pressure
Units	None	None	mol/m^3	mol/m^3	mol/m^3	mol/m^3	None	Pa
	1	1000	4.999E-01	1.724E-01	4.958E-01	1.725E-01	9.939E+02	2.485E+01
	2	1000	5.002E-01	1.459E-01	5.071E-01	1.459E-01	8.599E+02	2.150E+01
	3	1000	5.001E-01	1.262E-01	4.959E-01	1.262E-01	7.282E+02	1.820E+01
	4	1000	5.002E-01	1.094E-01	5.050E-01	1.094E-01	5.941E+02	1.485E+01
	5	1000	5.001E-01	9.522E-02	4.960E-01	9.524E-02	4.610E+02	1.153E+01
	6	1000	5.000E-01	8.273E-02	5.060E-01	8.276E-02	3.270E+02	8.175E+00
	7	1000	5.000E-01	7.215E-02	4.953E-01	7.218E-02	1.940E+02	4.850E+00
	8	1000	4.999E-01	6.304E-02	4.989E-01	6.304E-02	6.094E+01	1.523E+00
						Overall P	1.169E+03	2.923E+01

	Number of Mixers	Reynolds Number	$C_{mixingcup}$	$Var_{mixingcup}$	$C_{optical}$	$Var_{optical}$	Pressure'	Pressure
Units	None	None	mol/m^3	mol/m^3	mol/m^3	mol/m^3	None	Pa
	1	2	4.999E-01	1.507E-01	4.989E-01	1.507E-01	1.992E+03	4.980E+01
	2	2	5.002E-01	1.173E-01	5.051E-01	1.173E-01	1.724E+03	4.309E+01
	3	2	5.000E-01	9.351E-02	4.986E-01	9.352E-02	1.459E+03	3.649E+01
	4	2	5.001E-01	7.418E-02	5.027E-01	7.419E-02	1.191E+03	2.977E+01
	5	2	5.000E-01	5.908E-02	4.982E-01	5.909E-02	9.239E+02	2.310E+01
	6	2	5.000E-01	4.684E-02	5.035E-01	4.685E-02	6.553E+02	1.638E+01
	7	2	4.999E-01	3.731E-02	4.975E-01	3.732E-02	3.886E+02	9.715E+00
	8	2	4.998E-01	2.979E-02	4.978E-01	2.979E-02	1.219E+02	3.046E+00
						Overall P	2.345E+03	5.861E+01

	Number of Mixers	Reynolds Number	$C_{mixingcup}$	$Var_{mixingcup}$	$C_{optical}$	$Var_{optical}$	Pressure'	Pressure
Units	None	None	mol/m^3	mol/m^3	mol/m^3	mol/m^3	None	Pa
	1	10	4.991E-01	2.047E-01	5.523E-01	2.075E-01	1.070E+04	2.675E+02
	2	10	4.993E-01	1.889E-01	4.600E-01	1.905E-01	9.253E+03	2.313E+02
	3	10	4.987E-01	1.754E-01	5.477E-01	1.778E-01	7.823E+03	1.956E+02
	4	10	4.985E-01	1.616E-01	4.539E-01	1.674E-01	6.375E+03	1.594E+02
	5	10	4.984E-01	1.552E-01	5.446E-01	1.573E-01	4.934E+03	1.234E+02
	6	10	4.987E-01	1.470E-01	4.605E-01	1.484E-01	3.487E+03	8.719E+01
	7	10	4.982E-01	1.393E-01	5.391E-01	1.410E-01	2.047E+03	5.117E+01
	8	10	4.988E-01	1.321E-01	4.441E-01	1.351E-01	6.025E+02	1.506E+01
						Overall P	1.263E+04	3.157E+02

	Number of Mixers	Reynolds Number	$C_{mixingcup}$	$Var_{mixingcup}$	$C_{optical}$	$Var_{optical}$	Pressure'	Pressure
Units	None	None	mol/m^3	mol/m^3	mol/m^3	mol/m^3	None	Pa
	1	20	4.932E-01	2.155E-01	6.781E-01	2.497E-01	2.569E+04	6.423E+02
	2	20	4.961E-01	2.073E-01	3.386E-01	2.321E-01	2.219E+04	5.547E+02
	3	20	4.936E-01	1.973E-01	6.151E-01	2.120E-01	1.866E+04	4.665E+02
	4	20	4.940E-01	1.889E-01	3.509E-01	2.094E-01	1.515E+04	3.788E+02
	5	20	4.930E-01	1.774E-01	6.038E-01	1.897E-01	1.163E+04	2.908E+02
	6	20	4.933E-01	1.727E-01	3.550E-01	1.919E-01	8.129E+03	2.032E+02
	7	20	4.944E-01	1.658E-01	6.076E-01	1.786E-01	4.603E+03	1.151E+02
	8	20	4.933E-01	1.610E-01	3.412E-01	1.841E-01	1.047E+03	2.619E+01
						Overall P	3.046E+04	7.615E+02

Three-Dimensional

	Number of Mixers	Peclet Number	$C_{mixingcup}$	$Var_{mixingcup}$	$C_{optical}$	$Var_{optical}$	Pressure'	Pressure
Units	None	None	mol/m^3	mol/m^3	mol/m^3	mol/m^3	None	Pa
	1	100	4.975E-01	4.207E-02	5.112E-01	4.226E-02	1.505E+03	3.762E+01
	2	100	4.975E-01	1.044E-02	4.898E-01	1.050E-02	1.305E+03	3.262E+01
	3	100	5.005E-01	2.849E-03	5.035E-01	2.858E-03	1.114E+03	2.785E+01
	4	100	4.982E-01	5.901E-04	4.960E-01	5.907E-04	9.136E+02	2.284E+01
	5	100	4.969E-01	1.465E-04	4.974E-01	1.467E-04	7.162E+02	1.790E+01
	6	100	4.961E-01	3.042E-05	4.955E-01	3.083E-05	5.163E+02	1.291E+01
	7	100	4.954E-01	1.302E-05	4.952E-01	1.307E-05	3.202E+02	8.006E+00
	8	100	4.946E-01	1.834E-06	4.944E-01	1.908E-06	1.254E+02	3.134E+00
						Overall P	1.782E+03	4.455E+01

	Number of Mixers	Peclet Number	$C_{mixingcup}$	$Var_{mixingcup}$	$C_{optical}$	$Var_{optical}$	Pressure'	Pressure
Units	None	None	mol/m^3	mol/m^3	mol/m^3	mol/m^3	None	Pa
	1	500	4.852E-01	2.115E-01	4.956E-01	2.114E-01	1.505E+03	3.762E+01
	2	500	4.956E-01	9.943E-02	4.849E-01	9.955E-02	1.305E+03	3.262E+01
	3	500	5.044E-01	7.508E-02	5.085E-01	7.510E-02	1.114E+03	2.785E+01
	4	500	4.921E-01	4.856E-02	4.795E-01	4.872E-02	9.136E+02	2.284E+01
	5	500	4.910E-01	3.236E-02	4.936E-01	3.237E-02	7.162E+02	1.790E+01
	6	500	4.881E-01	2.415E-02	4.809E-01	2.421E-02	5.163E+02	1.291E+01
	7	500	4.920E-01	1.652E-02	4.915E-01	1.652E-02	3.202E+02	8.006E+00
	8	500	4.914E-01	1.181E-02	4.865E-01	1.183E-02	1.254E+02	3.134E+00
						Overall P	1.782E+03	4.455E+01

	Number of Mixers	Peclet Number	$C_{mixingcup}$	$Var_{mixingcup}$	$C_{optical}$	$Var_{optical}$	Pressure'	Pressure
Units	None	None	mol/m^3	mol/m^3	mol/m^3	mol/m^3	None	Pa
	1	1000	4.866E-01	1.660E-01	5.010E-01	1.662E-01	1.505E+03	3.762E+01
	2	1000	5.033E-01	1.343E-01	4.960E-01	1.344E-01	1.305E+03	3.262E+01
	3	1000	5.129E+05	1.174E-01	5.169E-01	1.174E-01	1.114E+03	2.785E+01
	4	1000	4.980E-01	8.905E-02	4.843E-01	8.924E-02	9.136E+02	2.284E+01
	5	1000	4.976E-01	7.046E-02	4.990E-01	7.046E-02	7.162E+02	1.790E+01
	6	1000	4.942E-01	6.081E-02	4.861E-01	6.088E-02	5.163E+02	1.291E+01
	7	1000	4.998E-01	4.816E-02	4.988E-01	4.816E-02	3.202E+02	8.006E+00
	8	1000	4.985E-01	4.052E-02	4.918E-01	4.057E-02	1.254E+02	3.134E+00
						Overall P	1.782E+03	4.455E+01

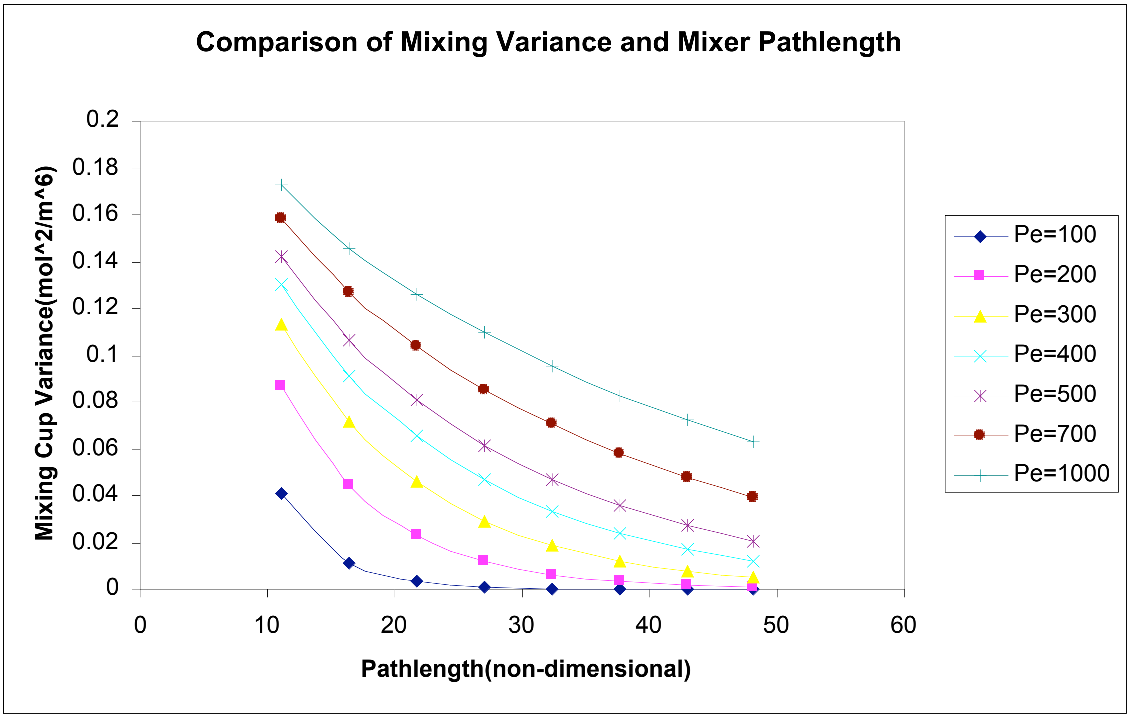

Figure 1. Mixing effectiveness described by varying the Peclet number.

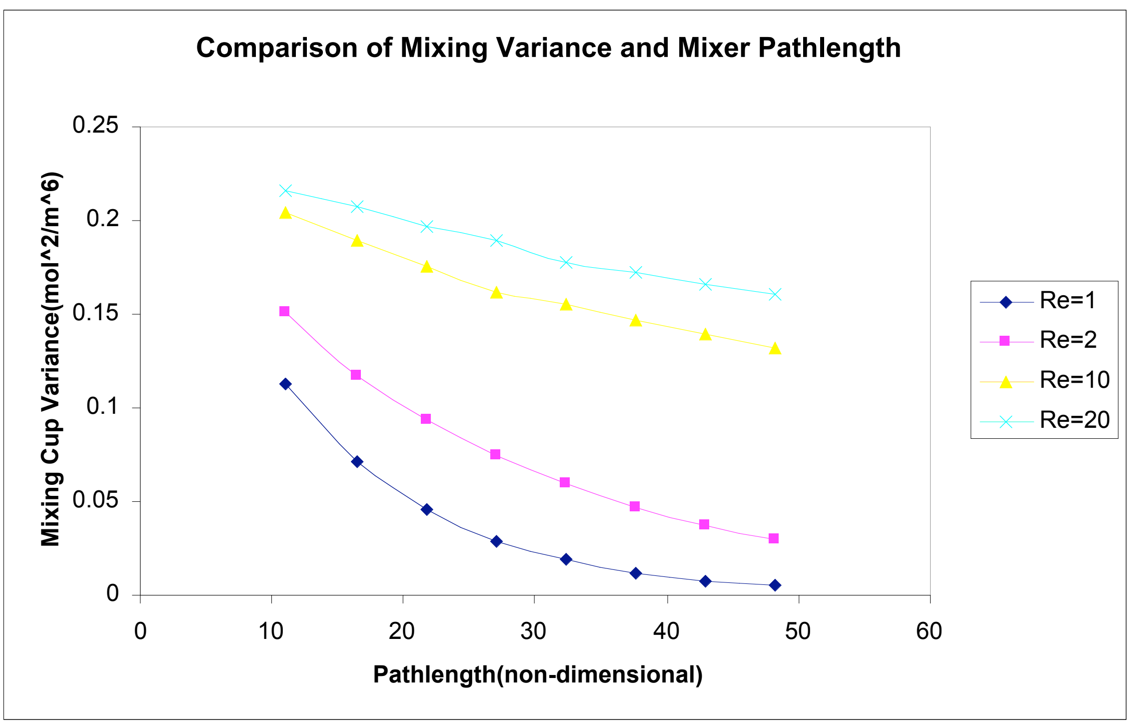

Figure 2. Mixing effectiveness described by varying the Reynolds number.

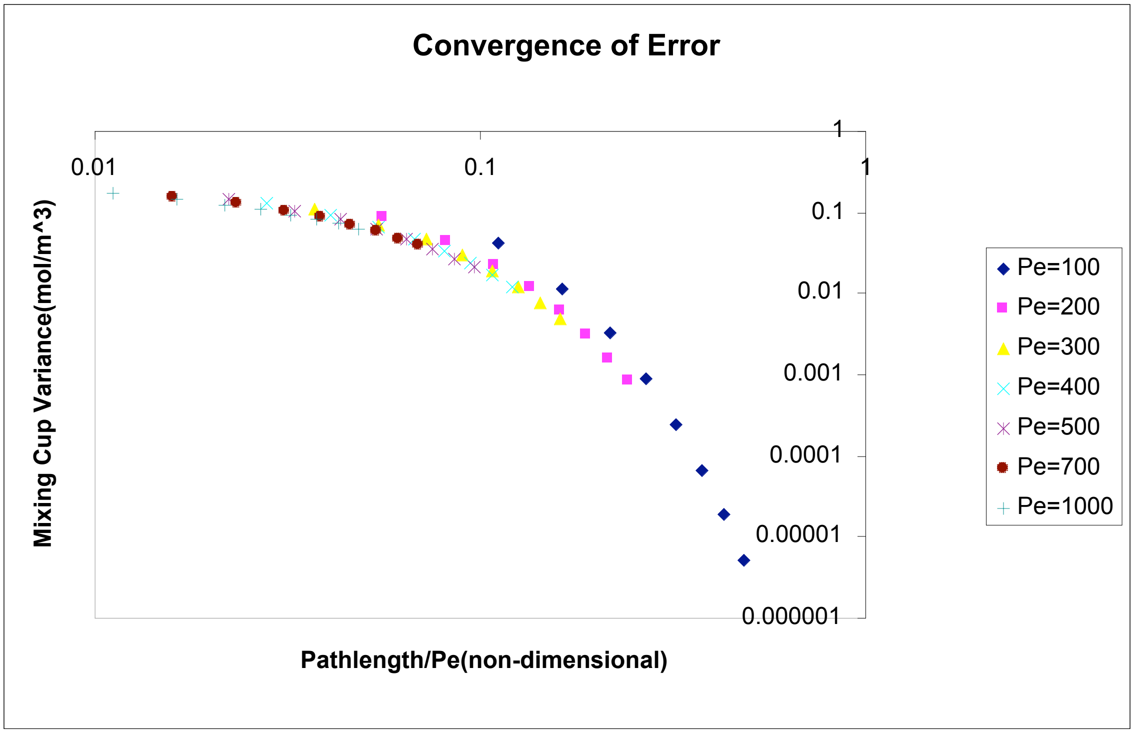

Figure 3. With the path-length in a ratio with the Peclet number, the variances for varying Peclet numbers converge to the same curve.

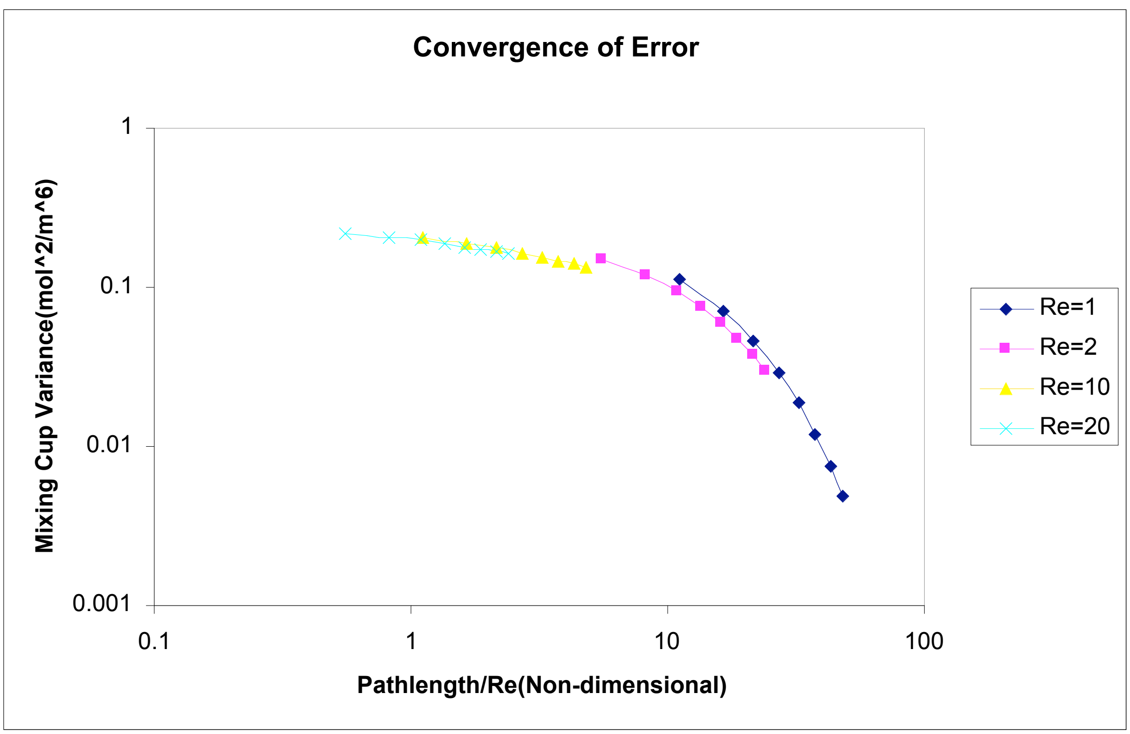

Figure 4. With the path-length in a ratio with the Reynolds number, the variances for varying Reynolds numbers converge to the same curve.

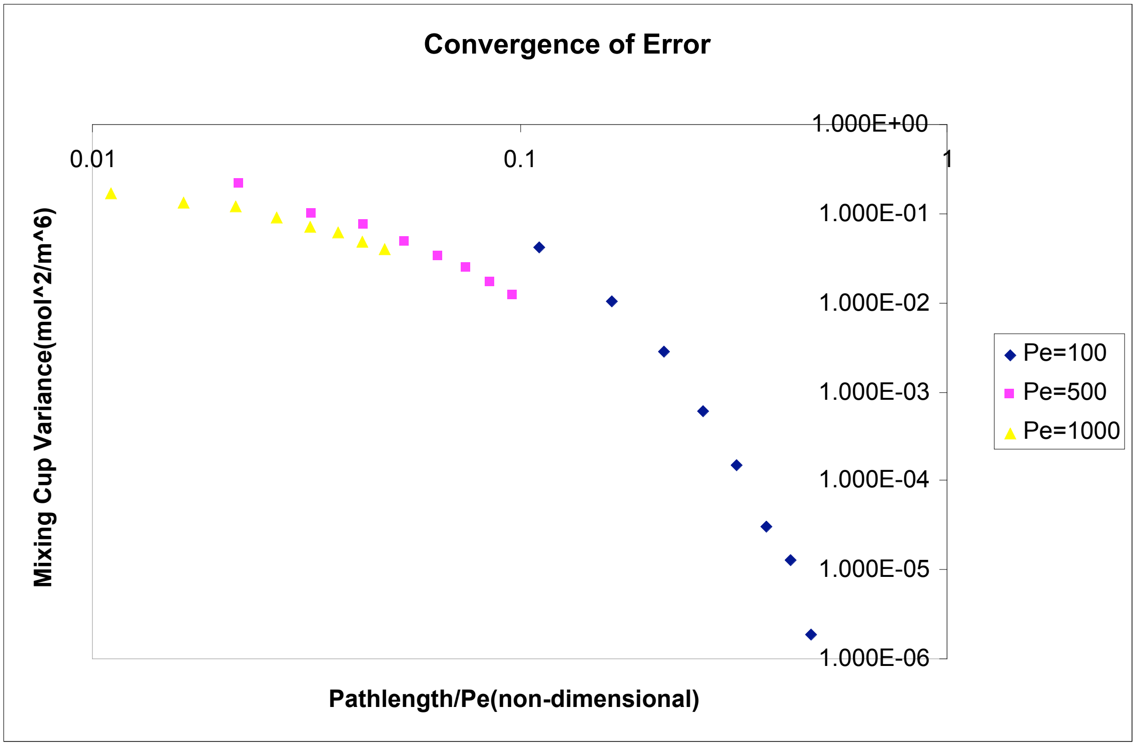

Figure 5. Convergence of variances for the three-dimensional cases calculated.

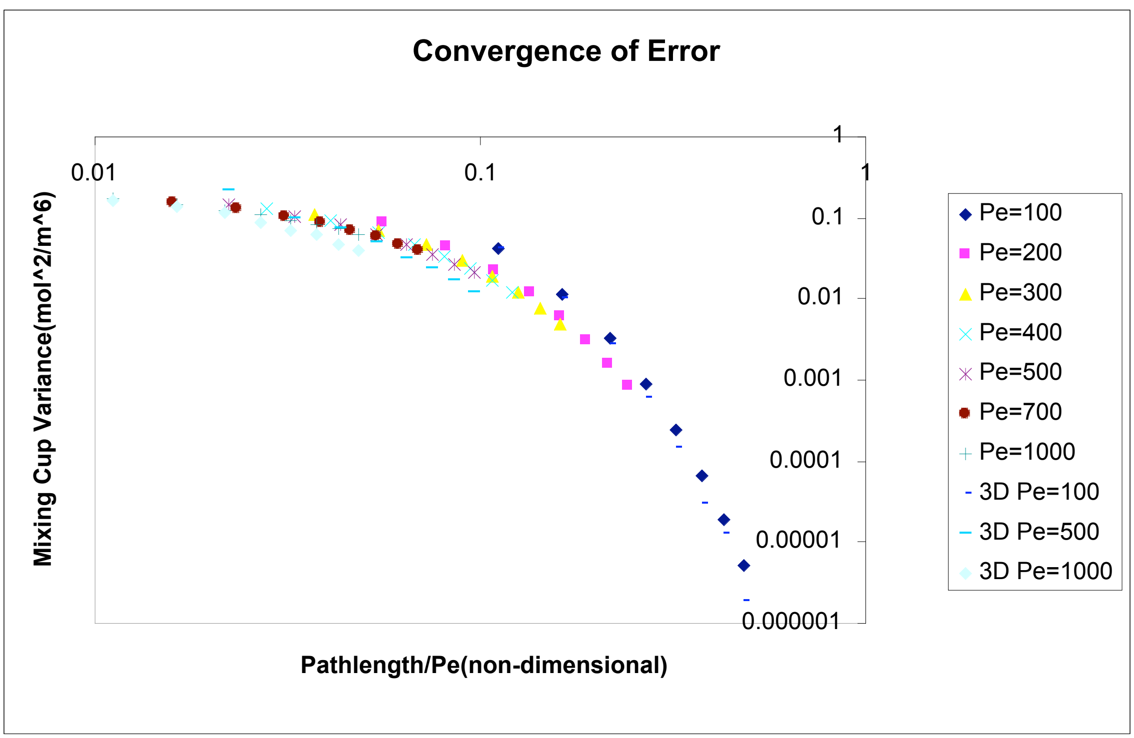

Figure 6. Convergence of errors for both the two and three dimensional cases, showing the similarities between the two geometries.

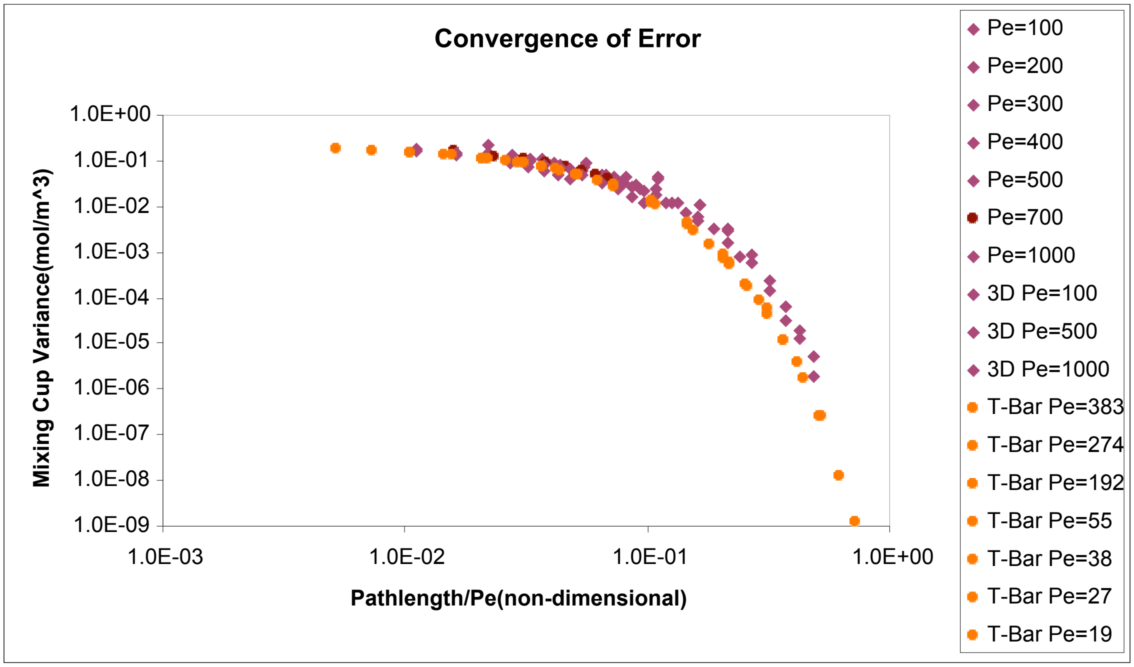

Figure 7. Comparison of the evaluated Tesla mixer and a T-shaped mixer.