

**Pressure Losses for Fluid Flow Through Abrupt Area
Contraction in Compact Heat Exchangers**

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Introduction

The goal of this experiment was to predict and improve on the results of Kays and London “entrance pressure-loss coefficient (K_c) for a multiple tube flat-duct heat exchanger core with an abrupt-contraction entrance.” [1] Kays and London determined the pressure-loss coefficient for various contraction ratios (σ) and graphed them as a line. They labeled this line as “Laminar”, but did not expound upon the range of Reynolds numbers the laminar flow line was applicable to. This experiment will graph the pressure loss coefficient for six contraction ratios and a range of Reynolds numbers to improve upon Kays’ and London’s “Laminar” flow graph.

Materials and Methods

This experiment used FEMLAB 3 version 2.3 to determine the pressure loss across the system and the maximum velocity in the outlet tube over a range of Reynolds numbers. The simulations were run in 2-D Navier-Stokes Incompressible Steady State Flow. Figure 1 shows an example of the drawing used in FEMLAB to determine this data; in this case it is a 1/3-contraction ratio. The contraction ratios analyzed in this experiment were, 0.90, 0.75, 0.67, 0.5, 0.33, and 0.2. The contraction ratio was calculated from the exit height over the inlet height. Height refers to the distance from the line of symmetry (boundary 2) to the respective boundary (boundary 3 for the entrance tube and boundary 5 for the exit tube). The height of the exit pipe was held constant at 0.25. The exit pipe height of 0.25 was chosen based on the hydraulic diameter for irregular shapes Eq. 1 [2].

$$D_H = \frac{4A}{C} \quad \text{Eq. 1}$$

The height of the entrance was varied to attain the desired contraction ratios. Figure 1 also shows the boundary conditions applied to some of the boundaries.

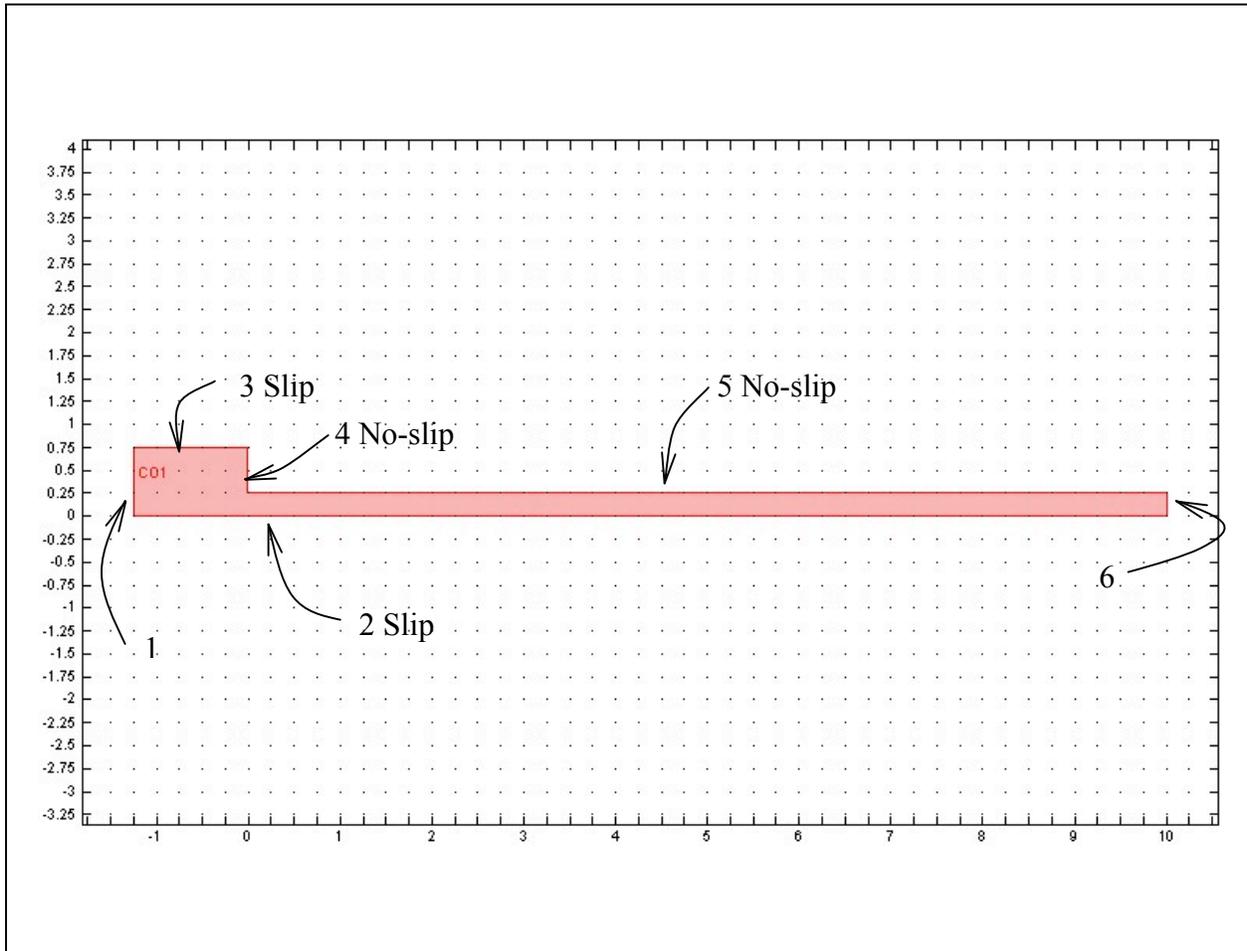


Fig. 1 2-D Abrupt Contraction Ratio 1/3

Boundary one was the inlet/outlet velocity. The x-direction velocity at boundary one was set equal to the contraction ratio; the contraction ratio was chosen so that the average velocity in the exit tube would be unity. Boundaries two and three were set as slip boundaries. This would create the desired symmetry to simulate the heat exchangers flow area contraction. Boundary six was set as the inlet/outlet pressure boundary and set equal to zero. The remaining boundaries, four and five, were set as no-slip boundaries. The length of the entrance analyzed was chosen to

be 1.25, to allow for fully developed flow entering the contraction area. The length of the exit tube was chosen to be 10, to allow for fully developed flow to exist at the exit boundary. In the sub-domain settings the density (ρ) was set equal to 10^5 . By varying the density the Reynolds number would also be varied. Using parametric solver “s” was varied from -3 to 3 by 0.1 increments. This would yield data for Reynolds number from 0.001 to 1000 .

Figure 2 shows the mesh applied to the drawing for the $1/3$ -contraction ratio. The mesh refinement for all the contraction ratios averaged 10642 elements with a standard deviation of 1726 elements, and a range from 9121 to $13,920$ elements. The mesh refinement was concentrated in the contraction area since this is the area with the most flow disturbances occurring and thus the most critical for analysis.

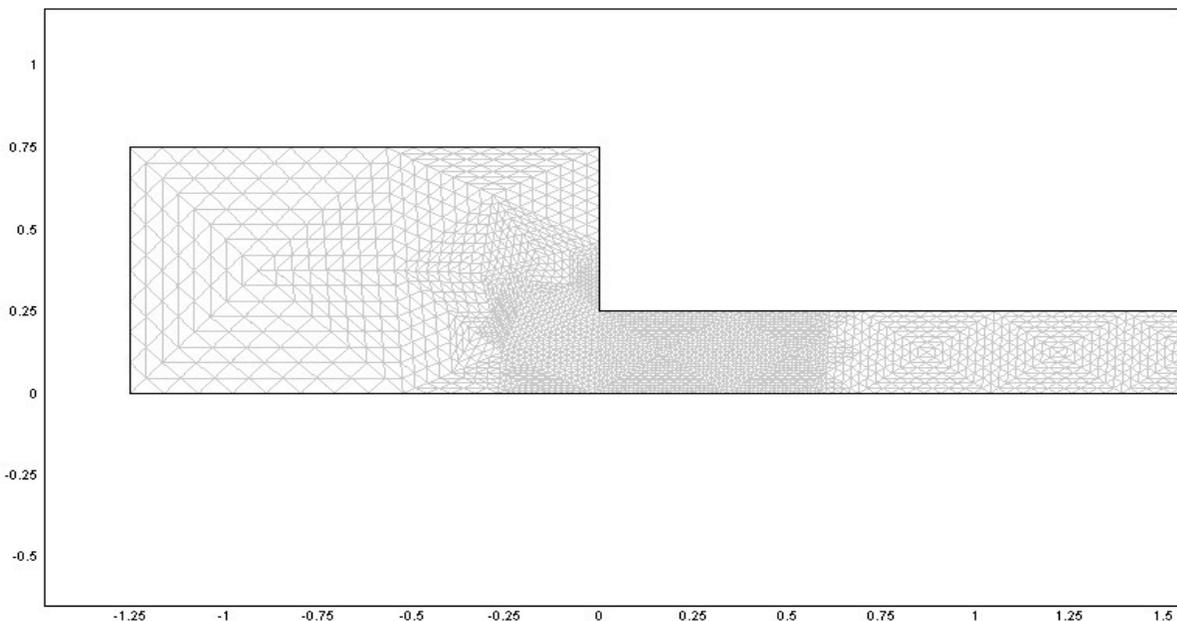


Fig. 2 Mesh Refinement for 2-D Abrupt Contraction Ratio $1/3$ (This is the abrupt contraction area. The small tube refinement repeats the same geometry from 1 to 10.)

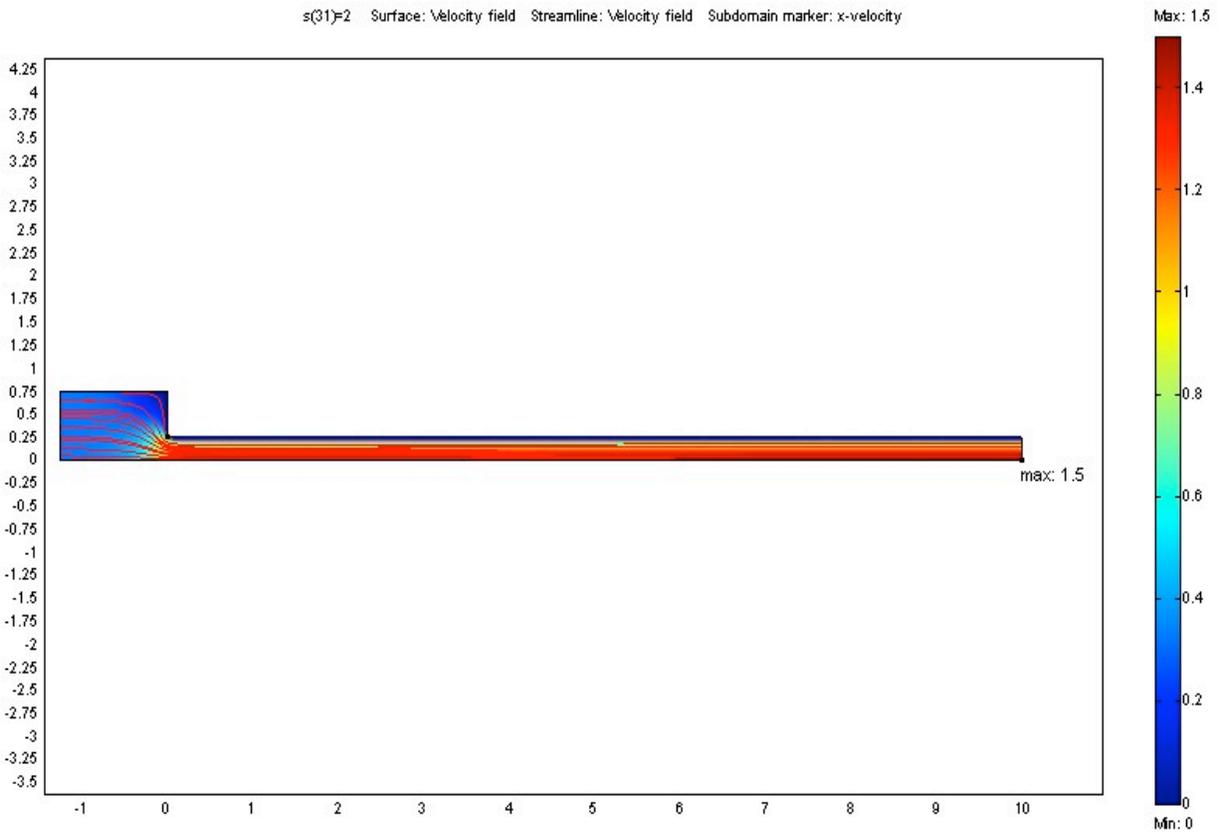


Fig. 3 Graphical Representation of 2-D Abrupt Contraction Ratio 1/3

Figure 3 shows a graphical representation for the results of a 1/3-contraction ratio. This is for a Reynolds number of 100. The x-velocity lines are graphed and the minimum and maximum x-velocities are displayed.

Figure 4 a is blow up of the throat area of Fig. 3.

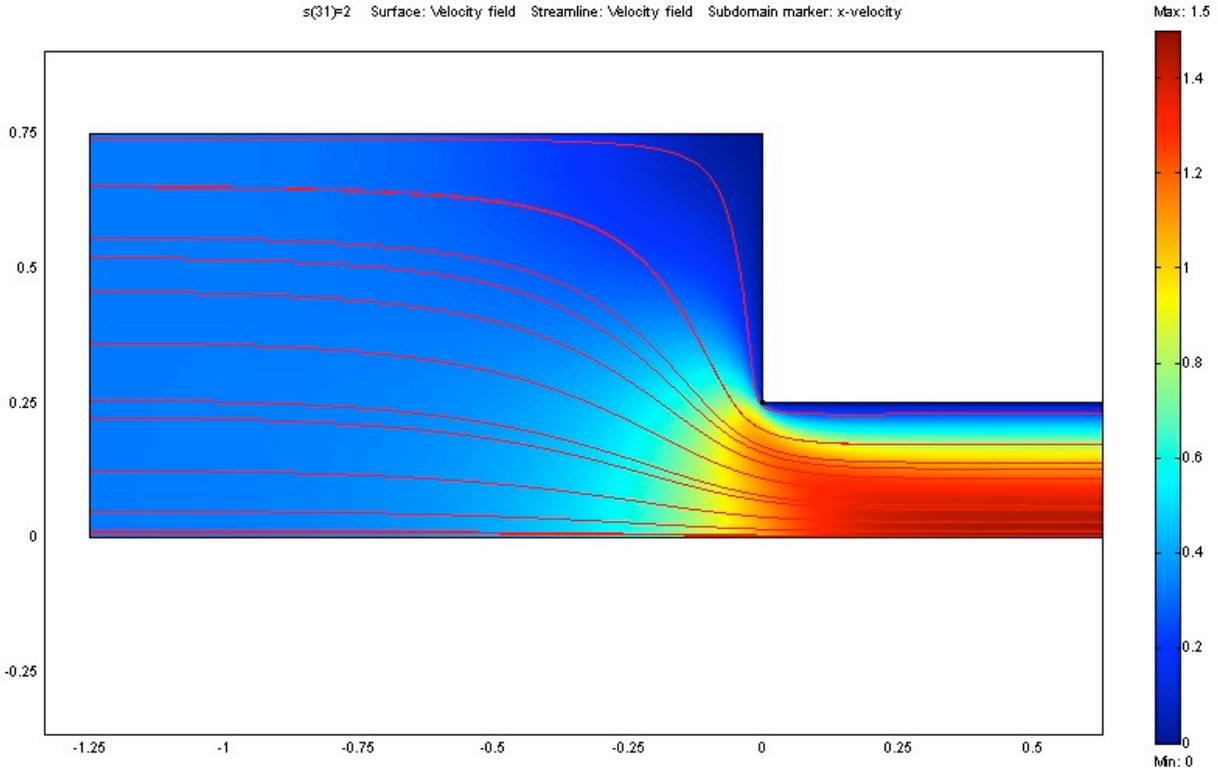


Fig. 4 Throat Area Graphical Representation of 2-D Abrupt Contraction Ratio 1/3

With the problem solved in FEMLAB, the pressure loss across the system was obtained from Eq. 2 and the FEMLAB data.

$$\Delta p_{system} = \frac{\int p^* dA}{\int dA} \quad \text{Eq. 2}$$

In FEMLAB this was accomplished by integrating for the pressure over boundary 1 and then dividing by the entrance height. The pressure loss due to friction in the entrance area, the area prior to contraction, was zero since there are no friction losses at the boundaries. The pressure loss in the small tube due to friction losses was calculated from Eq. 3.

$$\Delta p_{exitube} = \frac{2 * u_{max} * L * \mu}{H^2} \quad \text{Eq. 3}$$

Where:

u_{max} is the maximum x-velocity in the small tube

L is the length of the small tube

μ is the viscosity of the liquid

H is the height of the small tube (boundary 2 to boundary 5)

The maximum x-velocity was obtained from the FEMLAB results. The pressure loss due to contraction was then calculated from Eq. 4.

$$\Delta p_{contraction} = \Delta p_{system} - \Delta p_{entrance} - \Delta p_{exitube} \quad \text{Eq. 4}$$

Recall that the pressure loss due to friction in the entrance area was zero.

The difference in pressure due to contraction was converted to find the pressure-loss coefficient (K_{c_FEMLAB}). Equation 5 [1] is from Kays and London.

$$\frac{\Delta p_1}{\rho} = \frac{V^2}{2g_c} (1 - \sigma^2) + K_c \frac{V^2}{2g_c} \quad \text{Eq. 5}$$

Where the variable:

Δp_1 is the pressure loss due to contraction

ρ is the density of the fluid

V is the velocity in the small tube

σ is the contraction ratio (“core free-flow to frontal-area ratio”[1])

K_c is the pressure-loss coefficient due to abrupt contraction

Equation 5 was rearranged to solve for $\frac{\Delta p_1}{\rho V^2}$, this is Eq. 6.

$$\frac{\Delta p_1}{\rho V^2} = (1 - \sigma^2) + \frac{K_c}{2} \tag{Eq. 6}$$

Equation 6 was set equal to $\frac{\Delta p_{contraction}}{Re}$ and then solved for K_c yielding Eq. 7.

$$K_c = 2 * \left[\frac{\Delta p_{contraction}}{Re} - \frac{(1 - \sigma^2)}{2} \right] \tag{Eq. 7}$$

Equation 7 was used to calculate the pressure-loss coefficient based on the FEMLAB data. The pressure-loss coefficient calculated in Eq. 7 was graphed against Fig. 6 from Kays and London [1]. (The “Laminar” line on the graph for K_c is the line of concern for this experiment.)

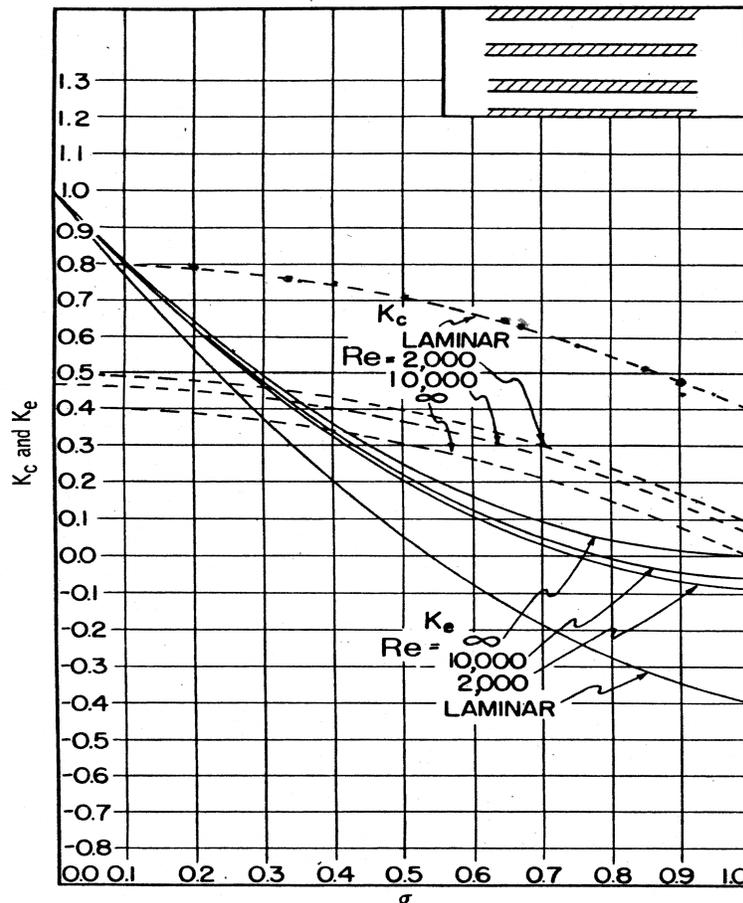


Fig. 6 Entrance and exit pressure-loss coefficients for a multiple-tube flat-duct heat exchanger core with abrupt-contraction entrance and abrupt-expansion exit. [1]

Equation 8 was used to compare the pressure loss calculated by the FEMLAB based pressure-loss coefficient and the pressure loss based on Kays and London pressure-loss coefficient for a given contraction ratio. These pressure losses were graphed vs. the Reynolds number.

$$\Delta p = \text{Re} \left[\frac{(1 - \sigma^2)}{2} + \frac{Kc}{2} \right] \quad \text{Eq. 8}$$

Equation 8 is a rearrangement of Eq. 7.

Results and Discussion

The FEMLAB based pressure-loss coefficient for a given Reynolds number was graphed for the various contraction ratios. In Fig. 7 the Reynolds numbers one and ten are graphed against the “Laminar” points from Fig. 1 [1].

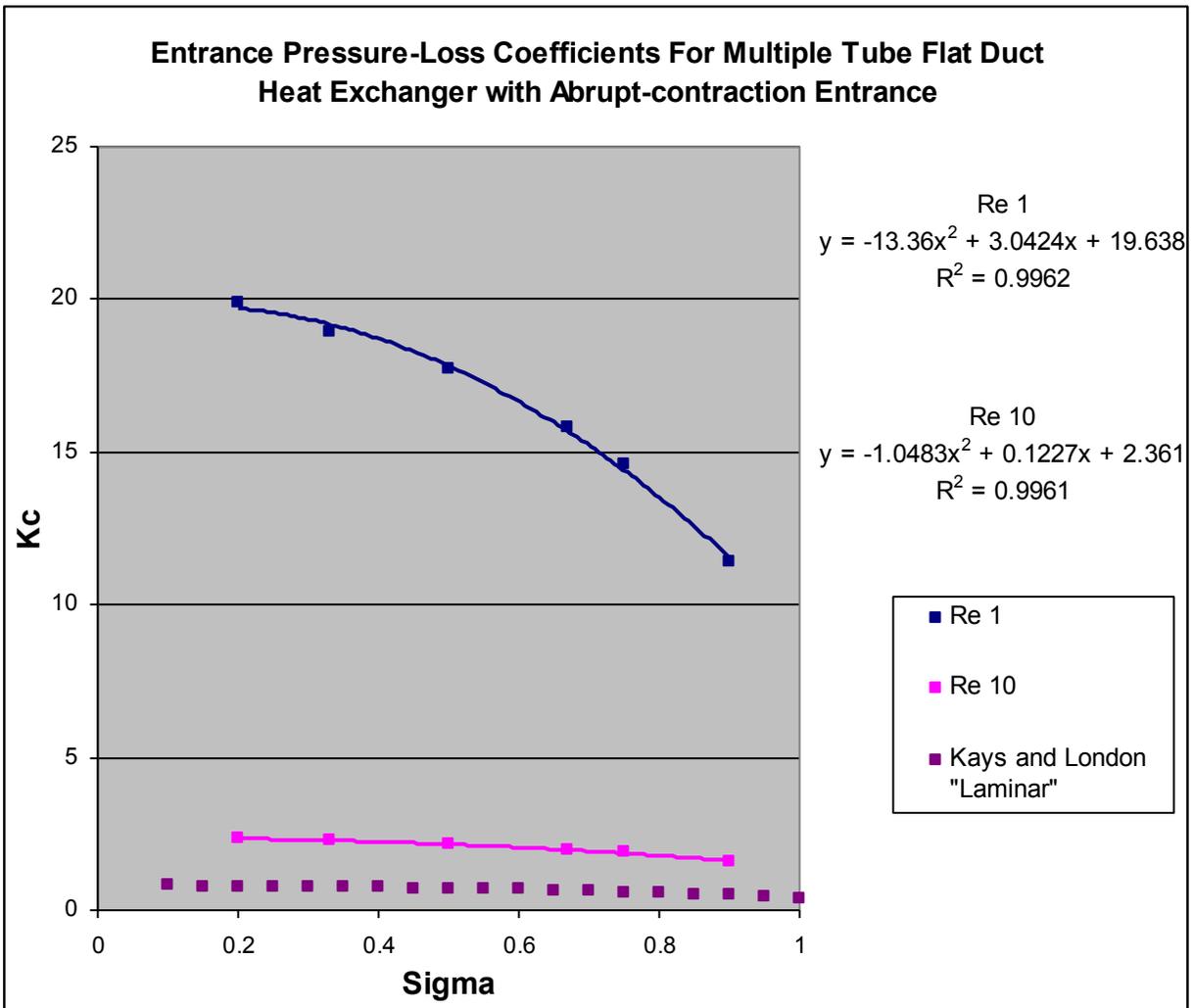


Fig. 7 Pressure-Loss Coefficient for Multiple Tube Flat Duct Heat Exchanger with Abrupt-Contraction Entrance for Kays and London “Laminar” and FEMLAB Reynolds Number One and Ten

The blue line in Fig. 2 is the best-fit line that represents the pressure-loss coefficient, based on the Reynolds number of 1. The pink line in Fig. 7 and Fig. 8 is the best-fit line that represents the pressure-loss coefficient, based on the Reynolds number of 10. Both of the best-fit lines are quadratic in form.

Figure 8 shows the Reynolds numbers 10, 100, and 1000 as they compare to “Laminar” points from Fig. 6 [1].

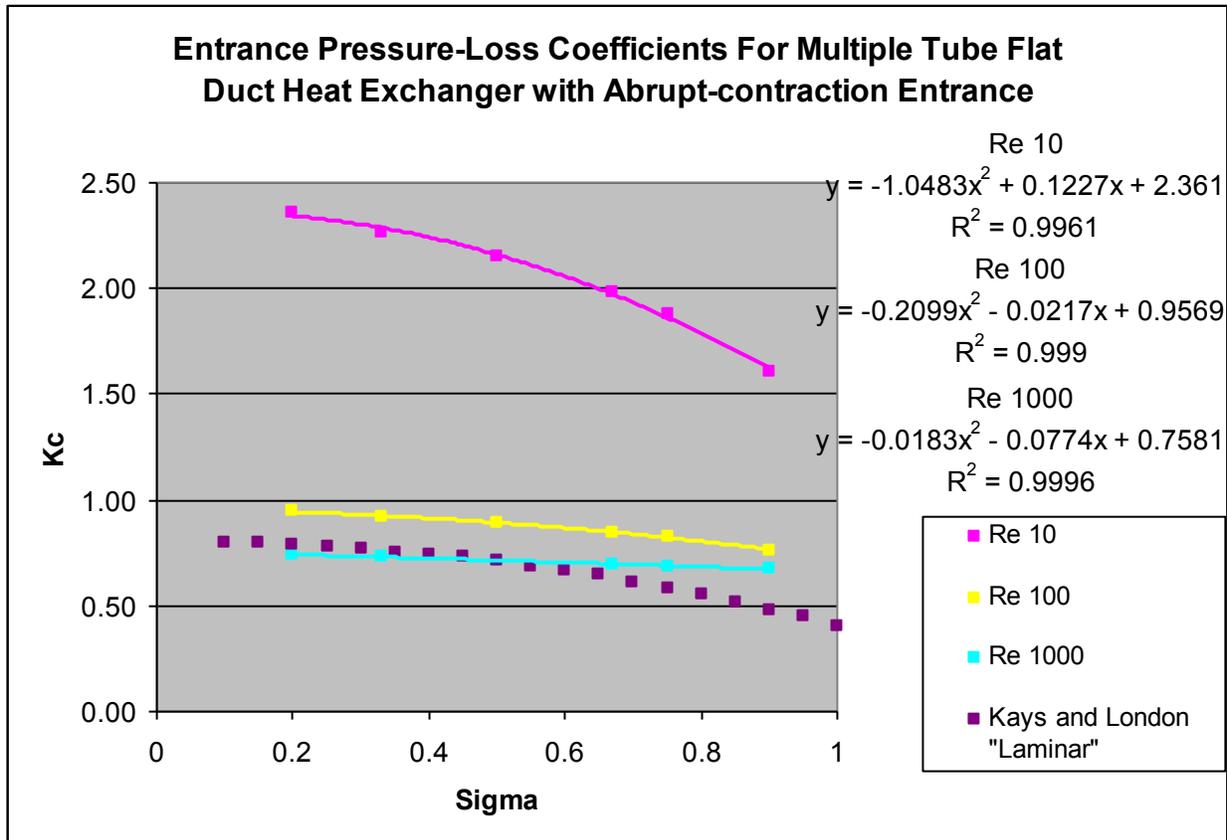


Fig. 8 Pressure-Loss Coefficient for Multiple Tube Flat Duct Heat Exchanger with Abrupt-Contraction Entrance for Kays and London “Laminar” and FEMLAB Reynolds Number 10, 100 and 1000

The pink line in Fig. 8 is the best-fit line that represents the pressure-loss coefficient, based on the Reynolds number of 10. The yellow line in Fig. 8 is the best-fit line that represents the pressure-loss coefficient, based on the Reynolds number of 100. The light blue line in Fig. 8 is the best-fit line that represents the pressure-loss coefficient, based on the Reynolds number of 1000. All three of these best-fit lines are quadratic in form.

The pressure-loss coefficient for Reynolds numbers less than 100 was significantly different, greater than 15 percent error, than Kays' and London's "Laminar" pressure-loss coefficient. The percent errors for the pressure-loss coefficients for Reynolds numbers 100 to 1000 are displayed in Table 1.

Table 1 Percent error between the FEMLAB based Kc and Kays' and London's Kc

% Error Between the FEMLAB Based Kc and Kays' and London's Kc						
Reynolds Number	Femlab Based Kc for the following σ					
	0.9	0.75	0.67	0.5	0.33	0.2
100	37	30	26	20	18	17
126	36	27	23	17	14	13
158	34	25	21	14	11	10
200	33	24	19	12	8	7
251	32	22	17	10	6	4
316	32	21	16	8	4	2
398	31	20	14	6	2	0
501	31	19	13	5	0	2
631	30	18	12	3	1	4
794	30	17	11	2	3	5
1000	29	16	10	1	4	7

Table one shows that the percent error difference for the contraction ratios 0.9 and 0.75 do not get below ten percent error within the analyzed Reynolds numbers. The remaining contraction ratios do come within ten percent error. The smaller contraction ratios 0.33 and 0.2 at Reynolds numbers higher than 158 come close and even match Kays and London.

Figure 9 shows the pressure losses due to contraction over the range of Reynolds numbers from 0.01 to 100 due to the contraction ratio of 1/3.

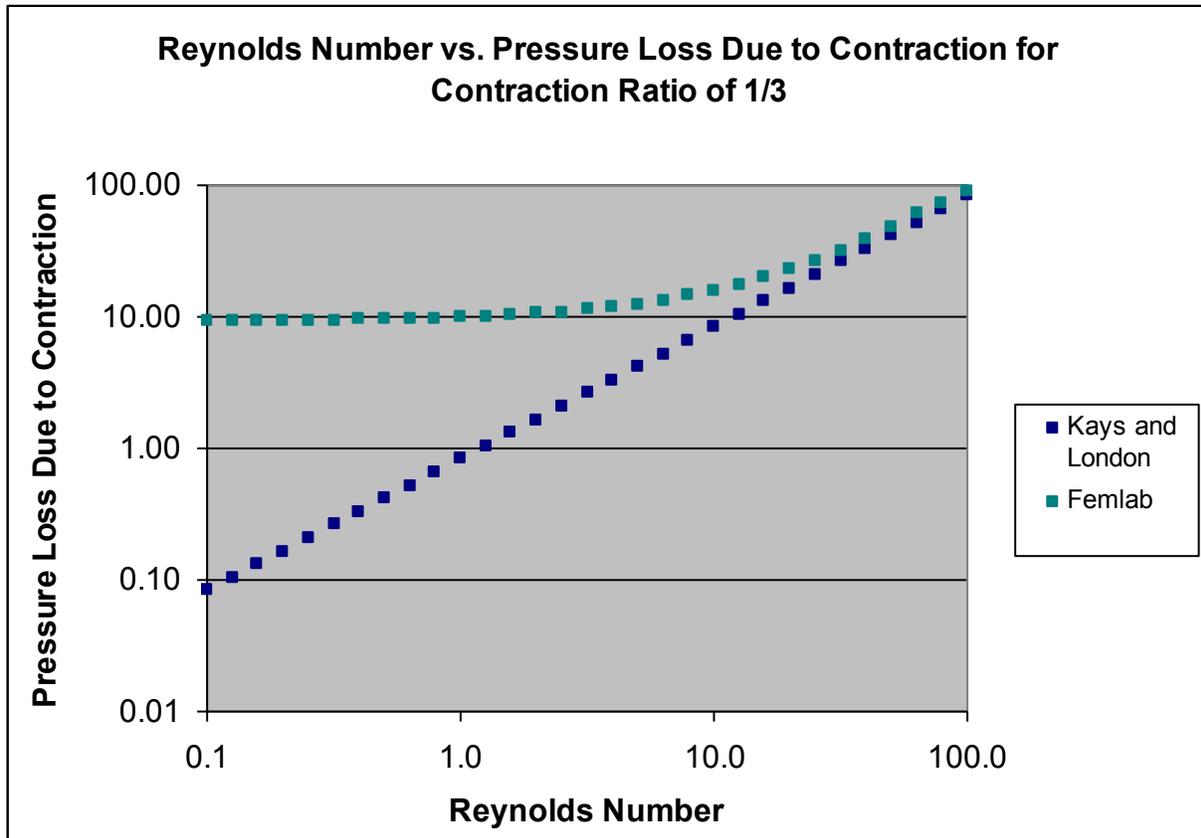


Fig. 9 Pressure Loss Due to the Abrupt Contraction Ratio of 1/3 vs Reynolds Number

Both of the axes on the graph are log rhythmic. The blue points are the pressure losses calculated from Eq. 8 for the Kays and London pressure-loss coefficient. The green points are the pressure losses calculated from Eq. 8 for the pressure-loss coefficient based on the FEMLAB data. Figure 9 shows that the pressure-loss coefficient is independent of the Reynolds number below a Reynolds number of 10.

Figure 10 shows the percent error between the pressure loss based on FEMLAB data and the Kays and London “Laminar” points. The percent errors were graphed over a range of Reynolds numbers. The range of Reynolds numbers in the graph were chosen to show where the percent error difference becomes small enough to be considered negligible.

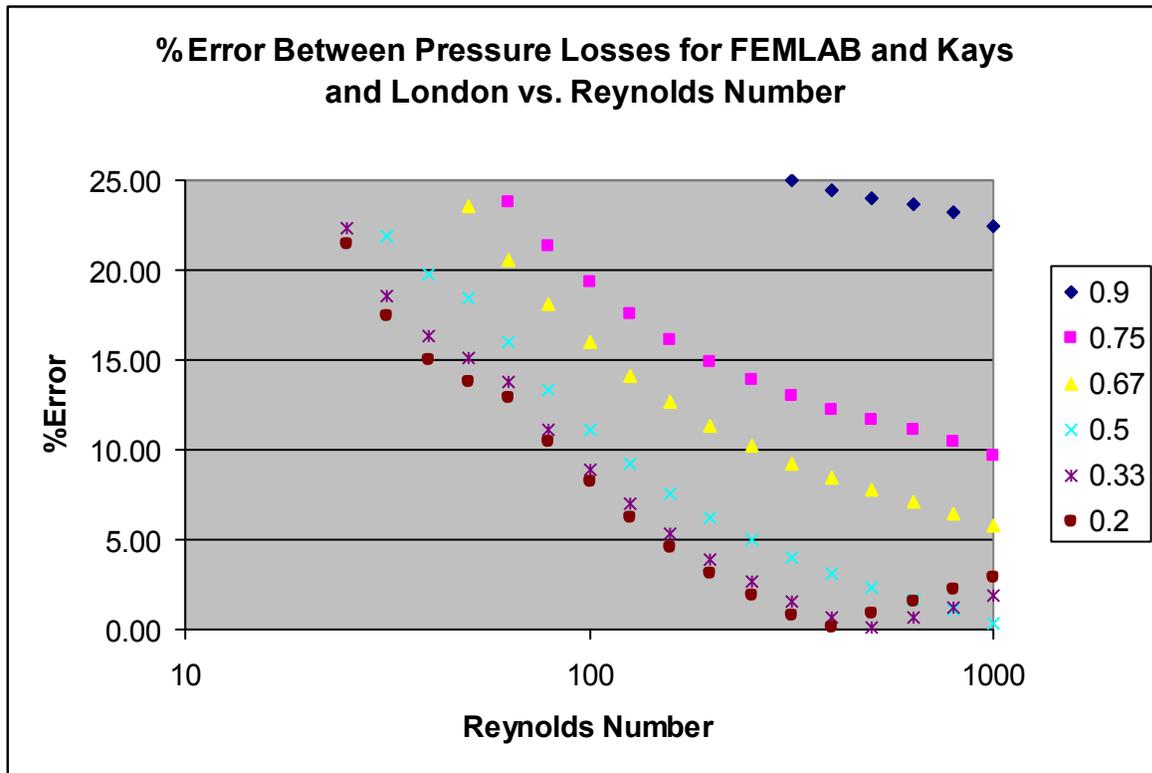


Fig. 10 Percent error losses between the pressure losses due to abrupt contraction for the FEMLAB and Kays and London over a range of Reynolds numbers. The Reynolds number axis is log rhythmic scale.

Note that percent error in pressure losses for the contraction ratios 0.5, 0.33 and 0.2 are the only ones that become small enough to be considered negligible. This occurs at Reynolds numbers greater than 200.

Conclusions and Recommendations

Kays' and London's "Laminar" pressure-loss coefficient were improved upon. The pressure-loss coefficients can be red for a larger range of laminar flow Reynolds numbers. This will provide more accurate pressure loss data.

The mesh for the 2/3 contraction ratio was 3000 elements larger than the average. I recommend refining the mesh so that it is closer to the average, or refining the mesh of the other contraction ratios to coincide with the 2/3 contraction ratio mesh.

I did not analyze the pressure changes due to changes in kinetic energy. I recommend that this be done to show where the pressure changes due to kinetic energy becomes significant.

I would also analyze more contractions ratios. This is easy enough using FEMLAB, just time consuming. I would analyze contraction ratios at 0.05 increments, since I was able to read off Kays and London pressure-loss coefficients at this interval.

Literature Cited

1. Kays, W. M., and London, A. L. Compact Heat Exchangers, 3rd edition, p. 109, 112, McGraw-Hill, New York, (1984).
2. Bird, R.B., et al. Transport Phenomena, 2nd edition, p. 183, John Wiley & Sons, New York, (2002).