# Surface Tension of Bubbles

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### Introduction

Surfactants are an important part of many of the things we use every day, including our lungs. Surfactants lower the surface tension of a fluid and are used in many adhesives and paints acting as a bridge between two phases. In addition surfactants are present in our lungs, however many prematurely born babies are born with insufficient amounts of these vital chemicals. Artificial surfactants are important to the survival of these infants until their bodies can naturally produce sufficient amounts of surfactant. The purpose of this study was to verify the results of Annelise Barron's experiment using Comsol. If the same experiment could be carried out using this computer program it would be much easier to initially test new surfactants and see how they compare to existing surfactants.

The experiment was a cuvet placed in the top of a container. This container was filled with a fluid that contained a surfactant. Air was then blown through the cuvet until an air bubble formed in the fluid. At the bottom of the container there was a pressure control, so that the pressure in the container can be controlled. By varying the pressure in the container the bubble size will also be varied. It is possible to measure the surface tension of this bubble, by knowing the size, shape, and pressure drop of the bubble. As the bubble varies in size the concentration of the surfactant changes and therefore the surface tension of the bubble. By oscillating the pressure and therefore the size of the bubble the relationship between concentration and surface tension was developed.

The experiment was set up in Comsol using the container as the reference frame and it unsuccessful in duplicating the results, because Comsol could not converge to a solution. After much trial and error the problem was instead solved in MatLab using the ode45 solver. These solutions are shown below and can be used to verify Comsol solutions.

While creating these solutions in MatLab a new reference frame was discovered and the problem in Comsol again but instead of focusing on the container as was done before, the problem was solved from the reference frame of the cuvet and a solution that matched the MatLab solution was obtained. In addition to the MatLab solution is the Comsol solution, geometry, and boundary conditions that were used to obtain the solution. These should make it possible to verify the physical surface tension problem in Comsol.

### MatLab Code Solution

Below is the code that was used to generate results in MatLab. This code integrates the differential equations until the diameter of the tube is reached. This diameter is set to 4mm or 0.04cm. Several results are given below in figures 1 and 2. These figures have a constant density of  $1g/cm^3$  and a surface tension of 70 dynes/cm.

```
%Units are in cgs
% run bubble3.m
function run bubble3
global pgas rho grav sigma
pgas = 500;
rho = 1;
grav = 980;
sigma =70;
ystop = 0.04;
options = odeset('Events',@events);
[t y te ye ie]=ode45(@bubble3,[0 .3],[0 0 0],options, ystop)
y(:,2) = y(:,2) - (max(y(:,2)))
plot(y(:,1),y(:,2),'-b');
8-----
% bubble shape rhs
function ydot=bubble3(t,y,ystop)
global pgas rho grav sigma
x = y(1);
yy = y(2);
theta = y(3);
ydot(1) = cos(theta);
ydot(2) = sin(theta);
ydot(3) = (pgas+rho*grav*yy)/sigma;
ydot=ydot';
§_____
% event function
function[value, isterminal, direction] = events(t, y, ystop)
value=y(1)-ystop;
isterminal = 1;
direction = 0;
```

#### Results



Below are the Matlab solutions that were solved using the above code.



# **Comsol Boundary Conditions**

Below is the solution to the problem using Comsol. This solution looks at the problem from the angle of the cuvet instead of the container the bubble is forming in. The solution is for a 2D model with 2 physical models, transient ALE and Navier Stokes. The geometry is a rectangle which is axial symmetric, with a length of ?, and a width of 4mm or .04cm. Below are the given boundary conditions needed to solve problem.

#### **ALE Boundary Conditions:**





#### **Navier Stokes Boundary Conditions:**

Below are the boundary conditions for the Navier Stokes equation set. The inlet condition should be set as Inlet pressure as 0 Pa. The open boundary at the bottom should be set such that pressure on this boundary is the pressure drop across the bubble. This pressure can be varied to create the oscillating bubble.



Open Boundary

# **Comsol and Matlab Solution**

Below are two graphs, these are the solutions from the identical problem solved in both Comsol and Matlab. This shows that the Comsol solution is indeed the correct solution. The first of these graphs is from Matlab and the second is from Comsol.

