Professor Finlayson

Andrew Nordmeier

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Introduction

Tear drop mixers are passive mixers that are useful for low Reynolds numbers in the range from 1 to 100. For higher Reynolds numbers, a different mixer should be chosen. The purpose of this research was to demonstrate the effects of changing the Peclet number.

Background

Both the Reynolds number and Peclet number are useful non-dimensional numbers in fluid flow problems. The Reynolds number is defined as:

$$\mathrm{Re} = \frac{\rho D u_s}{\mu} \tag{1}$$

where Re is the Reynolds number, ρ is the density, D is the characteristic length (in this case, diameter), u_s is the velocity, and μ is the viscosity. Defined in this way, the Reynolds number is the ratio of inertial to viscous forces. The Peclet number is defined as:

$$Pe = \frac{u_s * D}{D_m} \tag{2}$$

where Pe is the Peclet number and D_m is the mass diffusivity. The Peclet number is a ratio of the time for diffusion to the time of convection

The mixing cup concentration measures the amount of mixing. This is the concentration of the fluid if the flow emptied into a cup that was well stirred. It is defined as:

$$c_{mixing\ cup} = \frac{\int_A c(x,y,z)\, v(x,y)\, dxdy}{\int_A v(x,y)\, dxdy} \tag{3}$$

where $c_{mixing\ cup}$ is the mixing cup concentration, A is the area, c is the concentration, and v is the velocity.

The variance from the mixing cup concentration is then defined as:

$$\sigma_{mixing\ cup} = \frac{\int_A \left[c(x,y,z) - c_{mixing\ cup}\right]^2 v(x,y)\, dxdy}{\int_A v(x,y)\, dxdy} \tag{4}$$

The optical average concentration is defined as:

$$c_{optical} = \int_0^L c(x,y,z)\, dy \,/ \int_0^L dy \tag{5}$$

where L is the path length of the velocity. The optical average concentration may differ from the mixing cup concentration. The optical concentration gives the concentration perpendicular to the velocity, whereas the mixing cup concentration gives the concentration over the entire flow.

The variance from the optical concentration is then defined as:

$$\sigma_{optical} = \int_0^L \left[c\left(x,y,z\right) - c_{optical} \right]^2 dy \,/\, \int_0^L dy \qquad (6)$$

Boundary Conditions

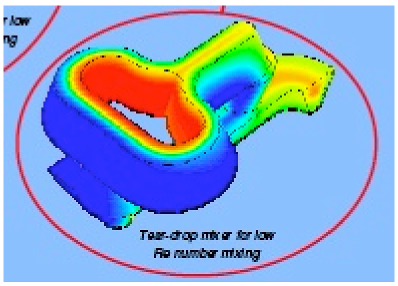

Figure 1. Tear drop mixer.

Figure 1 shows the shape of the tear drop mixer. It was modeled in Comsol as shown in Figure 2 (for the 2D case) and Figure 3(for the 3D case). For the 2D case, the fluid enters at the top of mixer, flows around, and exits at the bottom of the mixer. The 3D case is similar: the fluid enters at the bottom of the mixer, flows around the ellipse and through the mixer, and then exits through the straight portion of the mixer, again at the bottom.

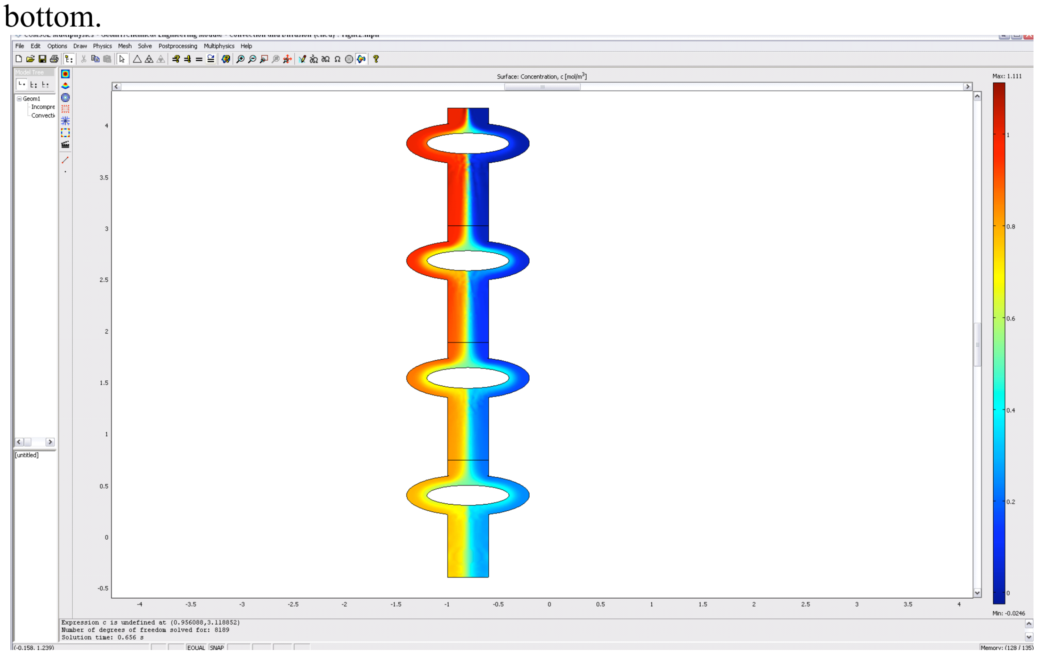
Figure 2. Comsol model for tear drop mixer. The concentration profile shown is for a Peclet of 500.

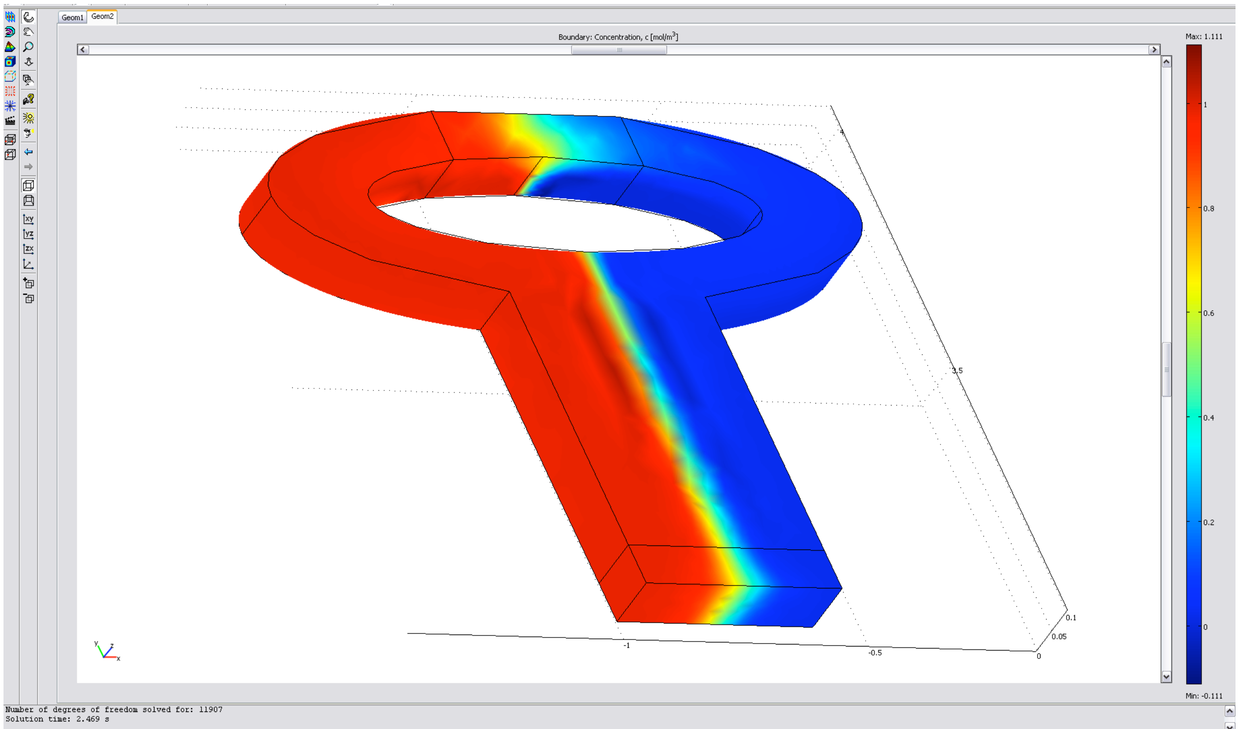

Figure 3. 3D shape in Comsol model. The concentration shown is for a Peclet number of 500.

For the velocity profile, the two main boundary conditions are the inlet and outlet. The outlet should be set to zero pressure, while the inlet has a velocity profile. Every other boundary is a wall with a no-slip condition. The density and viscosity should both be set to 1.

For convection, the outlet should be diffusive flux, the inlet should have a concentration profile that averages .5 (that is, 1 on one half of the inlet and 0 on the other half of the inlet), and all other boundaries should be insulated.

Results – 2D

Unless otherwise noted, all of these results were obtained with a mesh of 3856 elements and 8189 degrees of freedom. Table 1 shows results obtained for a Peclet number of 300. A complete table showing all results can be found in Appendix A.

Table 1. Results obtained for a Peclet number of 300. A table with all results can be found in Appendix A.

Pe	300									
Boundary	v	c*v	c_{mix}	var_{mix}	L	c_{opt}	var_{opt}	z	z/Pe	Pressu
2	0.39191	0.202125	0.515743	0.0105	0.4	0.515756	0.0132	6.55712	0.021857	
6	0.392093	0.202248	0.515816	0.022807	0.4	0.515503	0.0284	4.91784	0.016393	576.2
10	0.392052	0.201974	0.515171	0.049876	0.4	0.514449	0.0620	3.27856	0.010929	1151.9

Figure 4 shows a log-log plot of the variance versus the characteristic length/Pe ratio for Pe numbers equaling 100, 200, 300, 500, 700, and 1000.

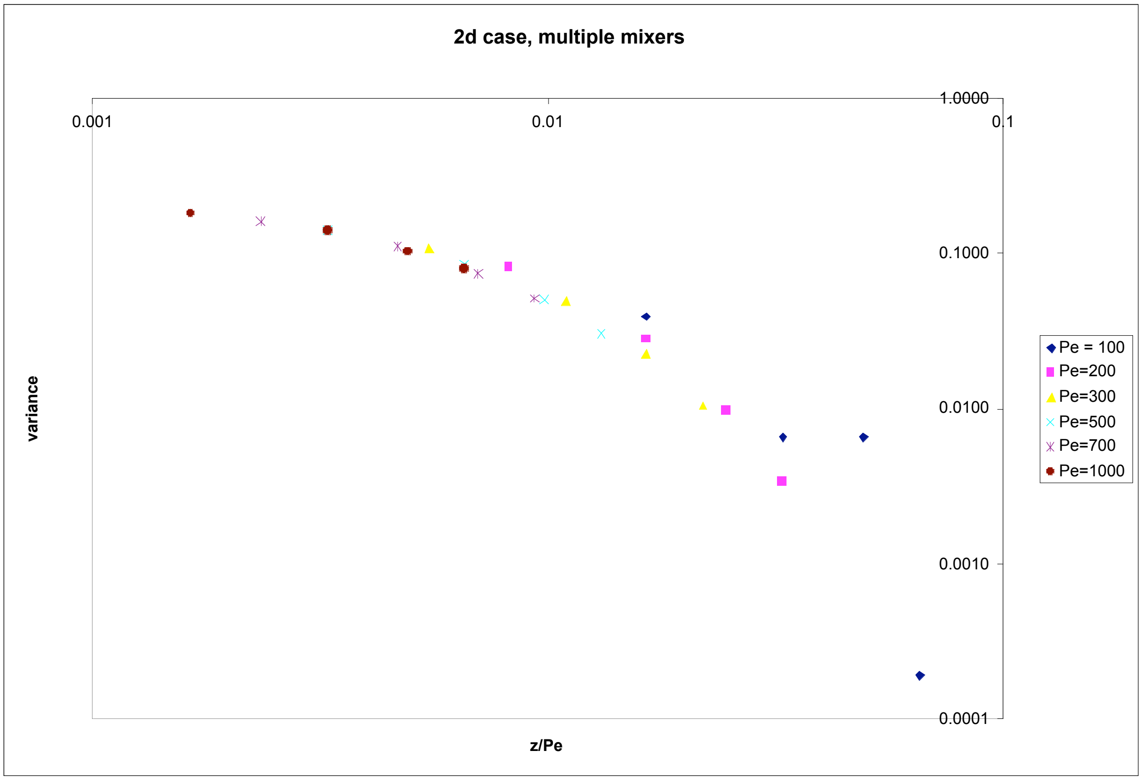

Figure 4. Log-log graph of the variance versus the characteristic length (z) to Pe ratio. The variance increases with increasing Pe number, meaning that the variance decreases as the z/Pe ratio increases.

As expected, the plot collapses onto one curve for all cases.

Results -2D with multiple meshes

For a Peclet number of 500, three different meshes were used. One mesh of 546 elements, one of 3856 elements, and one of 15424 elements. This was done to assess the possible errors. The results are summarized below in Table 2.

Table 2. A table comparing the same geometry and Peclet number for three different meshes.

Mesh number	DF	Boundary	v	c*v	c_{mix}	var_{mix}	L	c_{opt}	var_{opt}	z	z/Pe	Pressure
546	2928 (Navier Stokes)	2	0.388274	0.241374	0.621659	0.057647	0.4	0.62366	0.094304	6.55712	0.013114	0
	1281 (Convection)	6	0.381147	0.236343	0.620084	0.083224	0.4	0.620303	0.109312	4.91784	0.009836	534.8808
		10	0.381147	0.235145	0.61694	0.110287	0.4	0.615821	0.144838	3.27856	0.006557	1071.724
		14	0.381147	0.234441	0.615093	0.146832	0.4	0.605129	0.192181	1.63928	0.003279	1608.567
Mesh number	DF	Boundary	v	c*v	c_{mix}	var_{mix}	L	c_{opt}	var_{opt}	z	z/Pe	Pressure
3856	18543 (Navier Stokes)	2	0.39191	0.202221	0.515988	0.03038	0.4	0.51588	0.037021	6.55712	0.013114	0
	8189 (Convection)	6	0.392093	0.202373	0.516135	0.050366	0.4	0.5155	0.061873	4.91784	0.009836	576.236
		10	0.392052	0.201694	0.514457	0.084632	0.4	0.513167	0.103807	3.27856	0.006557	1151.991
		14	0.392022	0.201271	0.513418	0.141011	0.4	0.511438	0.167168	1.63928	0.003279	1726.433
Mesh number	DF	Boundary	v	c*v	c_{mix}	var_{mix}	L	c_{opt}	var_{opt}	z	z/Pe	Pressure
15424	71799 (Navier Stokes)	2	0.396058	0.207047	0.522769	0.030052	0.4	0.522751	0.036746	6.55712	0.013114	0
	31805 (Diffusion)	6	0.395999	0.207019	0.522777	0.050026	0.4	0.522678	0.061165	4.91784	0.009836	582.846
		10	0.396	0.207102	0.522985	0.083319	0.4	0.522223	0.101496	3.27856	0.006557	1165.373
		14	0.395996	0.20711	0.52301	0.138451	0.4	0.519833	0.163628	1.63928	0.003279	1747.947

Table 3 shows the percent error, using the mesh with 15,424 elements as the base to compare to.

Table 3. A table comparing the same geometry and Peclet number for three different meshes. In all cases, the mesh with 3856 elements (the mesh used for all other Peclet numbers) is within 3% of the mesh with 15424 elements, meaning that the 3856 element model is an adequate model.

Mesh number	Boundary	c_{mix}	% error	var_{mix}	% difference	c_{opt}	% difference	var_{opt}	% difference	Pressure	% difference
546	2	0.621659	18.92%	0.057647	91.82%	0.62366	19.30%	0.094304	156.64%	0	
	6	0.620084	18.61%	0.083224	66.36%	0.620303	18.68%	0.109312	78.72%	534.88078	-8.23%
	10	0.61694	17.97%	0.110287	32.37%	0.615821	17.92%	0.144838	42.70%	1071.7241	-8.04%
	14	0.615093	17.61%	0.146832	6.05%	0.605129	16.41%	0.192181	17.45%	1608.5674	-7.97%
Mesh number	**Boundary**	**c_{mix}**	**% error**	**var_{mix}**	**% difference**	**c_{opt}**	**% difference**	**var_{opt}**	**% difference**	**Pressure**	**% difference**
3856	2	0.515988	-1.30%	0.03038	1.09%	0.51588	-1.31%	0.037021	0.75%	0	
	6	0.516135	-1.27%	0.050366	0.68%	0.5155	-1.37%	0.061873	1.16%	576.23596	-1.13%
	10	0.514457	-1.63%	0.084632	1.58%	0.513167	-1.73%	0.103807	2.28%	1151.9909	-1.15%
	14	0.513418	-1.83%	0.141011	1.85%	0.511438	-1.61%	0.167168	2.16%	1726.433	-1.23%
Mesh number	**Boundary**	**c_{mix}**	**% error**	**var_{mix}**	**% difference**	**c_{opt}**	**% difference**	**var_{opt}**	**% difference**	**Pressure**	**% difference**
15424	2	0.522769	0.00%	0.030052	0.00%	0.522751	0.00%	0.036746	0.00%	0	
	6	0.522777	0.00%	0.050026	0.00%	0.522678	0.00%	0.061165	0.00%	582.84603	0.00%
	10	0.522985	0.00%	0.083319	0.00%	0.522223	0.00%	0.101496	0.00%	1165.3729	0.00%
	14	0.52301	0.00%	0.138451	0.00%	0.519833	0.00%	0.163628	0.00%	1747.9467	0.00%

In all cases, the mesh with 3856 elements is within 3% of the mesh with 15424 elements. The mesh with 546 errors is very far off in all cases. Because the mesh with 3856 elements is within 3% of the mesh with 15424 elements, it is considered a good enough model for the present purposes.

Results – 3D case

Because of memory issues, the 3D case was modeled as only a single mixer, with a mesh of 1955 elements. There were 15890 degrees of freedom in solving the Navier-Stokes equation, and 5540 degrees of freedom in the convection/diffusion problem. Table 4 shows the results for the 3D case for different Peclet numbers.

Table 4. Results for various Peclet numbers in the 3D geometry.

Pe	100								
Boundary	w	c*w	c_{mix}	var_{mix}	c_{opt}	var_{opt}	z	z/Pe	Pressure
16	0.022871	0.011742	0.513401	0.0546	0.497828	0.0600	1.68928	0.016893	
Pe	200								
Boundary	w	c*w	c_{mix}	var_{mix}	c_{opt}	var_{opt}	z	z/Pe	Pressure
	0.022871	0.011800	0.515937	0.1001	0.498984	0.1074	1.68928	0.008446	
Pe	300								
Boundary	w	c*w	c_{mix}	var_{mix}	c_{opt}	var_{opt}	z	z/Pe	Pressure
	0.022871	0.011824	0.516987	0.1252	0.499439	0.1318	1.68928	0.005631	
Pe	500								
Boundary	w	c*w	c_{mix}	var_{mix}	c_{opt}	var_{opt}	z	z/Pe	Pressure
	0.022871	0.011829	0.517205	0.1530	0.499275	0.1572	1.68928	0.003379	
Pe	700								
Boundary	w	c*w	c_{mix}	var_{mix}	c_{opt}	var_{opt}	z	z/Pe	Pressure
	0.022871	0.011822	0.516899	0.1688	0.498891	0.1713	1.68928	0.002413	
Pe	1000								
Boundary	w	c*w	c_{mix}	var_{mix}	c_{opt}	var_{opt}	z	z/Pe	Pressure
	0.022871	0.011812	0.516462	0.1835	0.498421	0.1842	1.68928	0.001689	

Figure 5 shows the variance versus the log of z/Pe plot. It was assumed that the flow entered through the bottom, went straight up, swirled around the ellipse, and exited out the other side to find the z value.

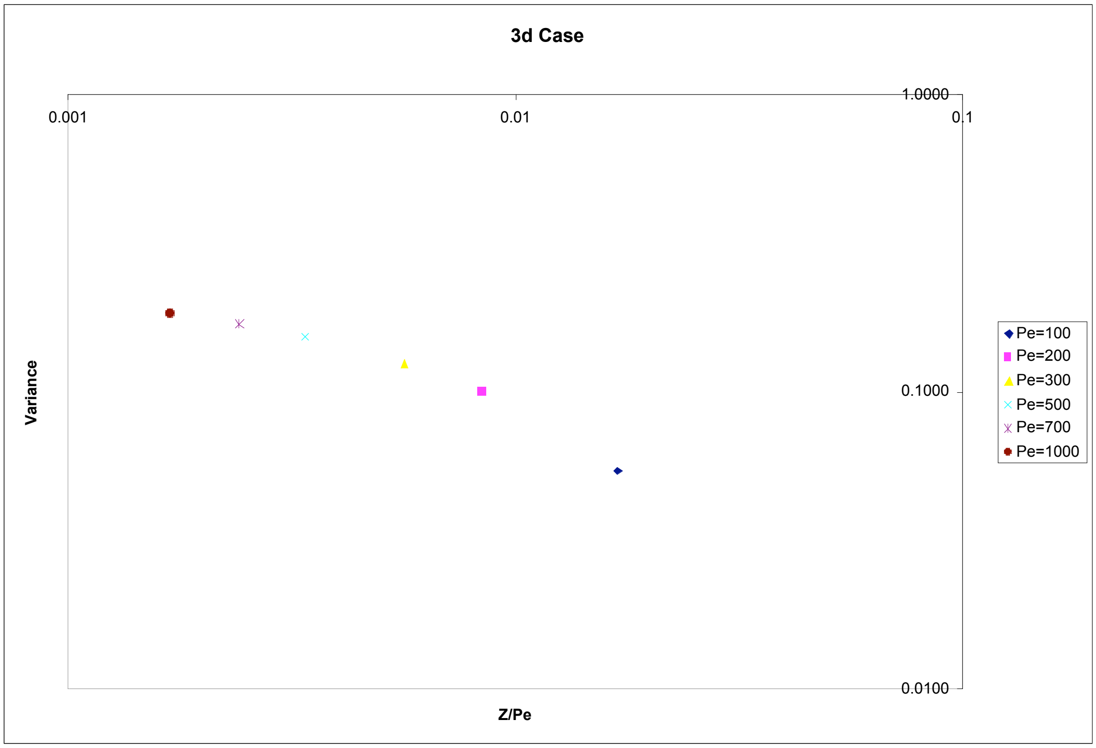

Figure 5. Results for the 3D case for the variance versus the ratio z/Pe.

Comparison – 2D to 3D case

The 2D and 3D case are very similar. As Figures 6 and 7 show, not only are they on the same order of magnitude, but are in fact very close numerically. As such, the 2D model models the 3D case more than adequately.

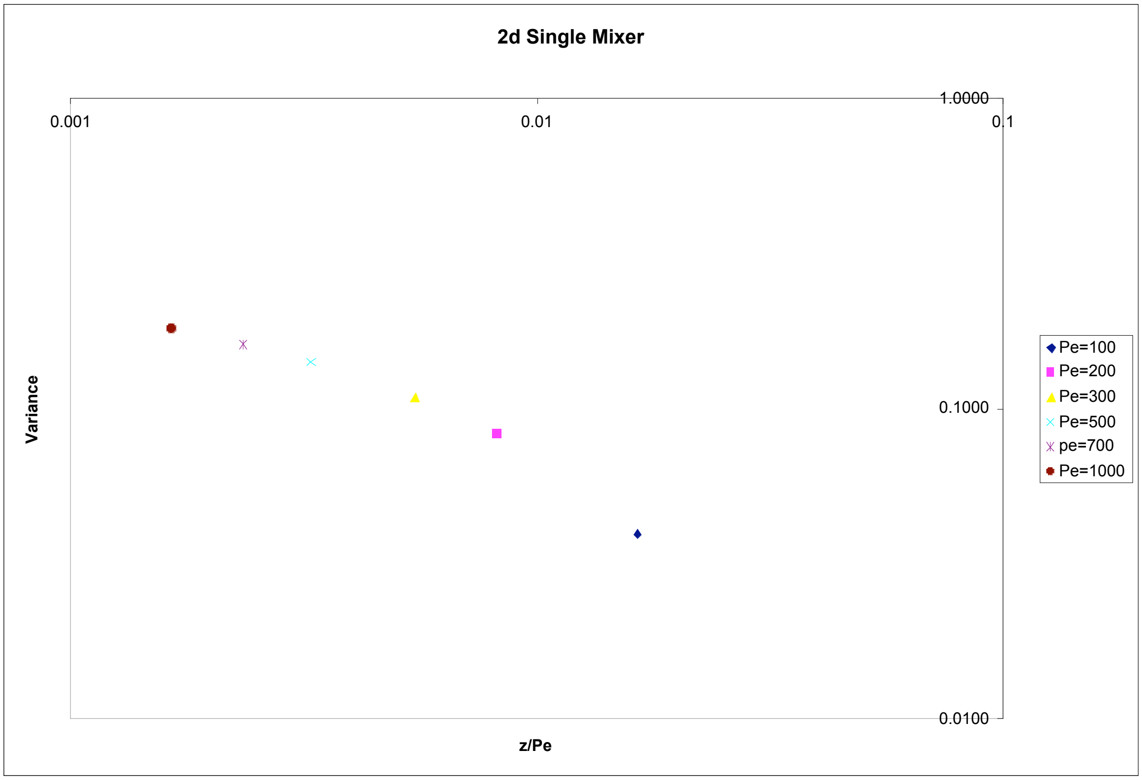

Figure 6. 2D mixer. Variance versus ratio of z/Pe.

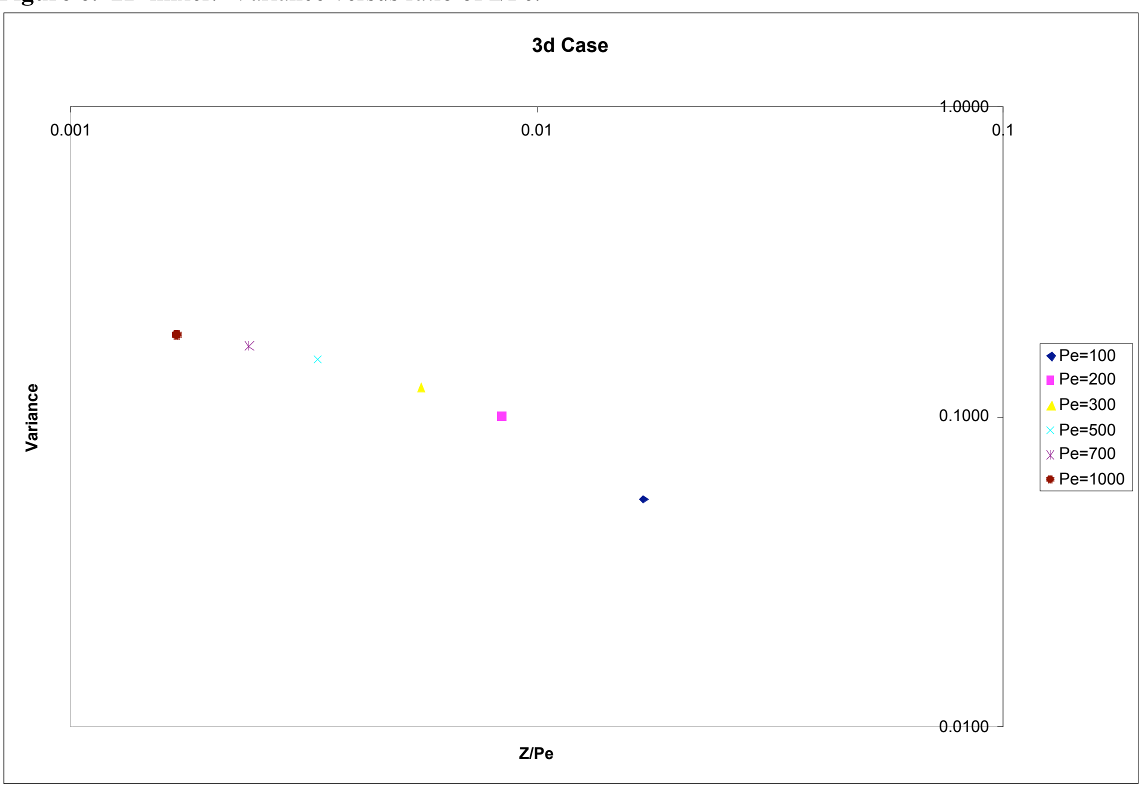

Figure 7. 3D mixer. Variance versus ratio of z/Pe.

Comparison to Literature

One obvious comparison is the amount of variance as a function of the number of mixers. This in turn reflects the amount of mixing (more variance means lesser mixing). This research is only for four mixers, whereas the literature goes up to 10. However, the variance can still be compared up to four mixers. Figure 8 shows the variance as function of number of mixers from literature, whereas Figure 9 shows from my research. In both cases, the Re number is 1. In Figure 9, the square root of the optical variance is presented versus the number of mixers.

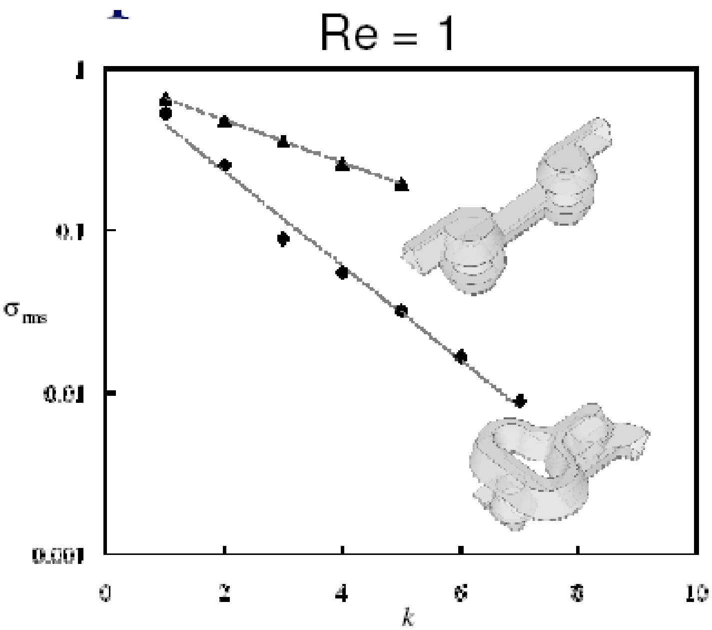

Figure 8. RMS versus the number of mixers (from micronet.com)

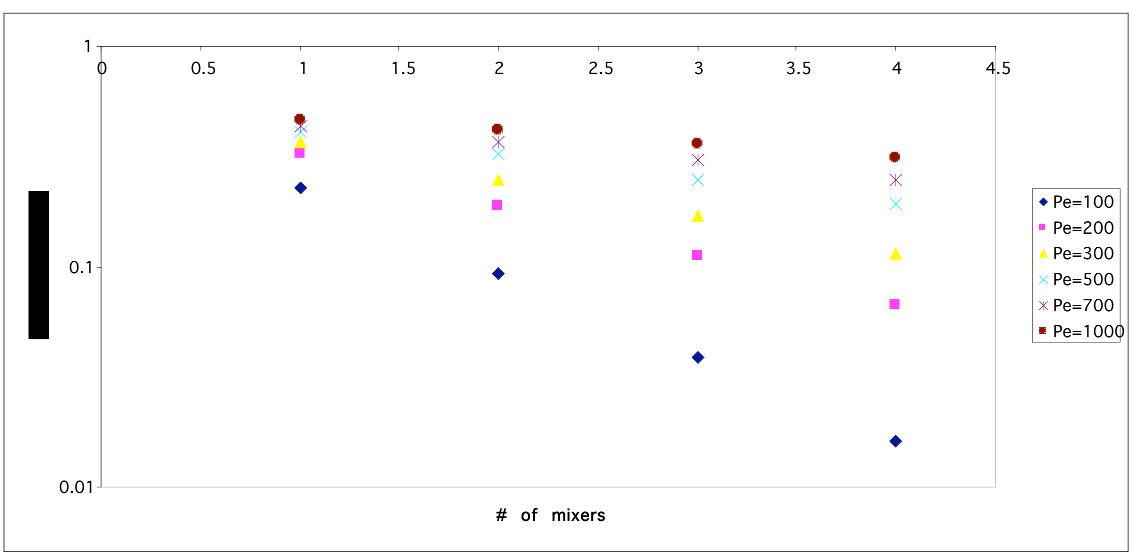

Figure 9. Square root of optical variance versus the number of mixers.

In both cases, the variance decreases as the number of mixers increases. Furthermore, in both cases, the variance is very nearly linear. It should be noted that the literature only has values for a Peclet number of 1000; my values are for a Peclet number that ranges from 100 to 1000. The variances for a Peclet number of a 1000 for my case range from .3 to .4; in the literature, after four mixers, the variance is below .1. In my case, a Peclet number of 300 or less is required to achieve a variance of .1 or less in four mixers. I do not know why the two values are so dissimilar.

Conclusions

As Figures 4 and 5 shows, the variance follows a pattern. For all z/Pe values, the variance forms one curve. In general, the variance decreases as the z/Pe ratio increases. This pattern is the same pattern as predicted by the T-sensor. This is true for both the 2D and 3D cases.

Table 2 shows both the optical and mixing cup concentrations for the 2D. In general, these two values are reasonably close to each other, usually agreeing to the fourth decimal place.

The mixing obviously increases as the number of mixers increases, as shown in Figure 10. The mixing also tends to be higher at lower Peclet numbers.

Further Research

There are some additional steps that could be taken with this research. One, the Reynolds number could be varied to determine what effect that has on mixing. Secondly, the 3D should be modeled as a series of mixers, to determine if it still follows the pattern established by the 2D model.

Appendix A – 2D Results

Re	1	z	1.63928

Pe 100

Boundary	v	c*v	c_{mix}	var_{mix}	L	c_{opt}	var_{opt}	z	z/Pe	Pressu
2	0.39191	0.201773	0.514845	0.0002	0.4	0.514849	0.0003	6.55712	0.065571	
6	0.392093	0.201871	0.514855	0.006641	0.4	0.51486	0.0015	4.91784	0.049178	576.2
10	0.392052	0.201894	0.514967	0.006642	0.4	0.51484	0.0088	3.27856	0.032786	1151.9
14	0.392022	0.201464	0.51391	0.0396	0.4	0.513509	0.0522	1.63928	0.016393	1726.4

Pe 200

Boundary	v	c*v	c_{mix}	var_{mix}	L	c_{opt}	var_{opt}	z	z/Pe	Pressu
2	0.39191	0.201898	0.515164	0.0034	0.4	0.51518	0.0044	6.55712	0.032786	
6	0.392093	0.202007	0.515202	0.009772	0.4	0.515104	0.0124	4.91784	0.024589	576.2
10	0.392052	0.201911	0.515011	0.028421	0.4	0.514519	0.0362	3.27856	0.016393	1151.9
14	0.392022	0.201328	0.513563	0.0827	0.4	0.51248	0.1049	1.63928	0.008196	1726.4

Pe 300

Boundary	v	c*v	c_{mix}	var_{mix}	L	c_{opt}	var_{opt}	z	z/Pe	Pressu
2	0.39191	0.202125	0.515743	0.0105	0.4	0.515756	0.0132	6.55712	0.021857	
6	0.392093	0.202248	0.515816	0.022807	0.4	0.515503	0.0284	4.91784	0.016393	576.2
10	0.392052	0.201974	0.515171	0.049876	0.4	0.514449	0.0620	3.27856	0.010929	1151.9
14	0.392022	0.201361	0.513647	0.1091	0.4	0.512247	0.1339	1.63928	0.005464	1726.4

Pe 500

Boundary	v	c*v	c_{mix}	var_{mix}	L	c_{opt}	var_{opt}	z	z/Pe	Pressu
2	0.39191	0.202221	0.515988	0.0304	0.4	0.51588	0.0370	6.55712	0.013114	
6	0.392093	0.202373	0.516135	0.050366	0.4	0.5155	0.0619	4.91784	0.009836	576.2
10	0.392052	0.201694	0.514457	0.084632	0.4	0.513167	0.1038	3.27856	0.006557	1151.9
14	0.392022	0.201271	0.513418	0.1410	0.4	0.511438	0.1672	1.63928	0.003279	1726.4

Pe 700

Boundary	v	c*v	c_{mix}	var_{mix}	L	c_{opt}	var_{opt}	z	z/Pe	Pressu
2	0.39191	0.201772	0.514843	0.0514	0.4	0.514395	0.0624	6.55712	0.009367	
6	0.392093	0.201958	0.515077	0.07454	0.4	0.514039	0.0916	4.91784	0.007025	576.2
10	0.392052	0.200868	0.51235	0.110979	0.4	0.509643	0.1366	3.27856	0.004684	1151.9
14	0.392022	0.200800	0.512216	0.1610	0.4	0.509124	0.1891	1.63928	0.002342	1726.4

Pe 1000

Boundary	v	c*v	c_{mix}	var_{mix}	L	c_{opt}	var_{opt}	z	z/Pe	Pressu
2	0.39191	0.200329	0.511161	0.0800	0.4	0.509465	0.0976	6.55712	0.006557	
6	0.392093	0.200601	0.511616	0.1036	0.4	0.509256	0.1272	4.91784	0.004918	576.2
10	0.392052	0.198993	0.507568	0.141145	0.4	0.501009	0.1759	3.27856	0.003279	1151.9
14	0.392022	0.199880	0.509869	0.1817	0.4	0.503145	0.2150	1.63928	0.001639	1726.4

Appendix B – 2D Results, multiple meshes

Mesh number	DF	Boundary	v	c*v	c_{mix}	var_{mix}	L	c_{opt}	var_{opt}	z	z/Pe	Pressure
546	2928 (Navier Stokes)	2	0.388274	0.241374	0.621659	0.057647	0.4	0.62366	0.094304	6.55712	0.013114	0
	1281 (Convection)	6	0.381147	0.236343	0.620084	0.083224	0.4	0.620303	0.109312	4.91784	0.009836	534.8808
		10	0.381147	0.235145	0.61694	0.110287	0.4	0.615821	0.144838	3.27856	0.006557	1071.724
		14	0.381147	0.234441	0.615093	0.146832	0.4	0.605129	0.192181	1.63928	0.003279	1608.567
Mesh number	DF	Boundary	v	c*v	c_{mix}	var_{mix}	L	c_{opt}	var_{opt}	z	z/Pe	Pressure
3856	18543 (Navier Stokes)	2	0.39191	0.202221	0.515988	0.03038	0.4	0.51588	0.037021	6.55712	0.013114	0
	8189 (Convection)	6	0.392093	0.202373	0.516135	0.050366	0.4	0.5155	0.061873	4.91784	0.009836	576.236
		10	0.392052	0.201694	0.514457	0.084632	0.4	0.513167	0.103807	3.27856	0.006557	1151.991
		14	0.392022	0.201271	0.513418	0.141011	0.4	0.511438	0.167168	1.63928	0.003279	1726.433
Mesh number	DF	Boundary	v	c*v	c_{mix}	var_{mix}	L	c_{opt}	var_{opt}	z	z/Pe	Pressure
15424	71799 (Navier Stokes)	2	0.396058	0.207047	0.522769	0.030052	0.4	0.522751	0.036746	6.55712	0.013114	0
	31805 (Diffusion)	6	0.395999	0.207019	0.522777	0.050026	0.4	0.522678	0.061165	4.91784	0.009836	582.846
		10	0.396	0.207102	0.522985	0.083319	0.4	0.522223	0.101496	3.27856	0.006557	1165.373
		14	0.395996	0.20711	0.52301	0.138451	0.4	0.519833	0.163628	1.63928	0.003279	1747.947

Mesh number	Boundary	c_{mix}	% error	var_{mix}	% difference	c_{opt}	% difference	var_{opt}	% difference	Pressure	% differenc
546	2	0.621659	18.92%	0.057647	91.82%	0.62366	19.30%	0.094304	156.64%	0	
	6	0.620084	18.61%	0.083224	66.36%	0.620303	18.68%	0.109312	78.72%	534.88078	-8.23
	10	0.61694	17.97%	0.110287	32.37%	0.615821	17.92%	0.144838	42.70%	1071.7241	-8.04
	14	0.615093	17.61%	0.146832	6.05%	0.605129	16.41%	0.192181	17.45%	1608.5674	-7.97
Mesh number	Boundary	c_{mix}	% error	var_{mix}	% difference	c_{opt}	% difference	var_{opt}	% difference	Pressure	% differenc
3856	2	0.515988	-1.30%	0.03038	1.09%	0.51588	-1.31%	0.037021	0.75%	0	
	6	0.516135	-1.27%	0.050366	0.68%	0.5155	-1.37%	0.061873	1.16%	576.23596	-1.13
	10	0.514457	-1.63%	0.084632	1.58%	0.513167	-1.73%	0.103807	2.28%	1151.9909	-1.15
	14	0.513418	-1.83%	0.141011	1.85%	0.511438	-1.61%	0.167168	2.16%	1726.433	-1.23
Mesh number	Boundary	c_{mix}	% error	var_{mix}	% difference	c_{opt}	% difference	var_{opt}	% difference	Pressure	% differenc
15424	2	0.522769	0.00%	0.030052	0.00%	0.522751	0.00%	0.036746	0.00%	0	
	6	0.522777	0.00%	0.050026	0.00%	0.522678	0.00%	0.061165	0.00%	582.84603	0.00
	10	0.522985	0.00%	0.083319	0.00%	0.522223	0.00%	0.101496	0.00%	1165.3729	0.00
	14	0.52301	0.00%	0.138451	0.00%	0.519833	0.00%	0.163628	0.00%	1747.9467	0.00

Appendix C –3D Results

Pe	100									
Boundary	**w**	**c*w**	**c_{mix}**	**var_{mix}**	**L**	**c_{opt}**	**var_{opt}**	**z**	**z/Pe**	**Pressure**
16	0.022871	0.011742	0.513401	0.0546	0.4	0.497828	0.0600	1.68928	0.016893	

Pe	200									
Boundary	**w**	**c*w**	**c_{mix}**	**var_{mix}**	**L**	**c_{opt}**	**var_{opt}**	**z**	**z/Pe**	**Pressure**
	0.022871	0.011800	0.515937	0.1001	0.4	0.498984	0.1074	1.68928	0.008446	

Pe	300									
Boundary	**w**	**c*w**	**c_{mix}**	**var_{mix}**	**L**	**c_{opt}**	**var_{opt}**	**z**	**z/Pe**	**Pressure**
	0.022871	0.011824	0.516987	0.1252	0.4	0.499439	0.1318	1.68928	0.005631	

Pe	500									
Boundary	**w**	**c*w**	**c_{mix}**	**var_{mix}**	**L**	**c_{opt}**	**var_{opt}**	**z**	**z/Pe**	**Pressure**
	0.022871	0.011829	0.517205	0.1530	0.4	0.499275	0.1572	1.68928	0.003379	

Pe	700									
Boundary	**w**	**c*w**	**c_{mix}**	**var_{mix}**	**L**	**c_{opt}**	**var_{opt}**	**z**	**z/Pe**	**Pressure**
	0.022871	0.011822	0.516899	0.1688	0.4	0.498891	0.1713	1.68928	0.002413	

Pe	1000									
Boundary	**w**	**c*w**	**c_{mix}**	**var_{mix}**	**L**	**c_{opt}**	**var_{opt}**	**z**	**z/Pe**	**Pressure**
	0.022871	0.011812	0.516462	0.1835	0.4	0.498421	0.1842	1.68928	0.001689	

Appendix D –Sample Calculations

All sample calculations done for a Peclet number of 500, and the boundary number 2.

$$c_{mixing\ cup} = \frac{\int_A c(x,y,z)\, v(x,y)\, dxdy}{\int_A v(x,y)\, dxdy}$$

First, find the velocity (.39191) and the concentration times velocity (.202221), both given in Comsol for the boundary. Then, c_{mix} is (.202221/.39191) = .515988.

$$\sigma_{mixing\ cup} = \frac{\int_A \left[c(x,y,z) - c_{mixing\ cup}\right]^2 v(x,y)\, dxdy}{\int_A v(x,y)\, dxdy}$$

Becomes $(c-.515988)^2 * v/.39191$ along the boundary in Comsol. It equals .0304.

$$c_{optical} = \int_0^L c(x,y,z)\, dy \,/ \int_0^L dy$$

First, get c/L from the Comsol boundary. In this case, L is .4, so c/.4 in Comsol gives you .51588.

$$\sigma_{optical} = \int_0^L \left[c(x,y,z) - c_{optical}\right]^2 dy \,/ \int_0^L dy$$

c_{opt} is .51588 (from above). In Comsol, $(c-.51588)^2/.4$ is .0370.