Financial Econometrics
Return Predictability

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Lecture Outline

- Market Efficiency

- The Forms of the Random Walk Hypothesis

- Testing the Random Walk Hypothesis

Reading

- FMUND, chapter 1

- MFTS, chapter 3, section 3.2.9

- APDVP, chapters 5 - 7.

Market Efficiency

An Intuitive Definition (Campbell, Lo and MacKinley, 1987, pg 20)

A capital market is said to be efficient if it fully and correctly reflects all relevant information in determining security prices. Formally, the market is said to be efficient with respect to some information set...if security prices would be unaffected by revealing that information to all participants. Moreover, efficiency with respect to an information set...implies that it is impossible to make economic profits by trading on the basis of [that information set].

Types of Market Efficiency

- *Weak Form*: Information set includes only the history of prices or returns

- *Semistrong Form*: The information set includes all publicly available information

- *Strong Form*: The information set contains all public and private information

Efficient Markets and the Law of Iterated Expectations

Samuelson's fameous result: Let $V^* =$ fundamental value of asset and assume P_t is a rational forecast. Then

$$
\begin{aligned}
P_t &= E[V^*|I_t] \\
P_{t+1} &= E[V^*|I_{t+1}] \\
E[P_{t+1} - P_t|I_t] &= E[E[V^*|I_{t+1}] - E[V^*|I_t]|I_t] \\
&= E[V^*|I_t] - E[V^*|I_t] = 0
\end{aligned}
$$

Thus realized changes in prices are unforecastable given information in the set I_t

Testing Market Efficiency

- Any test of market efficiency must assume an equilibrium model that defines normal security returns (e.g. CAPM)

- Perfect efficiency is unrealistic. Grossman and Stiglitz (1980) argue that you need some inefficiency to promote information gathering activity.

- The notion of relative efficiency - the efficiency of one market measured against another (e.g. NYSE vs. NASDAQ) may be a more useful concept than the all or nothing view taken by the traditional market efficiency literature

  - The concept of price discovery in multiple markets is an example of relative efficiency

The Random Walk Hypotheses

$$
\begin{aligned}
p_t &= \mu + p_{t-1} + \varepsilon_t, \ p_t = \ln(P_t) \\
&\implies r_t = \mu + \varepsilon_t, \ r_t = \Delta p_t
\end{aligned}
$$

- RW1: ε_t is independent and identically distributed (iid) $(0, \sigma^2)$. Not realistic

- RW2: ε_t is independent (allows for heteroskedasticity). Test using filter rules, technical analysis

- RW3: ε_t is uncorrelated (allows for dependence in higher moments). Test using autocorrelations, variance ratios, long horizon regressions

Autocorrelation Tests

Assume that r_t is covariance stationary and ergodic. Then

$$
\begin{aligned}
\gamma_k &= \text{cov}(r_t, r_{t-k}) \\
\rho_k &= \gamma_k/\gamma_0
\end{aligned}
$$

and sample estimates are

$$
\begin{aligned}
\hat{\gamma}_k &= \frac{1}{T}\sum_{t=1}^{T-k}(r_t - \bar{r})(r_{t+k} - \bar{r}), \ \hat{\rho}_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_0} \\
\bar{r} &= \frac{1}{T}\sum_{t=1}^{T} r_t
\end{aligned}
$$

Result (Hayashi, Proposition 2.9): Under RW1

$$
\begin{aligned}
E[\hat{\rho}_k] &= -\frac{T-k}{T(T-1)} + O(T^2), \ k = 1, 2, \ldots, p \\
\sqrt{T}\hat{\boldsymbol{\rho}} &\overset{A}{\sim} N(0, \mathbf{I}_p) \\
\hat{\boldsymbol{\rho}} &= (\ \hat{\rho}_1, \hat{\rho}_2, \ldots, \hat{\rho}_p)'
\end{aligned}
$$

Box-Pierce and Ljung-Box Q-statistics: Consider testing $H_0 : \rho_1 = \cdots = \rho_p = 0$. Under RW1, as $T \rightarrow \infty$

$$
\begin{aligned}
\text{BP} &= \sum_{k=1}^{m} \hat{\rho}_k^2 \overset{d}{\rightarrow} \chi^2(p) \\
\text{MQ} &= T(T+2)\sum_{k=1}^{p} \frac{\hat{\rho}_k^2}{T-k} \overset{d}{\rightarrow} \chi^2(p)
\end{aligned}
$$

Heteroskedasticity Robust Autocorrelation Tests

To test for autocorrelation in the raw returns when it is suspected that there are GARCH effects present (eg. RW3 is assumed), Diebold and Lopez (1995) suggested using the following heteroskedasticity robust version of MQ based on GMM estimation:

$$\text{MQ}^{HC}(m) = T(T+2) \sum_{j=1}^{m} \frac{1}{T-j} \left(\frac{\hat{\sigma}^4}{\hat{\sigma}^4 + \hat{\gamma}_j} \right) \hat{\rho}_j^2 \,\tilde{}\, \chi^2(m)$$

where $\hat{\sigma}^4$ is a consistent estimate of the squared unconditional variance of returns, and $\hat{\gamma}_j$ is the sample autocovariance of squared returns.

Variance Ratios

Intuition. Under RW1, the 2-period variance ratio satisfies

$$\text{VR}(2) = \frac{\text{var}(r_t(2))}{2 \cdot \text{var}(r_t)} = \frac{\text{var}(r_t + r_{t-1})}{2 \cdot \text{var}(r_t)} = \frac{2\sigma^2}{2\sigma^2} = 1$$

If r_t is a covariance stationary process then

$$\text{VR}(2) = \frac{\text{var}(r_t) + \text{var}(r_{t-1}) + 2\text{cov}(r_t, r_{t-1})}{2 \cdot \text{var}(r_t)}$$

$$= \frac{2\sigma^2 + 2\gamma_1}{2\sigma^2} = 1 + \rho_1$$

Three cases:

- $\rho_1 = 0 \implies \mathsf{VR}(2) = 1$

- $\rho_1 > 0 \implies \mathsf{VR}(2) > 1$ (mean aversion)

- $\rho_1 < 0 \implies \mathsf{VR}(2) < 1$ (mean reversion)

General $q - period$ variance ratio under stationarity

$$
\begin{aligned}
\mathsf{VR}(q) &= \frac{\mathsf{var}(r_t(q))}{q \cdot \mathsf{var}(r_t)} = 1 + 2 \sum_{k=1}^{q-1} \left(1 - \frac{k}{q}\right) \rho_k \\
r_t(q) &= r_t + r_{t-1} + \cdots + r_{t-q+1}
\end{aligned}
$$

Remark 1:

- Under RW1, $r_t \sim iid\,(0, \sigma^2)$, $\rho_k = 0$ for all k and so $\mathsf{VR}(q) = 1$.

Remark 2:

- For stationary and ergodic returns with a 1-summable Wold representation

$$r_t = \mu + \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j}, \ \varepsilon_t \sim iid(0, \sigma^2), \ \psi_0 = 1, \sum j|\psi_j| < \infty$$

  it can be shown that

$$\lim_{q \to \infty} \mathsf{VR}(q) = \frac{\sigma^2 \psi(1)^2}{\gamma_0} = \frac{\mathsf{lrv}(r_t)}{\mathsf{var}(r_t)} = \frac{\text{long-run variance}}{\text{short-run variance}}$$

Remark 3:

- Under RW2 and RW3, $\rho_k = 0$ for all k and $\mathsf{VR}(q) = 1$ provided

$$\frac{1}{T} \sum_{t=1}^{T} \mathsf{var}(r_t) \to \bar{\sigma}^2 > 0$$

Lo and MacKinlay's Test Statistics

Lo and MacKinlay (1988, 1989) developed a number of test statistics for testing the random walk hypothesis based on the estimated variance ratio using the sample one-period returns $\{r_1, \ldots, r_{Tq}\}$:

$$\widehat{\mathsf{VR}}(q) = \frac{\widehat{\mathrm{var}}(r_t(q))}{q \cdot \widehat{\mathrm{var}}(r_t)}$$

The form of the statistic depends on the particular random walk model (RW1, RW2 or RW3) assumed under the null hypothesis.

Under RW1, $\widehat{\mathsf{VR}}(q)$ is computed using

$$\widehat{\mathsf{VR}}(q) = \frac{\hat{\sigma}^2(q)}{q \cdot \hat{\sigma}^2}$$

where

$$\hat{\sigma}^2 = \frac{1}{Tq} \sum_{k=1}^{Tq} (r_k - \hat{\mu})^2, \ \ \hat{\sigma}^2(q) = \frac{1}{Tq^2} \sum_{k=q}^{Tq} (r_k(q) - q\hat{\mu})^2,$$

$$\hat{\mu} = \frac{1}{Tq} \sum_{k=1}^{Tq} r_k = \frac{1}{Tq}(p_{Tq} - p_0)$$

Lo and MacKinlay show that, under RW1,

$$\sqrt{Tq}(\widehat{\mathsf{VR}}(q) - 1) \overset{A}{\sim} N(0, 2(q-1))$$

$$\hat{\psi}(q) = \left(\frac{Tq}{2(q-1)}\right)^{1/2} (\widehat{\mathsf{VR}}(q) - 1) \overset{A}{\sim} N(0, 1))$$

Decision rule: reject RW1 at the 5% level if

$$|\hat{\psi}(q)| > 1.96$$

Remarks:

1. Very often $\widehat{VR}(q)$ and $\hat{\psi}(q)$ are computed for various values of q and $\widehat{VR}(q)$ is plotted against q with $\pm 2SE$ values based on its asymptotic distribution. Note that the SE bands do not represent a simultaneous test for all values of q considered.

2. Variance ratio tests are available in S+Finmetrics and the R package vrtest.

Lo and MacKinlay also derive a modified version variance ratio statistic based on the following bias corrected estimates of σ^2 and $\sigma^2(q)$:

$$\bar{\sigma}^2 = \frac{1}{Tq-1} \sum_{k=1}^{Tq} (r_k - \hat{\mu})^2, \ \bar{\sigma}^2(q) = \frac{1}{m} \sum_{k=q}^{Tq} (r_k(q) - q\hat{\mu})^2$$

$$m = q(Tq - q + 1)\left(1 - \frac{q}{Tq}\right)$$

Defining $\overline{VR}(q) = \bar{\sigma}^2(q)/\bar{\sigma}^2$, the biased corrected variance ratio statistic is

$$\bar{\psi}(q) = \left(\frac{3Tq^2}{2(2q-1)(q-1)}\right)^{1/2} (\overline{VR}(q) - 1) \overset{A}{\sim} N(0,1))$$

The variance ratio statistics $\hat{\psi}(q)$ and $\bar{\psi}(q)$ are not valid under the empirically relevant RW2 and RW3 models. For these models, Lo and MacKinlay derived the heteroskedasticity robust variance ratio statistic

$$\psi^*(q) = \hat{\Omega}(q)^{-1/2}(\overline{\text{VR}}(q) - 1) \overset{A}{\sim} N(0,1))$$

where

$$\hat{\Omega}(q) = \sum_{j=1}^{q-1}\left(\frac{2(q-j)}{j}\right)\hat{\delta}_j, \ \hat{\delta}_j = \frac{\sum_{t=j+1}^{Tq}\hat{\alpha}_{0t}\hat{\alpha}_{jt}}{\left(\sum_{j=1}^{Tq}\hat{\alpha}_{0t}\right)^2}$$

$$\hat{\alpha}_{jt} = (r_{t-j} - r_{t-j-1} - \hat{\mu})$$

Empirical Results from CLM

CML chapter 2, section 8. CRSP value-weighted (VW) and equal weighted (EW) indices, individual securities from 1962 - 1994

- Daily, weekly and monthly cc returns from VW and EW indices show significant 1st order autocorrelation

- $\widehat{\text{VR}}(q) > 1$ and $\psi^*(q)$ statistics reject RW3 for EW index but not VW index.

  - Market capitalization or size may be playing a role. In fact, $\widehat{\text{VR}}(q) > 1$ and $\psi^*(q)$ are largest for portfolios of small firms.

- For individual securities, typically $\widehat{\text{VR}}(q) < 1$ (negative autocorrelation) and $\psi^*(q)$ is not significant!!! How can portfolio $\widehat{\text{VR}}(q) > 1$ when individual security $\widehat{\text{VR}}(q) < 1$?

Traditional Views of Market Efficiency (Fama, circa 1970)

1. CAPM is a good measure of risk

2. Returns are close to unpredicatable

   (a) Stock, bond and foreign exchange bets are not predictable

   (b) Market volatility does not change much through time

3. Professional managers do not reliably outperform simple indices and passive portfolios once one corrects for risk

Modern Empirical Research (Cochrane, 2001)

1. There are assets, portfolios, funds, and strategies whose average returns cannot be explained by their market betas

2. Returns are predictable

   (a) dividend/price ratio and term premium can predict returns

   (b) Bond and foreign exchange returns are predictable

3. Some funds seem to outperform simple indices, even after controling for risk through market betas