Financial Econometrics and Volatility Models
Multivariate GARCH Models
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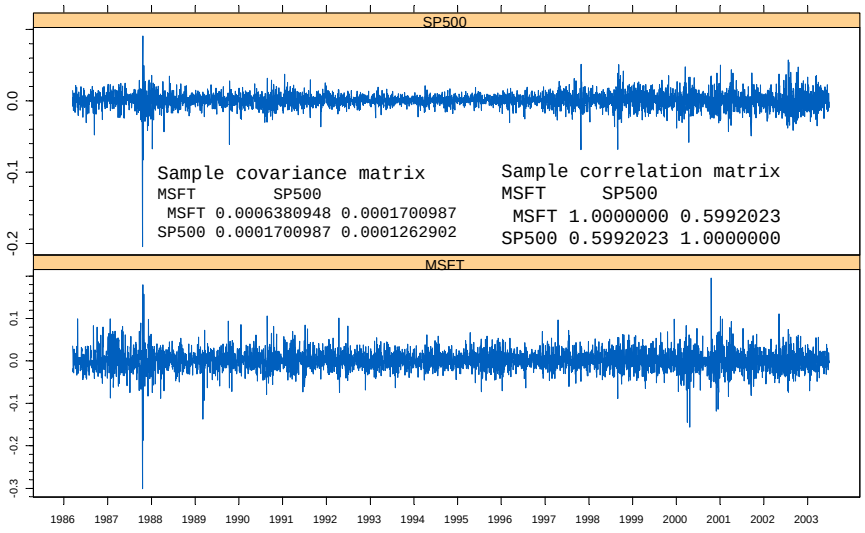
MSFT and S&P 500 Daily Returns



Sample covariance matrix

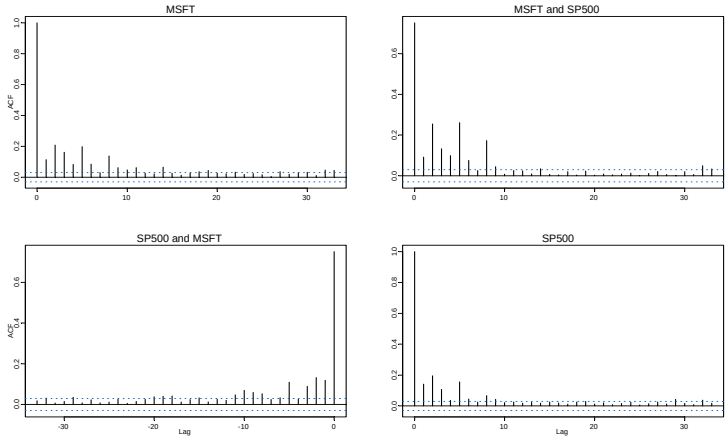
	MSFT	SP500
MSFT	0.0006380948	0.0001700987
SP500	0.0001700987	0.0001262902

Sample correlation matrix

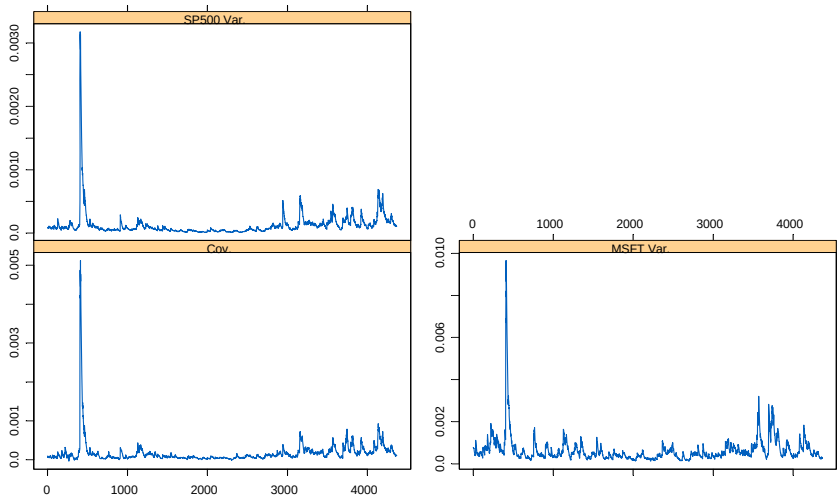
	MSFT	SP500
MSFT	1.0000000	0.5992023
SP500	0.5992023	1.0000000

Sample Cross-lag and Autocorrelations

Multivariate Series : msft.sp500.ts^2



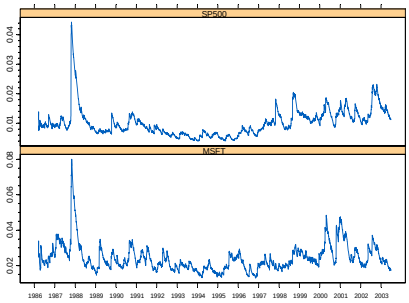
EWMA Covariance: lambda = 0.94



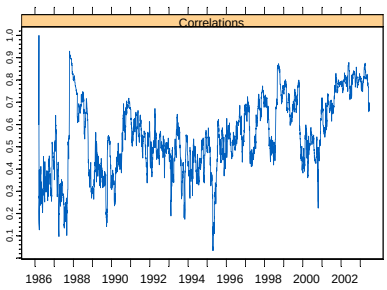
Estimating EWMA Covariance

```
> msft.sp500.ewma2=mgarch(msft.sp500.ts~1,~ewma2,trace=F)

Coefficients:

 C(1) 0.001804
 C(2) 0.000573
ALPHA 0.967368
```

EWMA Conditional Volatilities



EWMA Conditional Correlation

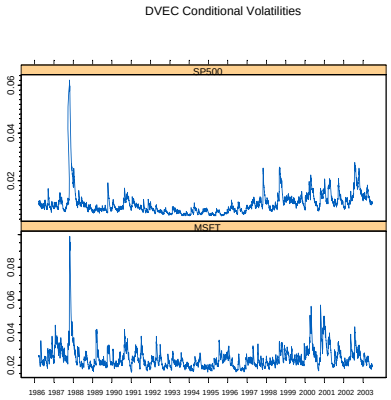


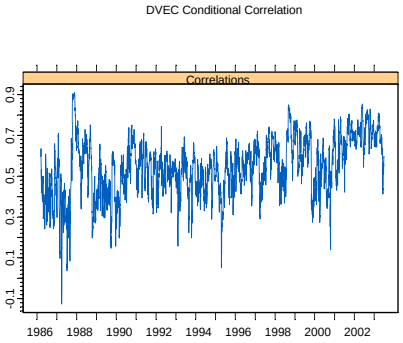
Estimated DVEC(1,1) Model

```
> msft.sp500.dvec = mgarch(msft.sp500.ts ~ 1,
                           ~ dvec(1, 1), trace = F)

                    Value  Std.Error t value   Pr(>|t|)
          C(1) 2.102e-003 3.137e-004   6.701 2.326e-011
          C(2) 6.812e-004 1.299e-004   5.246 1.629e-007
       A(1, 1) 1.710e-005 1.856e-006   9.212 0.000e+000
       A(2, 1) 2.801e-006 3.303e-007   8.481 0.000e+000
       A(2, 2) 1.650e-006 1.767e-007   9.338 0.000e+000
 ARCH(1; 1, 1) 6.930e-002 4.356e-003  15.910 0.000e+000
 ARCH(1; 2, 1) 6.758e-002 2.835e-003  23.840 0.000e+000
 ARCH(1; 2, 2) 7.724e-002 2.472e-003  31.251 0.000e+000
GARCH(1; 1, 1) 9.049e-001 5.899e-003 153.399 0.000e+000
GARCH(1; 2, 1) 9.137e-001 3.773e-003 242.157 0.000e+000
GARCH(1; 2, 2) 9.116e-001 3.906e-003 233.362 0.000e+000

AIC(11) = -50144
BIC(11) = -50073.81
```

Notice how the ARCH parameters are similar and the GARCH parameters are similar. This motivates the scalar DVEC specification

Conditional Volatilities and Correlations from DEV



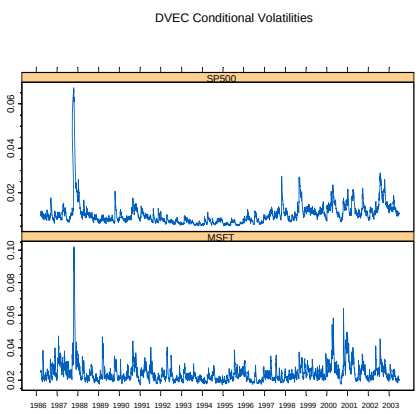


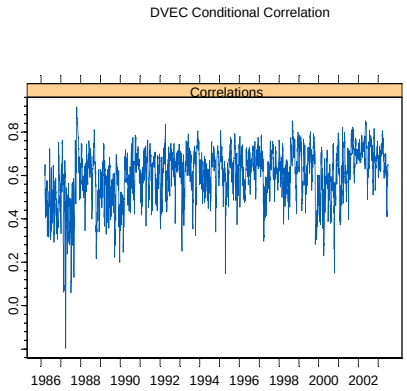
Estimated DVEC(1,1) Model with Covariance Targeting

```
> msft.sp500.dvecac=mgarch(msft.sp500.ts~1,~dvecac(1,1),trace=F)

                    Value Std.Error     t value   Pr(>|t|)
          C(1)  0.002004 0.0003201    6.261628 4.177e-010
          C(2)  0.000641 0.0001166    5.499571 4.025e-008
 ARCH(1; 1, 1)  0.302685 0.0064075   47.238892 0.000e+000
 ARCH(1; 2, 1)  0.288124 0.0085602   33.658335 0.000e+000
 ARCH(1; 2, 2)  0.108977 0.0206647    5.273580 1.403e-007
GARCH(1; 1, 1)  0.918609 0.0039952  229.927139 0.000e+000
GARCH(1; 2, 1)  0.942889 0.0053869  175.033595 0.000e+000
GARCH(1; 2, 2) -0.004230 1.0514770   -0.004023 9.968e-001

 AIC(8) = -49999.86
 BIC(8) = -49948.81
```

Strange result: probably converged to local minimum

Conditional Volatilities and Correlations from DEV with Cov Targeting



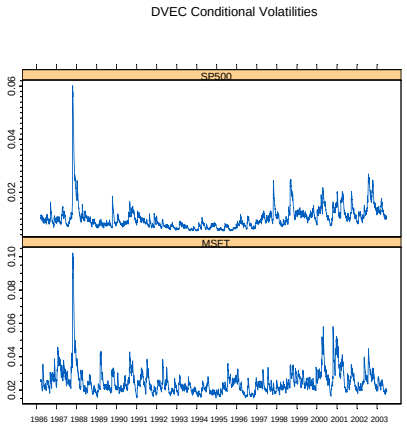


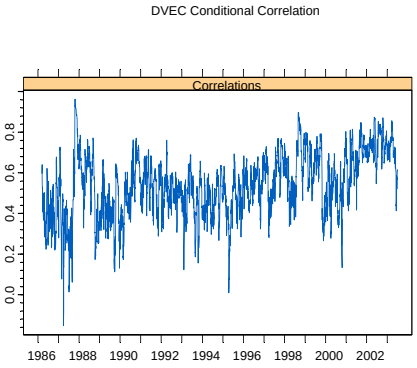
Estimated Scalar DVEC(1,1) Model

```
> msft.sp500.dvec.scalar = mgarch(msft.sp500.ts ~ 1,
                      ~ dvec.scalar.scalar(1, 1), trace = F)

             Value  Std.Error t value   Pr(>|t|)
    C(1) 0.0021088 0.00030660   6.878 6.934e-012
    C(2) 0.0007062 0.00013031   5.419 6.309e-008
 A(1, 1) 0.0035465 0.00016320  21.730 0.000e+000
 A(2, 1) 0.0007000 0.00005595  12.511 0.000e+000
 A(2, 2) 0.0011613 0.00005539  20.967 0.000e+000
 ARCH(1) 0.0719447 0.00184781  38.935 0.000e+000
GARCH(1) 0.9131034 0.00312740 291.969 0.000e+000

AIC(7) = -50131.18
BIC(7) = -50086.51
```

Common ARCH and common GARCH parameters

Conditional Volatilities and Correlations from Scalar DEV



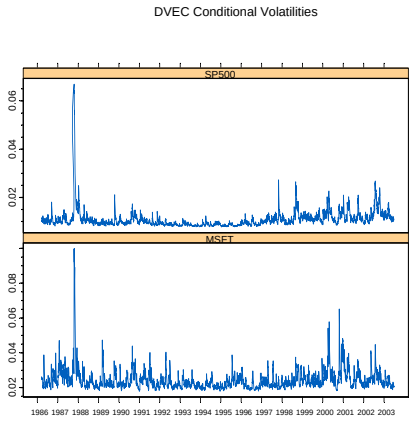


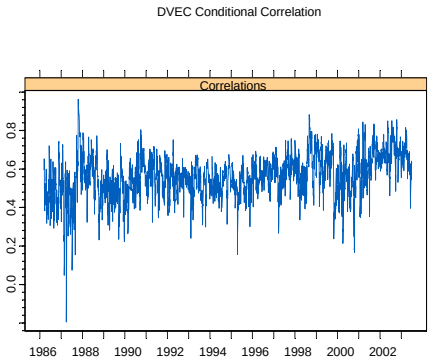
Estimated Matrix Diagonal Model

```
> msft.sp500.md=mgarch(msft.sp500.ts~1,
                       ~dvec.mat.mat(1,1),trace=F)

                   Value Std.Error    t value   Pr(>|t|)
          C(1) 0.0016874 0.0003368 5.011e+000 5.638e-007
          C(2) 0.0004430 0.0001633 2.713e+000 6.688e-003
       A(1, 1) 0.0073949 0.0002518 2.937e+001 0.000e+000
       A(2, 1) 0.0018198 0.0001419 1.283e+001 0.000e+000
       A(2, 2) 0.0025659 0.0001085 2.366e+001 0.000e+000
 ARCH(1; 1, 1) 0.3089436 0.0093939 3.289e+001 0.000e+000
 ARCH(1; 2, 1) 0.3056703 0.0081424 3.754e+001 0.000e+000
 ARCH(1; 2, 2) 0.0587813 0.0325585 1.805e+000 7.108e-002
GARCH(1; 1, 1) 0.9025885 0.0049981 1.806e+002 0.000e+000
GARCH(1; 2, 1) 0.9101793 0.0092980 9.789e+001 0.000e+000
GARCH(1; 2, 2) 0.0008398 7.2058057 1.165e-004 9.999e-001


AIC(11) = -49922.02
BIC(11) = -49851.83
```

Interpretation of all coefficients is not straightforward

Conditional Volatilities and Correlations from Matrix Diagonal Model





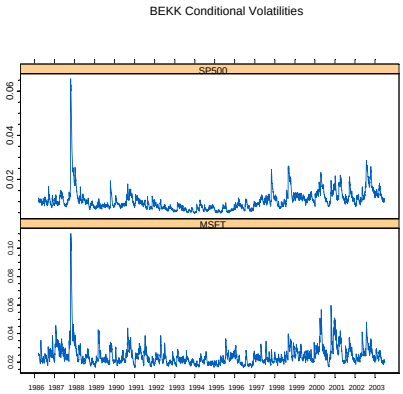
Estimated correlations are smoother in this model

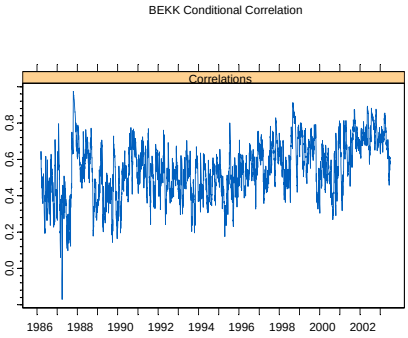
Estimated BEKK(1,1) Model

```
> msft.sp500.bekk=mgarch(msft.sp500.ts~1,~bekk(1,1),trace=F)
Value Std.Error   t value   Pr(>|t|)
          C(1)  0.0021320 0.0003160    6.7460 1.719e-011
          C(2)  0.0006983 0.0001301    5.3683 8.362e-008
       A(1, 1)  0.0048860 0.0002599   18.7960 0.000e+000
       A(2, 1)  0.0010324 0.0001032   10.0012 0.000e+000
       A(2, 2)  0.0005242 0.0001557    3.3674 7.655e-004
 ARCH(1; 1, 1)  0.2684892 0.0106286   25.2609 0.000e+000
 ARCH(1; 2, 1)  0.0301198 0.0047489    6.3425 2.491e-010
 ARCH(1; 1, 2)  0.0700258 0.0251728    2.7818 5.429e-003
 ARCH(1; 2, 2)  0.2501536 0.0073529   34.0213 0.000e+000
GARCH(1; 1, 1)  0.9389617 0.0046020  204.0335 0.000e+000
GARCH(1; 2, 1) -0.0142257 0.0017117   -8.3108 0.000e+000
GARCH(1; 1, 2)  0.0016594 0.0086175    0.1926 8.473e-001
GARCH(1; 2, 2)  0.9707394 0.0026330  368.6812 0.000e+000

AIC(13) = -50165
BIC(13) = -50082.05
```

Not straightforward to interpret parameters

Conditional Volatilities and Correlations from BEKK Model



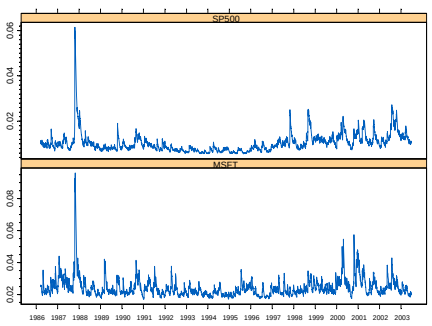


Estimated CCC Model

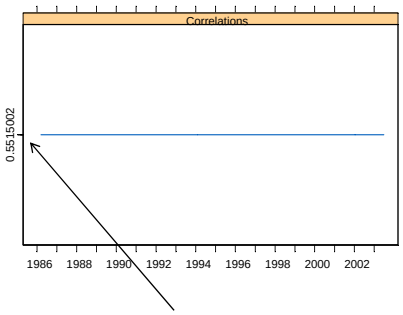
```
> msft.sp500.ccc = mgarch(msft.sp500.ts ~ 1,  ~ ccc(1, 1),
                          trace = F)
Value  Std.Error t value   Pr(>|t|)
         C(1) 2.106e-003 3.340e-004   6.304 3.188e-010
         C(2) 6.397e-004 1.342e-004   4.767 1.929e-006
      A(1, 1) 2.898e-005 3.026e-006   9.577 0.000e+000
      A(2, 2) 2.098e-006 2.065e-007  10.159 0.000e+000
 ARCH(1; 1, 1) 7.087e-002 5.148e-003  13.768 0.000e+000
 ARCH(1; 2, 2) 7.583e-002 2.910e-003  26.057 0.000e+000
GARCH(1; 1, 1) 8.816e-001 8.757e-003 100.678 0.000e+000
GARCH(1; 2, 2) 9.079e-001 4.650e-003 195.222 0.000e+000

Estimated Conditional Constant Correlation Matrix:
MSFT  SP500
 MSFT 1.0000 0.5515
SP500 0.5515 1.0000

AIC(8) = -49971.55
BIC(8) = -49920.5
```

Conditional Volatilities and Correlations from CCC Model

CCC Conditional Volatilities



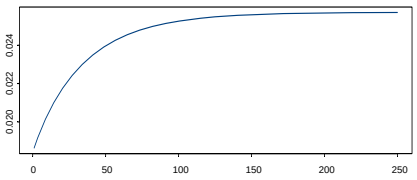
CCC Conditional Correlation



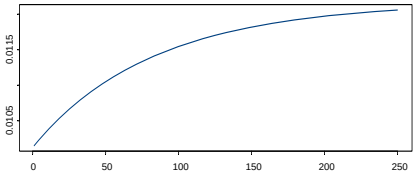
Conditional correlation restricted to sample correlation

Volatility and Correlation Predictions from DVEC(1,1)

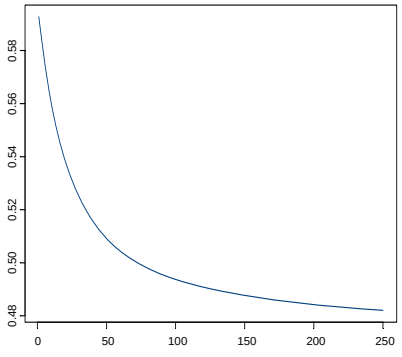
Predicted volatility for MSFT



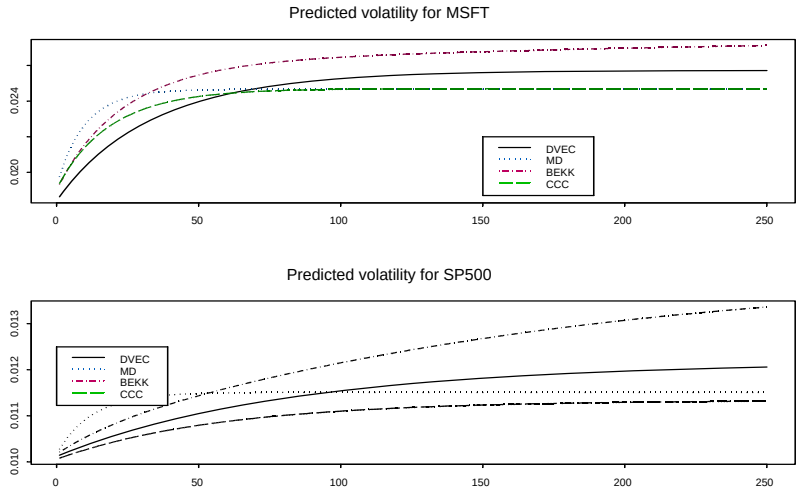
Predicted volatility for SP500



Predicted correlation b/w MSFT and SP500



Comparison of Predicted Volatilities



Comparison of Correlation Predictions