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# On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks

LAWRENCE R. GLOSTEN, RAVI JAGANNATHAN, and DAVID E. RUNKLE*

**ABSTRACT**

We find support for a negative relation between conditional expected monthly return and conditional variance of monthly return, using a GARCH-M model modified by allowing (1) seasonal patterns in volatility, (2) positive and negative innovations to returns having different impacts on conditional volatility, and (3) nominal interest rates to predict conditional variance. Using the modified GARCH-M model, we also show that monthly conditional volatility may not be as persistent as was thought. Positive unanticipated returns appear to result in a downward revision of the conditional volatility whereas negative unanticipated returns result in an upward revision of conditional volatility.

THE TRADEOFF BETWEEN RISK and return has long been an important topic in asset valuation research. Most of this research has examined the tradeoff between risk and return among different securities within a given time period. The intertemporal relation between risk and return has been examined by several authors—Fama and Schwert (1977), French, Schwert, and Stambaugh (1987), Harvey (1989), Campbell and Hentschel (1992), Nelson (1991), and Chan, Karolyi, and Stulz (1992), to name a few. This paper extends that research.

There is general agreement that investors, within a given time period, require a larger expected return from a security that is riskier. However, there is no such agreement about the relation between risk and return across time. Whether or not investors require a larger risk premium on average for investing in a security during times when the security is more risky remains an open question. At first blush, it may appear that rational risk-averse

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investors would require a relatively larger risk premium during times when the payoff from the security is more risky. A larger risk premium may not be required, however, because time periods which are relatively more risky could coincide with time periods when investors are better able to bear particular types of risk. Further, a larger risk premium may not be required because investors may want to save relatively more during periods when the future is more risky. If all the productive assets available for transferring income to the future carry risk and no risk-free investment opportunities are available, then the price of the risky asset may be bid up considerably, thereby reducing the risk premium.[1] Hence a positive as well as a negative sign for the covariance between the conditional mean and the conditional variance of the excess return on stocks would be consistent with theory. Since there are conflicting predictions about this aspect of the tradeoff between risk and return, it is important to empirically characterize the nature of the relation between the conditional mean and the conditional variance of the excess return on stocks as a group.

The empirical literature on this topic has attempted to characterize the nature of the linear relation between the conditional mean and the conditional variance of the excess return on stocks. However, the reported findings are conflicting. For example, Campbell and Hentschel (1992) and French, Schwert, and Stambaugh (1987) conclude that the data are consistent with a positive relation between conditional expected excess return and conditional variance, whereas Fama and Schwert (1977), Campbell (1987), Pagan and Hong (1991), Breen, Glosten, and Jagannathan (1989), Turner, Startz, and Nelson (1989), and Nelson (1991) find a negative relation. Chan, Karolyi, and Stulz (1992) find no significant variance effect for the United States, but implicitly find one of the world market portfolio. Harvey (1989) provides empirical evidence suggesting that there may be some time variation in the relation between risk and return.

Most of the support for a zero or positive relation has come from studies that use the standard GARCH-M model of stochastic volatility.[2] Other studies, using alternative techniques, have documented a negative relation between expected return and conditional variance. In order to resolve this conflict we examine the possibility that the standard GARCH-M model may not be rich enough to capture the time series properties of the monthly excess return on stocks. We consider a more general specification of the GARCH-M model. In particular, (1) we incorporate dummy variables in the GARCH-M model to capture seasonal effects using the procedure first suggested by Glosten, Jagannathan, and Runkle (1988), (2) we allow for asymmetries in the conditional variance equation, following the suggestions of Glosten,

[1] Abel (1988), Backus and Gregory (1992), Gennotte and Marsh (1993), and Glosten and Jagannathan (1987) have shown that the risk premium on the market portfolio of all assets could, in equilibrium, be lower during relatively riskier times.

[2] See Bollerslev, Chou, and Kroner (1992) for an extensive survey of GARCH and GARCH-M models in finance.

Jagannathan, and Runkle (1988), (3) we include the nominal interest rate in the conditional variance equation, and (4) we consider the EGARCH-M specification suggested by Nelson (1991) with the modifications mentioned in (1) through (3) above. These models suggest a weak but statistically significant negative relation between conditional variance and expected return.

Two of our findings are somewhat at odds with the existing literature. First, our data provide little evidence to support the belief that the conditional volatility of the monthly excess return on stocks is highly persistent, while Nelson (1991) finds high persistence in the volatility of daily returns. There are no theoretical reasons for the properties of the monthly and daily returns to be the same. In particular, Nelson (1991) argues that as the frequency at which data are sampled becomes very high, persistence should become larger. Second, both unexpected positive and negative excess returns on stocks change the next period's conditional volatility of the excess return on stocks. Unexpected positive returns result in a downward revision while unexpected negative returns result in an upward revision. In contrast, Nelson (1991) and Engle and Ng (1993), using daily data on stock index returns, find that large positive as well as negative unanticipated returns lead to an upward revision in the conditional volatility, although negative shocks of similar magnitude lead to larger revisions. Hence the time series properties of monthly excess returns are somewhat different from those of daily returns reported in Nelson (1991) and Engle and Ng (1993), and our results for monthly data along with the results for daily data reported by others provide a more complete characterization of the time series properties of stock index returns.

The remainder of the paper proceeds as follows. Section I describes the model that forms the basis for our empirical analysis. Section II discusses the econometric issues involved and our estimation methods. Section III contains the empirical results. Section IV concludes.

## I. The Relation between the Conditional Mean and the Conditional Variance of the Excess Return on Stocks

Consider the relation between conditional variance and conditional mean given by

$$E_t[x_{t+1}] = \beta\sigma_t^2. \tag{1}$$

When $x_{t+1}$ is the excess return on the aggregate wealth portfolio, and $\sigma_t^2$, its variance assessed at time $t$, captures most of the economic uncertainty that agents care about, the model in (1) is the approximation to the true risk-return relation derived by Merton (1980).

In our empirical work, we assume that (1) holds even for nominal returns. We consider the following general model for estimation:

$$E[x_{t+1} \mid F_t] = \alpha + \beta \operatorname{Var}(x_{t+1} \mid F_t), \tag{2}$$

where $F_t$ denotes the information set of agents. Campbell (1993) provides sufficient conditions for the relation given in (2) to hold approximately in equilibrium, where $x_t$ is the excess return on the market index portfolio. However, $\beta$ is not in general a measure of the risk aversion coefficient of the representative agent and $\alpha$ is not in general equal to zero. The relation in (2) forms the basis for our empirical work.

## II. Estimating the Model

### A. Econometric Issues

The parameter $\beta$ in the model given by (2) cannot be estimated without specifying how variances change over time, since $\mathrm{Var}(x_{t+1}|F_t)$ is not directly observed by the econometrician. To appreciate the difficulties involved, project both sides of (2) on $G_t$, the econometrician's information set, which is a strict subset of the agents' information set $F_t$. With $v_{t-1} = \mathrm{Var}(x_t|F_{t-1})$, we get

$$E[x_t \mid G_{t-1}] = \alpha + \beta E[v_{t-1} \mid G_{t-1}]. \tag{3}$$

Hence, we can write

$$x_t = \alpha + \beta E[v_{t-1} \mid G_{t-1}] + \eta_t,$$

where

$$\eta_t = u_{t-1} + \epsilon_t, \quad u_{t-1} = \beta(v_{t-1} - E[v_{t-1} \mid G_{t-1}]),$$

and

$$\epsilon_t = x_t - E[x_t \mid F_{t-1}].$$

Since, by definition

$$E\left[\epsilon_t^2 \mid F_{t-1}\right] = v_{t-1},$$
$$u_{t-1} = \beta\left[E\left[\epsilon_t^2 \mid F_{t-1}\right] - E\left[\epsilon_t^2 \mid G_{t-1}\right]\right].$$

Note that

$$E[\eta_t \mid G_{t-1}] = E[u_{t-1} \mid G_{t-1}] = E[u_{t-1}\epsilon_t \mid G_{t-1}] = 0.$$

Therefore,

$$E\left[\eta_t^2 \mid G_{t-1}\right] = E\left[u_{t-1}^2 \mid G_{t-1}\right] + E\left[\epsilon_t^2 \mid G_{t-1}\right]. \tag{4}$$

The term on the left in equation (4) is the variance of the error in forecasting $x_t$ based on the econometrician's information set. The first term on the right is the variance of the measurement error, $(v_{t-1} - E[v_{t-1}|G_{t-1}])$, and the second term is the expected value of the conditional variance $v_{t-1}$ based on the econometrician's information set $G_{t-1}$. Unless the variance of the measurement error is a constant, we cannot obtain a consistent estimate of $\beta$. This problem was first pointed out by Pagan and Ullah (1988). Also notice that the intercept term $\alpha$ in equation (2) is not identifiable based on the smaller information set available to the econometrician, since $E[v_{t-1}|G_{t-1}]$ may involve a constant term.

To see this estimation problem more clearly, consider now the special case

$$E[v_{t-1} \mid G_{t-1}] = b_0 + b_1 z_{t-1}$$

for some $z \in G$ where $b_1$ is a row vector and $z_{t-1}$ is a column vector. Then

$$E\left[\eta_t^2 \mid z_{t-1}\right] = \beta^2 \operatorname{Var}(v_{t-1} \mid G_{t-1}) + (b_0 + b_1 z_{t-1}).$$

The left side is the variance of the excess return, conditional on observing the instrument $z$ alone. The first term on the right side is the variance of the measurement error $\beta(v_{t-1} - E[v_{t-1}|G_{t-1}])$, and the second term is the variance of $\epsilon_t$ given $z_{t-1}$.

There have been several approaches to the estimation of the general econometric model in (3) through (4). One approach was suggested by Campbell (1987) and assumes that $\operatorname{Var}(v_{t-1}|G_{t-1})$ is an arbitrary constant, while $z_{t-1}$ is a vector of observable variables. If $\operatorname{Var}(v_{t-1}|G_{t-1})$ is a constant, then we can test whether $\beta$ is positive. We can estimate the regression equations

$$x_t = c_0 + c_1 z_{t-1} + \eta_t \tag{5}$$

and

$$\eta_t^2 = d_0 + d_1 z_{t-1} + \zeta_t. \tag{6}$$

Since the estimated slope coefficient $c_1$ is a consistent estimate of $\beta b_1$, and $d_1$ provides a consistent estimate of $b_1$, the ratio of any two corresponding elements of $c_1$ and $d_1$ provides a consistent estimate of $\beta$. If $z_{t-1}$ is not a scalar, then we may impose the constraint that the slope coefficients in (5) and the slope coefficients in (6) differ only by the scale factor, $\beta$. Such a restriction also provides a natural test for the validity of the model specification. We call this approach *Campbell's Instrumental Variable Model.* Another approach, the GARCH-M model, assumes that $\operatorname{Var}(v_{t-1}|G_{t-1})$ is identically zero, and that $z_{t-1}$ consists of innovations and variances that, while unobservable, can be estimated by the econometrician. A generalization of the GARCH-M approach maintains the assumption that $\operatorname{Var}(v_{t-1}|G_{t-1})$ is zero but allows $z_{t-1}$ to consist both of observable instruments and lagged values of estimated variances and innovations. We refer to this approach as the *Modified GARCH-M Model.* Since the specification of the information set is crucial for the Modified GARCH-M Model, we will first describe the information set used in this study and then proceed to describe the GARCH-M models we examine.

### *B. Specification of the Econometrician's Information Set*

Estimating any model of the intertemporal relation between risk and return requires taking a stand on the variables that make up the instrument vector, $z_t$. In our investigations we focus attention on the volatility information in the following variables: (a) the nominal interest rate, (b) October and

January seasonal dummies, and (c) the unanticipated part of the excess return on stocks. In what follows we provide some justification as to why we focus our attention on these variables.

The use of nominal interest rates in conditional variance models has some intuitive appeal. It has been well known since Fischer (1981) that the variance of inflation increases with its level. To the extent that short-term nominal interest rates embody expectations about inflation, they could be a good predictor of future volatility in excess returns. Using the information contained in nominal interest rates, Fama and Schwert (1977), Campbell (1987), and Breen, Glosten, and Jagannathan (1989) have demonstrated that it is possible to forecast time periods when the excess return on stocks is relatively large and significantly less volatile. Giovannini and Jorion (1989) and Singleton (1989) also examine the ability of nominal interest rates to predict changes in the volatility of, respectively, foreign exchange and stock returns.

Including deterministic seasonal dummies is motivated by the seasonal patterns reported in Lakonishok and Smidt (1988) and Keim (1985). Table I presents the summary statistics for the monthly excess returns on the Center for Research in Security Prices (CRSP) value-weighted stock index portfolio during the post–Treasury Accord period for the months of October, January, and other calendar months.[3] An apparent increase in October and January volatility is suggested by results presented in Panels A and C.

During the period 1951:4 to 1989:12, monthly excess continuously compounded returns on the CRSP value-weighted index of stocks (Panel C), during months other than October and January, had a mean of 0.48 percent and a standard deviation of 3.83 percent. The standard deviation of January excess returns is 5.19 percent (i.e., 1.35 times that in other months) and the standard deviation of October excess returns is 6.17 percent (1.61 times that in other months). While October and January are both months of relatively larger volatility, October, unlike January, has relatively lower excess returns on average than other months.

There are several potential contributing explanations for the excess volatility of January excess stock returns. Relatively more news arrives in January since most firms (almost two-thirds) use the calendar year as their fiscal year. Such firms close their books on December 31. The annual reports are typically more informative since they are done more carefully and are audited. Information from such reports starts leaking in during the month of January. Further, consumer sales exhibit pronounced quarterly seasonal patterns. This pattern arises because the fourth quarter is the important

[3] A careful examination of the data in Lakonishok and Smidt (1988) suggests that the monthly seasonal patterns in volatility are unlikely to be captured adequately by treating months other than October and January as being similar. However, our objective in this paper is limited to showing how to model seasonals in a way different than what has been done in the literature. Characterizing the nature of the monthly patterns in volatility is left as an exercise for the future.

**Table I**

**Summary Statistics for Monthly Data Recorded in the Period 1951:4 to 1989:12**

| | Number of Observations | | |
|---|---|---|---|
| | Oct. | Jan. | Other Months |
| | 39 | 38 | 388 |
| **Panel A. Continuously Compounded Monthly Return on the CRSP Value-weighted Index of Status on the NYSE** | | | |
| Mean ($\times 100$) | 0.25 | 1.77 | 0.91 |
| Standard Deviation ($\times 100$) | 6.16 | 5.18 | 3.80 |
| Skewness | −1.35 | 0.21 | −0.44 |
| Kurtosis | 6.59 | −0.41 | 0.64 |
| **Panel B. Continuously Compounded Monthly Return on Treasury Bills from Ibbotson & Associates** | | | |
| Mean ($\times 100$) | 0.46 | 0.43 | 0.43 |
| Standard Deviation ($\times 100$) | 0.26 | 0.22 | 0.26 |
| Skewness | 0.83 | 0.66 | 0.96 |
| Kurtosis | 0.54 | −0.03 | 0.90 |
| **Panel C. Excess of the Continuously Compounded Monthly Return on the CRSP Value-weighted Index of Status on the NYSE over That on Treasury Bills** | | | |
| Mean ($\times 100$) | −0.21 | 1.34 | 0.48 |
| Standard Deviation ($\times 100$) | 6.17 | 5.19 | 3.83 |
| Skewness | −1.40 | 0.18 | −0.48 |
| Kurtosis | 6.70 | −0.43 | 0.65 |
| Mean/variance | 0.56 | 0.50 | 3.28 |

holiday season. Comprehensive and reliable information about consumer spending during this period typically becomes available during January.

One has to stretch to provide an explanation for October volatility, and had this study been done prior to October 1987, we probably would not have singled out October. Yet, there is the perception that October has excess volatility. Laurie Cohen, writing in a *Wall Street Journal* article, attributes to Mark Twain the observation, "October is one of the peculiarly dangerous months to speculate in stocks. The others are: July, January, September, April, November, May, March, June, December, August, and February." Our purpose is to illustrate a methodology for dealing with seasonal patterns rather than to provide a detailed analysis of them. Though the seasonal variables are statistically significant, our general conclusions from the model estimations do not depend upon the inclusion of the seasonal variables. Rather, their significance suggests the value of further investigations of seasonal patterns.

We also allow positive and negative innovations to returns to have different impacts on conditional variance. To see why this is desirable, suppose discount rates are constant and have no relationship to anticipated future volatility. Any unanticipated decrease in expected future cash flows decreases the stock price. If the variance of the future cash flows remains the same or does not fall proportionately to the fall in stock prices, the variance of future cash flows per dollar of stock price will rise and future returns will be more volatile. Hence, if most of the fluctuations in stock prices are caused by fluctuations in expected future cash flows and the riskiness of future cash flows does not change proportionally when investors revise their expectations, then unanticipated changes in stock prices and returns will be negatively related to unanticipated changes in future volatility. Black (1976) and Christie (1982) have suggested a different reason for the negative effect of current returns on future variance: a decrease in today's stock price changes a firm's capital structure by increasing leverage. This increased leverage causes higher expected variance in the future. Both Black and Christie find support for their predictions in the relation between expected return and variance for individual stocks.

### *C. Modified GARCH-M Model of the Variance*

#### *C.1. Model Specifications*

The GARCH model assumes that the information set of investors and the econometrician coincide. The general Modified GARCH-M model can be written

*Equation for the conditional mean:*

$$E[x_{t+1}|G_t] = \mu(G_t)$$

where $\mu(\cdot)$ is a function that describes the nature of the dependence of the conditional mean on the elements of the information set $G_t$. Hence, we can write

$$x_{t+1} = \mu(G_t) + \epsilon_{t+1}, \quad \text{with } E[\epsilon_{t+1}|G_t] = 0.$$

*Equation for the conditional variance:*

$$\mathrm{Var}(x_{t+1}|G_t) \equiv \mathrm{Var}(\epsilon_{t+1}|G_t) = V(G_t)$$

where $V(.)$ is a function that describes the nature of the dependence of the conditional variance on the elements of the information set $G_t$. It is convenient to assume that the conditional variance function can be decomposed in the following way:[4]

$$V(G_t) = f_m(G_{t-1}) + f(G_t \setminus G_{t-1})$$

where $f_m(G_{t-1})$ is that part of the conditional variance, $V(G_t)$, that depends only on information known as of date $t-1$, and $f(G_t \setminus G_{t-1})$ is that part of

[4] In the case of the EGARCH-M models, $V(.)$ is the log of the variance.

the conditional variance that depends on the new information, $G_t \setminus G_{t-1}$, that becomes available at date $t$. Our analysis of various specifications focuses on the function $f(.)$.

Now consider the standard GARCH-M process suggested by Bollerslev (1986) for stock excess return, $x_t$, given by

*Model 1:*

$$x_t = a_0 + a_1 v_{t-1} + \epsilon_t \tag{7}$$

$$v_{t-1} = b_0 + b_1 v_{t-2} + g_1 \epsilon_{t-1}^2 \tag{8}$$

where $E_{t-1}[\epsilon_t] = 0$ and $E_{t-1}[\epsilon_t^2] = v_{t-1}$. The GARCH-M model specifies the conditional mean function, $\mu(G_{t-1}) = a_0 + a_1 v_{t-1}$, and the conditional variance function, $V(G_{t-1}) = b_0 + b_1 v_{t-2} + g_1 \epsilon_{t-1}^2$. That is, $f_m(G_{t-2}) = b_0 + b_1 v_{t-2}$ and $f(G_{t-1} \setminus G_{t-2}) = g_1 \epsilon_{t-1}^2$. The univariate GARCH-M model assumes that the econometrician's information set consists only of the past innovations to the excess return, $x_t$. Hence, the only new information that becomes available at date $t-1$ is $\epsilon_{t-1}$. The model further assumes that the function $f(G_{t-1} \setminus G_{t-2}) = g_1 \epsilon_{t-1}^2$. As we have argued earlier, there are a priori reasons to suspect that this assumption may not be reasonable.

If future variance is not a function only of the squared innovation to current return, then a simple GARCH-M model is misspecified and any empirical results based on it alone are not reliable. In Model 2 we assume that the impact of $\epsilon_{t-1}^2$ on conditional variance $v_{t-1}$ is different when $\epsilon_{t-1}$ is positive (i.e., when the indicator or dummy variable $I_{t-1}$ in (9) is 1) than when $\epsilon_{t-1}$ is negative (i.e., when the indicator or dummy variable $I_{t-1}$ in (9) is 0). This leads to

*Model 2:*

$$v_{t-1} = b_0 + b_1 v_{t-2} + g_1 \epsilon_{t-1}^2 + g_2 \epsilon_{t-1}^2 I_{t-1}. \tag{9}$$

In Models 3 through 5 we relax the assumption that the information set, $G_t$, consists only of past realizations of the excess return on the portfolio. Including the risk-free interest rate, $r_{ft}$, leads to

*Model 3:*

$$v_{t-1} = b_0 + b_1 v_{t-2} + b_2 r_{ft} + g_1 \epsilon_{t-1}^2 + g_2 \epsilon_{t-1}^2 I_{t-1}. \tag{10}$$

Given the results of Table I and for reasons mentioned earlier, we introduce January and October seasonal dummies in the variance of stock index excess returns. For this purpose we assume that the seasonal effects amplify the underlying *fundamental volatility* (which does not by definition exhibit any seasonal patterns) in the months of October and January by a constant month-specific scale factor. We also assume that the fundamental volatility next period depends only on the fundamental part of the excess return innovation.

In particular, we assume that we can write the excess return innovation in any calendar month as a scale multiple of some underlying fundamental innovation that does not exhibit any seasonal patterns, as follows:

$$\epsilon_t = (1 + \lambda_1 \mathrm{OCT}_t + \lambda_2 \mathrm{JAN}_t)\eta_t$$

where $\eta_t$ does not exhibit any deterministic seasonal behavior. Let $h_{t-1} = E_{t-1}[\eta_t^2]$ denote the conditional variance of $\eta_t$. We postulate that $h_t$ evolves over time according to

*Model 4:*

$$h_{t-1} = b_0 + b_1 h_{t-2} + g_1 \eta_{t-1}^2 + g_2 \eta_{t-1}^2 I_{t-1}; \tag{11}$$

or, when the risk-free rate is included,

*Model 5:*

$$h_{t-1} = b_0 + b_1 h_{t-2} + b_2 r_{ft} + g_1 \eta_{t-1}^2 + g_2 \eta_{t-1}^2 I_{t-1}. \tag{12}$$

Notice that Model 1 is obtained from Model 5 by imposing the restriction that $\lambda_1 = \lambda_2 = b_2 = g_2 = 0$. Similarly, Models 2, 3, and 4 can be considered as restricted versions of Model 5.

Our approach to modelling seasonals is different from the one used by Baillie and Bollerslev (1989). In our specification, we assume that we can deseasonalize the excess return innovation, $\epsilon$, to get $\eta$. The realized value of the deseasonalized innovation, $\eta$, influences the conditional variance of the distribution from which the deseasonalized innovation for the next period is drawn from. In contrast, in Baillie and Bollerslev (1989), the seasonal part of the innovation to this period's return affects the variance of the deseasonalized innovation next period.

Because inference in GARCH-M models depends on the correct specification of the information set and the validity of the functions used to represent the conditional mean and the conditional variance, we estimate three additional models to check our specification. First, we check for nonlinearity in the mean equation by adding $v_{t-1}^{1/2}$ to Model 2 and Model 4. These models are then called Model 6 and Model 7. If the coefficient on $v_{t-1}^{1/2}$ is significantly different from zero, that difference is evidence of misspecification.

In the above models, there are a priori reasons to suspect that the coefficient $g_2$ as well as $g_1 + g_2$ are negative, since empirical evidence suggests that a positive innovation to stock return is associated with a decrease in return volatility. However, if $g_1 + g_2$ is negative, conditional variance can potentially become negative for some realization of $\epsilon$. Hence we also follow the suggestions of Engle (1982) and Nelson (1991) and consider the exponential form for the law of motion for conditional variance, as given below:

*Model 2-L:*

$$\log(h_{t-1}) = b_0 + b_1 \log(h_{t-2}) + g_1 \eta_{t-1} \Big/ \sqrt{h_{t-2}} + g_2 \eta_{t-1} I_{t-1} \Big/ \sqrt{h_{t-2}}. \tag{13}$$

Following Nelson (1991), we use $\eta_{t-1}/\sqrt{h_{t-2}}$ instead of functions of $\eta_{t-1}^2$ in equation (13) to minimize the impact of extremely large realizations in absolute value so that the stochastic process for $h_t$ will be well behaved.[5] Model 1-L is the same as Model 2-L but with $g_2$ restricted to be zero.

Since we also want to test whether the risk-free rate, $r_{ft}$, helps predict conditional variance using the log specification, we also estimate

*Model 3-L:*

$$\log(h_{t-1}) = b_0 + b_1 \log(h_{t-2}) + b_2 r_{ft} + g_1 \eta_{t-1}\Big/\sqrt{h_{t-2}} + g_2 \eta_{t-1} I_{t-1}\Big/\sqrt{h_{t-2}}. \tag{14}$$

Models 4-L and 5-L add deterministic seasonals to the variance equation of Models 2-L and 3-L in the manner adopted for the level specification. Two additional models were estimated to test the specification of the EGARCH-M model. Model 6-L adds $v_{t-1}^{1/2}$ to the mean equation for Model 4-L.

## *C.2. Estimation and Inference and Diagnostic Tests*

We estimate all models discussed in this section by maximizing the log-likelihood function for the model, assuming that $\epsilon_t$ is conditionally normally distributed. Even if this assumption is incorrect, as long as the conditional means and variances are correctly specified, the quasi-maximum likelihood estimates will be consistent and asymptotically normal, as pointed out by Glosten, Jagannathan, and Runkle (1988) and Bollerslev and Wooldridge (1992). All our inference is based on robust standard errors from the quasi-maximum likelihood estimation, employing the procedures described in Bollerslev and Wooldridge (1992) and Glosten, Jagannathan, and Runkle (1988). We compute robust standard errors using two-sided numerical derivatives.[6]

We also use a variety of diagnostic tests to determine whether various aspects of our different models are correctly specified. First, we examine whether the residuals of the estimated models display excess skewness and kurtosis. Properly specified GARCH-M and EGARCH-M models should be able to significantly reduce the excess skewness and kurtosis evident in nominal excess returns. We test for excess skewness and kurtosis, under the

[5] Potential negative values for the constructed conditional variances are not the only possible reason for using the log specification. It may also be true that the log model simply models the true conditional variance better than the level model. For more on this issue, see Engle and Ng (1993).

[6] Since we use dummy variables which take the value of one or zero, it may appear as though we may be violating the differentiability assumptions underlying the derivation of the robust standard errors. Note, however, that since the dummy variables are multiplied by the corresponding squared innovations, the differentiability conditions will be satisfied for the modified GARCH-M models we consider. Although the differentiability conditions will be violated for the modified versions of Nelson's E-GARCH model we consider, this is unlikely to be an issue since points at which the differentiability assumptions are not satisfied will occur with zero probability, and the numerical derivatives we compute are always bounded.

null hypothesis that the errors are drawn from a conditional normal distribution. These tests have been previously applied to GARCH-M models by Campbell and Hentschel (1992).

Second, we examine whether the squared standardized residuals from the estimated models, $(\epsilon_t/\sqrt{v_{t-1}})^2$, are independent and identically distributed. We use the three tests proposed by Engle and Ng (1993): the Sign Bias Test, the Negative Size Bias Test, and the Positive Size Bias Test as well as a joint test of all three.

In the Sign Bias Test, the squared standardized residuals are regressed on a constant and a dummy variable, denoted $S_t^-$, that takes a value of one if $\epsilon_{t-1}$ is negative and zero otherwise. The Sign Bias Test Statistic is the $t$-statistic for the coefficient on $S_t^-$. This test shows whether positive and negative innovations affect future volatility differently from the prediction of the model.

In the Negative Size Bias Test, the squared standardized residuals are regressed on a constant and $S_t^- \epsilon_{t-1}$. The Negative Size Bias Test Statistic is the $t$-statistic for the coefficient on $S_t^- \epsilon_{t-1}$. This test shows whether larger negative innovations are correlated with larger biases in predicted volatility.

In the Positive Size Bias Test, the squared standardized residuals are regressed on a constant and $S_t^+ \epsilon_{t-1}$, where $S_t^+ = 1 - S_t^-$. The Positive Size Bias Test Statistic is the $t$-statistic for the coefficient on $S_t^+ \epsilon_{t-1}$. This test shows whether larger positive innovations are correlated with larger biases in predicted volatility.

There is one additional comparison that we make among the models, although it is not formally a diagnostic test. Because the parameterization of the models differs so much, it is hard to compare the amount of persistence in variance that these models predict. One way to compare persistence in variance across models is to regress $h_t$ on a constant and $h_{t-1}$. We report the slope coefficient and its standard error (it is one over the square root of the number of observations) of the regression for each model.

## III. Empirical Results

Our objective is to examine the role of model specification in determining the estimated relation between risk and return. The discussion above suggests that we can estimate this relation using either Campbell's Instrumental Variable Model or a variety of Modified GARCH-M and EGARCH-M models. While our focus is on the latter, we first present results using the first apprcach and find that Campbell's general conclusions are replicated in our data. We then present the results for the various GARCH-M models.

### A. Campbell's Instrumental Variable Model

Table II provides the empirical results obtained when the CRSP value-weighted index of stocks on the New York Stock Exchange (NYSE) is used as

**Table II**

**Monthly Risk-Return Relation on the CRSP Value-weighted Index of NYSE Equities: Campbell's Instrumental Variable Approach, 1951:4 to 1989:12**

The variable, $x_t$, is the differential between the continuously compounded monthly return on the CRSP value-weighted index of equities on the NYSE and $r_{ft}$, the continuously compounded monthly return on Treasury bills from Ibbotson & Associates. The variable $\text{JAN}_t$ takes the value one in January and zero otherwise, and $\text{OCT}_t$ takes the value one in October and zero otherwise. The $t$-statistics are computed using the procedures in Hansen (1982) which allows for conditional heteroskedasticity. The reported $t$-statistics are for 20 lags.

Model A

Mean Equation: $x_t = c_0 + c_1\ \text{OCT}_t + c_2\ \text{JAN}_t + c_3 r_{ft} + \epsilon_t$
Variance Equation: $\epsilon_t^2 = d_0 + d_1\ \text{OCT}_t + d_2\ \text{JAN}_t + d_3 r_{ft} + \zeta_t$

| | $d_0 \times 10^4$ | $d_1 \times 10^4$ | $d_2 \times 10^4$ | $d_3$ | $c_0 \times 100$ | $c_1 \times 100$ | $c_2 \times 100$ | $c_3$ |
|---|---|---|---|---|---|---|---|---|
| Coefficient | 7.25 | 21.68 | 13.11 | 0.18 | 1.48 | −0.63 | 0.86 | −2.31 |
| $t$-Statistic | 2.70 | 1.35 | 2.25 | 2.74 | 5.21 | −0.56 | 1.01 | −3.42 |

Model B

Restricted Mean Equation: $x_t = c_0 + \beta d_1\ \text{OCT}_t + \beta d_2\ \text{JAN}_t + \beta d_3 r_{ft} + \epsilon_t$
Variance Equation: $\epsilon_t^2 = d_0 + d_1\ \text{OCT}_t + d_2\ \text{JAN}_t + d_3 r_{ft} + \zeta_t$

| | $d_0 \times 10^4$ | $d_1 \times 10^4$ | $d_2 \times 10^4$ | $d_3$ | $c_0 \times 100$ | $\beta$ | ChiSQ(2) | $p$-Value |
|---|---|---|---|---|---|---|---|---|
| Coefficient | 8.25 | 9.23 | 4.19 | 0.15 | 1.44 | −12.75 | 3.22 | 0.20 |
| $t$-Statistic | 3.49 | 0.87 | 1.42 | 2.90 | 5.02 | −2.43 | | |

the stock index portfolio. We limit attention to 1951:4 to 1989:12, which is the post–Treasury Accord period. The estimated value of the slope coefficient for the risk-free rate in equation (5) for expected excess return is $-2.31$ ($t = -3.42$.). The estimated value of the slope coefficient for the risk-free rate in the variance equation given by (6) is 0.18 ($t = 2.74$).

Note that the residuals in equations (5) and (6) can be serially correlated, since the econometrician's information set may be strictly smaller than that of economic agents. Therefore, the $t$-statistics were computed using the procedures suggested by Newey and West (1987). Since there is substantial persistence in the residuals of equation (6), we report the $t$-statistics corresponding to a lag length of 20. In this sample, the $t$-statistics decrease as the number of lags increases, but stabilize at about 10 lags.

We also estimate the model by imposing the constraint that the slope coefficients in equation (5) be scalar multiples of the slope coefficients in equation (6). The estimated value of the scalar, $\beta$, is $-12.75$ ($t = -2.43$). With this restriction are two over-identifying restrictions. The null hypothesis that the over-identifying restrictions are not binding lead to a chi-square (D.F. = 2) value of 3.22 with an associated $p$-value of 0.20. Hence, based on these results, we cannot reject the hypothesis that there is a negative relation between the conditional mean and conditional variance of the excess return on stocks.

The natural question that arises at this stage is why the findings reported by French *et al.* (1987) for the standard GARCH-M model are different from the conclusions in this section. We address this issue in the next section.

### *B. Modified GARCH-M Models*

Tables III and IV present the estimates for Models 1 through 7. A comparison of Model 1 and Model 2 illustrates the restrictive nature of the standard GARCH-M specification for the conditional variance equation. Model 1 presents the results for a standard GARCH-M model. Both positive and negative innovations to excess returns result in upward revisions of the conditional variance ($g_1$ is positive). Also, time periods with relatively large variances are associated, on average, with relatively larger returns ($a_1$ is positive). However, the association is weak and not statistically significant at conventional levels.

These relations change as soon as positive and negative unanticipated returns are allowed to have different effects on conditional variance. Model 2 allows for such a difference by estimating the parameter $g_2$. A simple specification test comparing Model 1 and Model 2 shows that the standard GARCH-M model is too restrictive. If the parameters that the two models share are compared using a generalized specification test, computed using robust standard errors, the value of the test statistic is 12.829.[7] Under the null hypothesis that Model 1 is correctly specified, this test statistic should be asymptotically distributed as a $\chi_5^2$ random variable. Thus, we can reject the null hypothesis at the 5 percent level. Note that in Model 2 an unexpected negative return sharply increases conditional variance of the next period excess return, while an unexpected positive return decreases conditional variance. Model 1 does not allow for such a possibility.

There is another important difference between Model 1 and Model 2. Table IV shows that for Model 2, the estimated persistence of variance from one period to the next, as measured by the first-order autoregressive coefficient for $h_t$, is smaller than it is for Model 1.

Even though Model 2 seems less restrictive than Model 1, there are two reasons that we should not be satisfied with it. First, the robust standard errors suggest that the coefficient $g_2$ is imprecisely estimated. In fact, it is not significantly different from zero. Second, Model 2 does little better than Model 1 in all of the diagnostic tests.

Models 3, 4, and 5 attempt to solve the deficiencies in Model 2 by including the effect of the risk-free interest rate and deterministic seasonals on conditional variance. Each of these models results in statistically significant asymmetry in the conditional variance equation (i.e., $g_2$ is not equal to zero).

[7] For more on generalized specification tests, see Newey (1985).

## Table III

## Monthly Risk-Return Relation of the CRSP Value-weighted Index of NYSE Equities: Modified GARCH-M, 1951:4 to 1989:12 (Model Estimates)

With $x_t$ the differential between the continuously compounded monthly return on the CRSP value-weighted index of equities on the NYSE and $r_{ft}$, the continuously compounded monthly return on Treasury bills from Ibbotson & Associates, the models are defined by

$$x_t = a_0 + a_1 v_{t-1} + a_2 v_{t-1}^{1/2} + \epsilon_t; \quad \epsilon_t = (1 + \lambda_1 \mathrm{OCT} - \lambda_2 \mathrm{JAN})\eta_t; \quad v_{t-1} = \mathrm{Var}_{t-1}(\epsilon_t);$$

$$h_{t-1} = \mathrm{Var}_{t-1}(\eta_t); \quad I_{t-1} = 1 \ \text{ if } \ \eta_{t-1} > 0, \ \text{ and 0 otherwise;}$$

$$h_{t-1} = b_0 + b_1 h_{t-2} + b_2 r_{ft} + g_1 \eta_{t-1}^2 + g_2 \eta_{t-1}^2 I_{t-1};$$

where OCT takes the value one in October and zero otherwise and JAN takes the value one in January and zero otherwise. Robust $t$-statistics (in brackets) are calculated using the procedure in Bollerslev and Wooldridge (1992).

| | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 |
|---|---|---|---|---|---|---|---|
| $a_0$ | −0.453 | 1.064 | 1.850 | 1.071 | 1.854 | 5.730 | 7.782 |
| (×100) | [−0.575] | [1.947] | [4.232] | [2.398] | [4.419] | [2.064] | [2.289] |
| $a_1$ | 5.926 | −2.843 | −7.625 | −3.165 | −8.019 | 16.893 | 30.349 |
| | [1.307] | [−0.878] | [−2.621] | [−1.131] | [−2.828] | [1.163] | [1.652] |
| $b_0$ | 0.016 | 0.074 | 0.035 | 0.026 | 0.030 | 0.059 | 0.018 |
| (×100) | [2.058] | [1.653] | [1.595] | [2.467] | [2.477] | [1.902] | [3.295] |
| $b_1$ | 0.842 | 0.483 | 0.334 | 0.769 | 0.506 | 0.623 | 0.848 |
| | [16.758] | [1.572] | [0.824] | [8.801] | [2.927] | [2.830] | [17.299] |
| $b_2$ | | | 0.159 | | 0.078 | | |
| | | | [1.433] | | [2.221] | | |
| $g_1$ | 0.070 | 0.257 | 0.188 | 0.153 | 0.177 | 0.161 | 0.121 |
| | [2.541] | [1.709] | [2.109] | [2.590] | [2.268] | [1.502] | [2.734] |
| $g_2$ | | −0.340 | −0.248 | −0.227 | −0.252 | −0.267 | −0.207 |
| | | [−2.270] | [−2.602] | [−3.570] | [−3.272] | [−2.683] | [−3.749] |
| $\lambda_1$ | | | | 0.677 | 0.454 | | 0.606 |
| | | | | [3.638] | [3.120] | | [3.600] |
| $\lambda_2$ | | | | 0.269 | 1.254 | | 0.306 |
| | | | | [1.816] | [1.795] | | [1.936] |
| $a_2$ | | | | | | −1.983 | −3.123 |
| | | | | | | [−1.576] | [−1.947] |
| Log likelihood | 1248.202 | 1252.917 | 1266.288 | 1268.422 | 1276.518 | 1254.276 | 1271.824 |

In Model 3, the risk-free rate is included as an explanatory variable in the conditional variance equation in Model 2. Note that $a_1$ and $g_2$ both become statistically significant. The coefficient on the risk-free rate itself, $b_2$, is positive, but not significant, and may be imprecisely measured. Table IV shows that excess skewness and kurtosis remain quite severe after the risk-free rate is included.

**Table IV**

**Monthly Risk-Return Relation on the CRSP Value-weighted Index of NYSE Equities: Modified GARCH-M, 1951:4 to 1989:12 (Diagnostic Tests)**

Skewness and Kurtosis are the estimated skewness and kurtosis of the estimated standardized residuals from the mean equation. The Sign bias, Negative size bias, Positive size bias, and Joint tests are those suggested by Engle and Ng (1993). We report the slope coefficient and $t$-statistic from the regression of the squared standardized residual on (respectively) (1) an indicator variable which takes the value one if the residual is negative and zero otherwise, (2) the product of this indicator variable and the residual, and (3) the product of the residual and an indicator variable that takes the value one if the residual is positive and zero otherwise. The AR(1) coefficient is the slope coefficient from the regression of the fitted deseasonalized variance at time $t$ on the fitted deseasonalized variance at time $t - 1$.

| | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 |
|---|---|---|---|---|---|---|---|
| Skewness | −0.781 | −0.701 | −0.463 | −0.455 | −0.289 | −0.709 | −0.478 |
| $t$-Statistic | −6.855 | −6.157 | −4.065 | −3.992 | −2.532 | −6.225 | −4.190 |
| Kurtosis | 3.177 | 3.359 | 1.918 | 0.927 | 0.557 | 3.527 | 1.154 |
| $t$-Statistic | 13.880 | 14.675 | 8.380 | 4.052 | 2.431 | 15.408 | 5.040 |
| Sign bias | 0.745 | 0.988 | 0.776 | 0.710 | 0.429 | 0.841 | 0.673 |
| $t$-Statistic | 2.280 | 2.992 | 2.756 | 2.932 | 1.895 | 2.515 | 2.613 |
| Negative size bias | 0.013 | 8.117 | 5.758 | 5.539 | 3.861 | 4.243 | 5.477 |
| $t$-Statistic | 0.0028 | 1.612 | 1.329 | 1.483 | 1.052 | 0.837 | 1.389 |
| Positive size bias | −3.866 | 2.138 | 1.342 | 0.015 | −1.220 | 1.988 | −0.826 |
| $t$-Statistic | −0.596 | 0.345 | 0.256 | 0.0035 | −0.314 | 0.316 | −0.166 |
| Joint test | 16.797 | 11.854 | 10.851 | 13.050 | 6.459 | 9.415 | 11.997 |
| Significance level | 0.0008 | 0.0079 | 0.013 | 0.0045 | 0.091 | 0.024 | 0.0074 |
| AR(1) Coefficient on deseasonalized conditional variance (std. error = 0.0464) | 0.897 | 0.494 | 0.609 | 0.265 | 0.374 | 0.677 | 0.355 |

In Model 4, deterministic seasonals are added to Model 2. A Wald test that the October and January seasonal effects are jointly significant (i.e., the null that $\lambda_1 = \lambda_2 = 0$) is estimated to be 6.38. Under the null, this statistic should be asymptotically distributed as a $\chi_2^2$ random variable. Thus, we can reject the hypothesis that $\lambda_1 = \lambda_2 = 0$ at the 5 percent level. There are three other important characteristics of this model worth noting. First, with the inclusion of deterministic seasonals, both $g_1$ and $g_2$ are statistically significant (as they were in Model 3). Second, this method of modelling seasonals in variance greatly reduces the excess kurtosis in the residuals. Finally, note that the amount of persistence in the conditional variance, as measured by the first-order autoregressive coefficient for $h_t$, is much smaller than in any of the previous models.

Model 5 adds both the risk-free rate and deterministic seasonals to Model 2. As in Model 4, both $g_1$ and $g_2$ are significantly different from zero. Unlike in Model 3, $b_2$, the coefficient on the risk-free rate, is significantly positive. Note that the serial correlation in the estimated conditional variances is much smaller than the standard GARCH-M model, Model 1.

Table IV shows that the level of excess skewness and kurtosis have been significantly reduced—although the null hypothesis of no excess skewness or kurtosis can be rejected at the 5 percent level. Model 5 is also the first model that does not fail the Sign Bias Test at the 5 percent level. In addition, it is the first model not to fail the joint test of sign bias, negative size bias, and positive size bias at the 5 percent level. These diagnostics suggest that Model 5 is the most satisfactory model considered thus far.

Despite its success, Model 5 is still quite fragile. We attempted to check the robustness of the specification by adding $v_{t-1}^{1/2}$ to the mean equation. Even with great effort, we were not able to get the parameter estimates from that model to converge. Models 6 and 7 show the effects of adding $v_{t-1}^{1/2}$ to the mean equation in Models 2 and 4, respectively. In neither case was the coefficient on $v_{t-1}^{1/2}$ statistically significant. Note also that the estimated coefficients in the conditional variance equation are relatively close in all of these models.

We therefore come to the following five conclusions from our examination of the seven different GARCH-M specifications.

1. The relation between conditional mean and conditional variance is negative and statistically significant;
2. the risk-free rate contains information about future volatility, within the Modified GARCH-M framework;
3. the October and January seasonals in volatility are statistically significant;
4. conditional volatility of the monthly excess return is not highly persistent; and
5. negative residuals are associated with an increase in variance, while positive residuals are associated with a slight decrease in variance.

With the exception of the third conclusion, these results hold with or without the inclusion of seasonals.

Because even the best of these models displays excess skewness and kurtosis, we also estimated different EGARCH-M models. The estimates for Models 1-L and 6-L are shown in Tables V and VI. Models 1-L and 2-L are based on the EGARCH-M model proposed by Nelson (1991). However, the results in these models, using monthly data, are quite different from those found by Nelson. Note that $g_2$, the coefficient detecting asymmetry in the conditional variance equation, has a very small $t$-statistic. Table V shows that both Model 1-L and Model 2-L display excess skewness and kurtosis, and that both fail the Sign Bias Test. Note also that the first-order serial correlation of the $h_t$'s in both models is quite low.

Unlike in the level models, the coefficient on $g_2$ remains insignificant, regardless of which additional variables are included in the conditional variance equation. As a result, we do not report these regressions. Instead,

**Table V**

**Monthly Risk-Return Relation on the CRSP Value-weighted Index of NYSE Equities: Modified EGARCH-M, 1951:4 to 1989:12 (Model Estimates)**

With $x_t$ the differential between the continuously compounded monthly return on the CRSP value-weighted index of equities on the NYSE and $r_{ft}$, the continuously compounded monthly return on Treasury bills from Ibbotson & Associates, the models are defined by

$$x_t = a_0 + a_1 v_{t-1} + a_2 v_{t-1}^{1/2} + \epsilon_t; \quad \epsilon_t = (1 + \lambda_1 \mathrm{OCT} + \lambda_2 \mathrm{JAN})\eta_t; \quad v_{t-1} = \mathrm{Var}_{t-1}(\epsilon_t);$$

$$h_{t-1} = \mathrm{Var}_{t-1}(\eta_t); \quad H_{t-1} = \log(h_{t-1}); \quad I_{t-1} = 1 \ \text{ if } \eta_{t-1} > 0, \ \text{ and 0 otherwise};$$

$$H_{t-1} = b_0 + b_1 H_{t-2} + b_2 r_{ft} + g_1\left(\eta_{t-1}/\sqrt{h_{t-2}}\right) + g_2\left(\eta_{t-1}/\sqrt{h_{t-2}}\right)I_{t-1};$$

where OCT takes the value one in October and zero otherwise, and JAN takes the value one in January and zero otherwise. Robust $t$-statistics (in brackets) are calculated using the procedure in Bollerslev and Wooldridge (1992).

| | Model 1-L | Model 2-L | Model 3-L | Model 4-L | Model 5-L | Model 6-L |
|---|---|---|---|---|---|---|
| $a_0$ | 1.195 | 1.199 | 1.604 | 1.097 | 1.536 | 3.890 |
| (×100) | [2.932] | [2.290] | [4.176] | [2.657] | [4.158] | [1.231] |
| $a_1$ | −3.947 | −4.022 | −6.387 | −3.486 | −6.119 | 12.157 |
| | [−1.532] | [−1.537] | [−2.483] | [−1.305] | [−2.426] | [0.707] |
| $a_2$ | | | | | | −1.366 |
| | | | | | | [−0.908] |
| $b_0$ | −5.583 | −5.567 | −5.728 | −5.035 | −5.102 | −4.479 |
| (×100) | [−6.476] | [−6.472] | [−5.719] | [−4.513] | [−4.390] | [−2.654] |
| $b_1$ | 0.133 | 0.133 | 0.183 | 0.235 | 0.281 | 0.321 |
| | [0.999] | [0.992] | [1.271] | [1.386] | [1.701] | [1.248] |
| $b_2$ | | | 99.600 | | 81.002 | |
| | | | [3.624] | | [3.127] | |
| $g_1$ | −0.456 | −0.427 | −0.383 | −0.378 | −0.338 | −0.355 |
| | [−6.448] | [−3.599] | [−5.195] | [−5.249] | [−4.811] | [−3.738] |
| $g_2$ | | −0.052 | | | | |
| | | [−0.274] | | | | |
| $\lambda_1$ | | | | 0.426 | 0.349 | 0.451 |
| | | | | [2.573] | [2.392] | [2.579] |
| $\lambda_2$ | | | | 0.318 | 0.299 | 0.346 |
| | | | | [2.027] | [2.035] | [2.017] |
| Log likelihood | 1262.937 | 1262.274 | 1272.081 | 1270.970 | 1279.081 | 1271.538 |

we try to address the deficiencies in Model 1-L in Models 3-L, 4-L, and 5-L by including the effect of the risk-free interest rate and deterministic seasonals on conditional variance. Note that Models 3-L through 5-L impose the restriction that $g_2 = 0$.

Model 3-L adds the risk-free rate to the conditional variance equation in Model 1-L. In contrast to Model 3, the coefficient on the risk-free rate has a

**Table VI**

**Monthly Risk-Return Relation on the CRSP Value-weighted Index of NYSE Equities: Modified EGARCH-M, 1951:4 to 1989:12 (Diagnostic Tests)**

Skewness and Kurtosis are the estimated skewness and kurtosis of the estimated standardized residuals from the mean equation. The Sign bias, Negative size bias, Positive size bias, and Joint tests are those suggested by Engle and Ng (1993). We report the slope coefficient and $t$-statistic of the regression of the squared standardized residual on (respectively) (1) an indicator variable which takes the value one if the residual is negative and zero otherwise, (2) the product of this indicator variable and the residual, and (3) the product of the residual and an indicator variable that takes the value one if the residual is positive and zero otherwise. The AR(1) coefficient is the slope coefficient in the regression of the fitted deseasonalized variance at time $t$ on the fitted deseasonalized variance at time $t - 1$.

| | Model 1-L | Model 2-L | Model 3-L | Model 4-L | Model 5-L | Model 6-L |
|---|---|---|---|---|---|---|
| Skewness | −0.484 | −0.491 | −0.400 | −0.416 | −0.337 | −0.436 |
| $t$-Statistic | −4.249 | −4.306 | −3.506 | −3.654 | −2.954 | −3.825 |
| Kurtosis | 1.372 | 1.409 | 1.095 | 0.620 | 0.441 | 0.722 |
| $t$-Statistic | 5.995 | 6.157 | 4.786 | 2.710 | 1.925 | 3.153 |
| Sign bias | 0.555 | 0.577 | 0.364 | 0.507 | 0.409 | 0.549 |
| $t$-Statistic | 2.109 | 2.180 | 1.446 | 2.183 | 1.833 | 2.311 |
| Negative size bias | 4.111 | 3.794 | 3.244 | 3.676 | 3.575 | 4.342 |
| $t$-Statistic | 1.004 | 0.919 | 0.816 | 1.018 | 1.011 | 1.192 |
| Positive size bias | 7.469 | 8.257 | 3.054 | 5.982 | 3.449 | 4.910 |
| $t$-Statistic | 1.535 | 1.693 | 0.660 | 1.409 | 0.861 | 1.103 |
| Joint test | 4.632 | 5.074 | 2.139 | 4.813 | 2.407 | 5.408 |
| Significance level | 0.201 | 0.166 | 0.544 | 0.186 | 0.333 | 0.144 |
| AR(1) Coefficient on deseasonalized conditional variance (std. error = 0.0464) | 0.078 | 0.089 | 0.430 | 0.062 | 0.335 | 0.102 |

large $t$-statistic, even without deterministic seasonals. However, excess skewness and kurtosis are still a problem in this model. Note that there is no significant sign bias in this model. In fact, none of the Engle-Ng tests show any evidence of misspecification in this model.

Model 4-L adds deterministic seasonals to Model 1-L, using the same method adopted for the level models. A Wald test that the October and January seasonal effects are jointly significant (i.e., the test of the null that $\lambda_1 = \lambda_2 = 0$) is estimated to be 7.66. Under the null, this statistic should be asymptotically distributed as a $\chi_2^2$ random variable. Thus, we can reject the hypothesis that $\lambda_1 = \lambda_2 = 0$ at the 5 percent level. Although the amount of excess kurtosis is much lower for Model 4-L than for any of the previous models, we can still reject the hypothesis of no excess kurtosis at the 5 percent level. Note also, that Model 4-L appears to have sign bias.

Model 5-L adds both the risk-free interest rate and deterministic seasonals to Model 1-L. The coefficients on all of those terms are statistically significant. The Wald test statistic for the hypothesis that $\lambda_1 = \lambda_2 = 0$ is 7.31, while the test statistic for the hypothesis that $\lambda_1 = \lambda_2 = b_2 = 0$ is 15.53. Thus, we can reject both hypotheses at the 5 percent level. Table V shows that we cannot reject the hypothesis that there is no excess kurtosis in the estimated residuals from Model 5-L. Model 5-L also shows no signs of sign bias, negative size bias, or positive size bias.

Since Model 5-L passed more of the diagnostic tests than any other model, it is our preferred specification. As a further check, we estimated Model 6-L, by adding $v_{t-1}^{1/2}$ to the conditional mean equation. The coefficient on $v_{t-1}^{1/2}$ is not statistically significant, and the results for the conditional variance equation are qualitatively the same as for Model 5-L. However, the diagnostic tests show that Model 6-L performs worse than Model 5-L in some important ways. Model 6-L has a statistically significant amount of excess kurtosis, and it fails the Sign Bias Test. This suggests there is little evidence of misspecification in Model 5-L, and that the model should be the preferred specification. Note also that the first-order serial correlation of $h_t$ in Model 5-L is still relatively low at 0.3358.[8]

Given the essentially exploratory nature of our seasonal analysis, it is important to note that the coefficient estimates for Model 3-L (without seasonals) and Model 5-L are very close. Thus, general conclusions about the nature of stochastic volatility are invariant to the inclusion and exclusion of the January and October seasonals. Notice also that the conclusions derived from Model 5 (the "levels" model with the risk-free rate and seasonals) and Model 5-L are the same.

Since the finding of low persistence of conditional variance is so different from results reported in the literature (except for Campbell and Hentschel 1992), it needs some explanation. At this point, we can only speculate. Perhaps there are regimes in which variance is relatively persistent, but there are frequent and relatively unpredictable regime shifts.[9] Thus, the data are characterized by both persistence and random changes in variance. This explanation is suggested by the fact that the likelihood function of Model 2-L has two local maxima (we report the global maximum results). The local (not global) maximum is characterized by variance estimates that are highly persistent, but produces residuals that exhibit substantial skewness and kurtosis. It is possible that the two local maxima are merely an artifact of the relatively small postwar sample. On the other hand, the likelihood function

[8] We also tested the robustness of our conclusions in two other ways. First, we estimated Model 5-L with a sample ending in December 1986 to see whether the October 1987 stock market crash had an undue influence on our estimates. Second, we estimated Model 5-L using equally weighted returns. Both sets of results were qualitatively similar to those for Model 5-L.

[9] Models of "regime shifting" have been examined in Hamilton and Susmel (1992), and Cai (1993).

may be suggesting that there are two ways to fit the data, and the fit with lower persistence is slightly better.

## IV. Conclusion

There is a positive but insignificant relation between the conditional mean and conditional volatility of the excess return on stocks when the *standard* GARCH-M framework is used to model the stochastic volatility of stock returns. On the other hand, Campbell's Instrumental Variable Model estimates a negative relation between conditional mean and conditional volatility. In this paper we empirically show that the standard GARCH-M model is misspecified and alternative specifications provide a reconciliation between these two results. When the model is modified to allow positive and negative unanticipated returns to have different impacts on the conditional variance, we find a negative relation between the conditional mean and the conditional variance of the excess return on stocks. This relation becomes stronger and statistically significant when conditional variance is allowed to have deterministic monthly seasonals and to depend on the nominally risk-free interest rate. Hence our results are consistent with the negative relation between volatility and expected return reported in Fama and Schwert (1977), Campbell (1987), Breen, Glosten, and Jagannathan (1989), and Harvey (1991). We show that our conclusions do not change when we use Nelson's EGARCH-M model modified to include the risk-free rate or seasonals or both.

We also find that the time series properties of monthly excess returns are substantially different from the reported properties of daily excess returns. First, persistence of conditional variance in excess returns is quite low in monthly data while Nelson (1991) finds persistence high in daily data. Second, positive and negative unexpected returns have vastly different effects on future conditional variance; the expected impact of a positive unexpected return is negative. In contrast, Nelson (1991) and Engle and Ng (1993) find different effects for positive and negative unexpected returns, but both lead to variance increases.

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