

UW

Econ 584
Time Series Econometrics

Cointegration

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Spurious Regression

$y_{it} = y_{it-1} + \varepsilon_{it}, \varepsilon_{it} \sim iid N(0,1)$
 $cov(\varepsilon_{1t}, \varepsilon_{2t}) = 0$

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1 cointegrating vector, 1 common trend

Simulated bivariate cointegrated system

1 cointegrating vector, 1 common trend

Cointegrating residual

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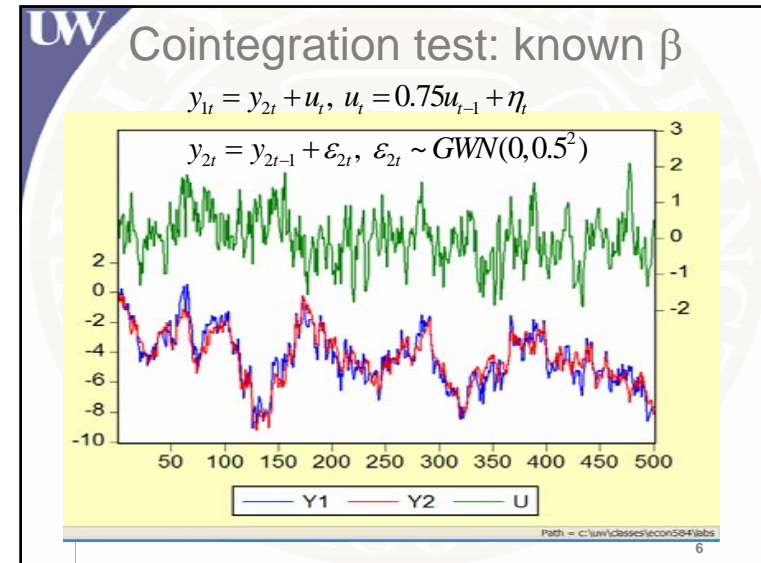
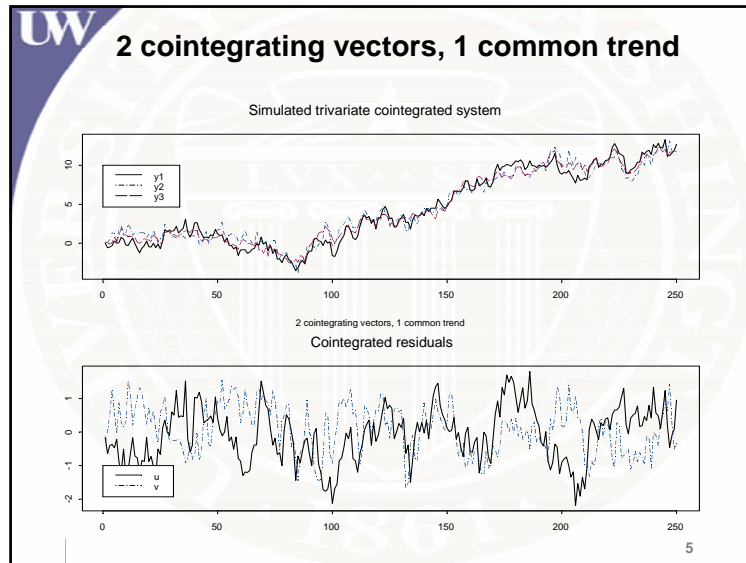
1 cointegrating vector, 2 common trends

Simulated trivariate cointegrated system

1 cointegrating vector, 2 common trends

Cointegrating residual

4



ADF test on u_t

Null Hypothesis: U has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic based on SIC, MAXLAG=5)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-9.097037	0.0000
Test critical values:		
1% level	-3.443228	
5% level	-2.867112	
10% level	-2.569800	

*MacKinnon (1996) one-sided p-values.

Reject H_0 : y_1 and y_2 are not cointegrated at 1% level

Cointegration Test: Estimate β

Step 1: estimate β by OLS

Dependent Variable: Y1
Method: Least Squares
Date: 05/24/06 Time: 13:30
Sample: 1 500
Included observations: 500

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.015124	0.084955	-0.178019	0.8588
Y2	0.993235	0.017938	55.36959	0.0000

R-squared	0.860261	Mean dependent var	-4.382984
Adjusted R-squared	0.859981	S.D. dependent var	1.884345
S.E. of regression	0.705106	Akaike info criterion	2.143056
Sum squared resid	247.5931	Schwarz criterion	2.159914
Log likelihood	-533.7639	F-statistic	3065.792
Durbin-Watson stat	0.572230	Prob(F-statistic)	0.000000

Step 2: ADF test on OLS residual

$$\Delta \hat{u}_t = \pi \hat{u}_{t-1} + \eta_t$$

Unit Root Test

Test type: Augmented Dickey-Fuller

Test for unit root in:

- Level
- 1st difference
- 2nd difference

Log length:

- Automatic selection: Modified Akaike
- User specified: 0

Maximum lags: 17

Include in test equation:

- Intercept
- Trend and intercept
- None

No intercept

OK Cancel

Use PO Critical Values

Null Hypothesis: UHAT has a unit root
Exogenous: None
Lag Length: 0 (Fixed)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-9.105974	0.0000
Test critical values:		
1% level	-2.569604	
5% level	-1.941459	← Do not use!
10% level	-1.616273	

*MacKinnon (1996) one-sided p-values.

N-1 = 1, constant in 1st step regression
PO 5% critical value is -3.36

Stock-Watson DOLS

$$y_{1t} = \alpha + \beta y_{2t} + \psi_1 \Delta y_{2t+1} + \psi_0 \Delta y_{2t} + \psi_{-1} \Delta y_{2t-1} + \varepsilon_t$$

Dependent Variable: Y1
Method: Least Squares
Date: 05/24/06 Time: 14:05
Sample (adjusted): 3 499
Included observations: 497 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=5)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.019388	0.167521	-0.115705	0.9079
Y2	0.991895	0.035268	28.12417	0.0000
D(Y2(1))	0.103777	0.067753	1.531687	0.1262
D(Y2)	0.064669	0.064762	0.998574	0.3185
D(Y2(-1))	0.080376	0.069807	1.151412	0.2501

R-squared	0.858116	Mean dependent var	-4.393828
Adjusted R-squared	0.856963	S.D. dependent var	1.864151
S.E. of regression	0.705027	Akaike info criterion	2.148847
Sum squared resid	244.5548	Schwarz criterion	2.191187
Log likelihood	-528.9885	F-statistic	743.9081
Durbin-Watson stat	0.583469	Prob(F-statistic)	0.000000

Stock-Watson DGLS

$$y_{1t} = \alpha + \beta y_{2t} + \psi_1 \Delta y_{2t+1} + \psi_0 \Delta y_{2t} + \psi_{-1} \Delta y_{2t-1} + u_t$$

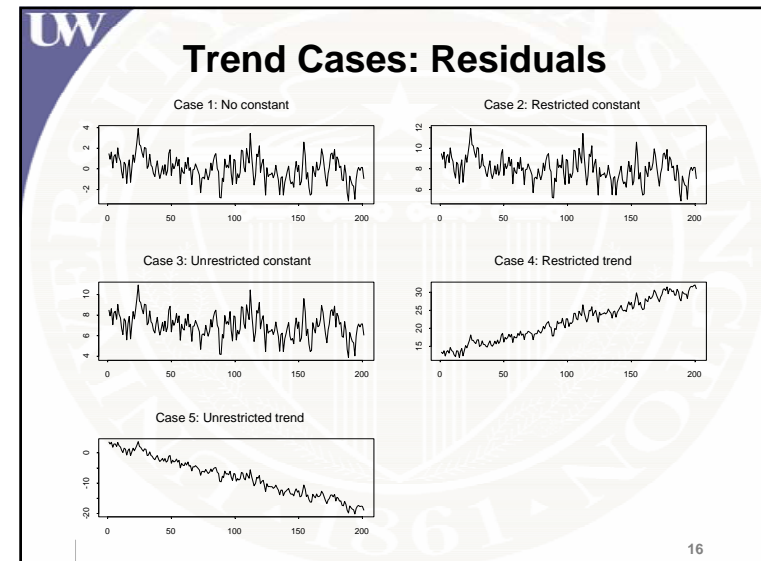
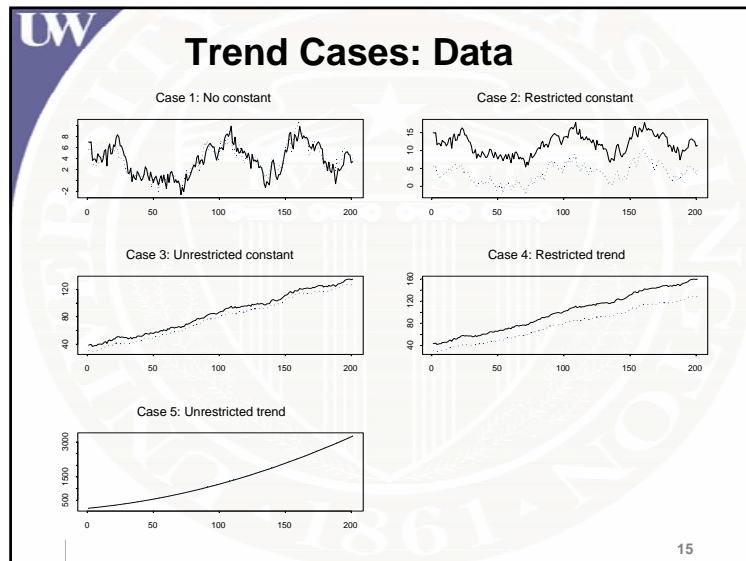
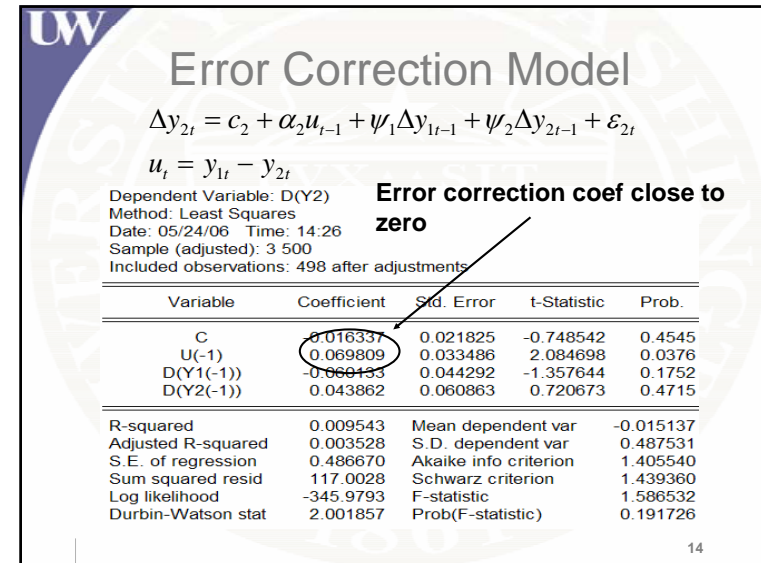
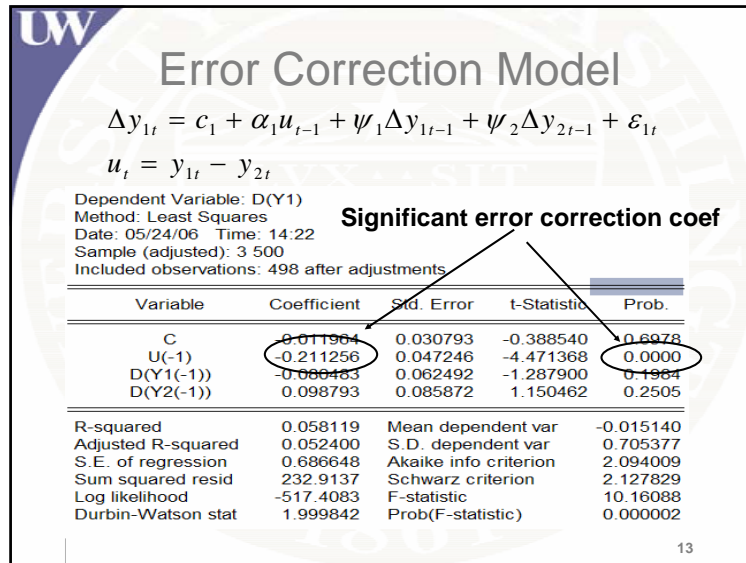
$$u_t = \rho u_{t-1} + \varepsilon_t$$

Dependent Variable: Y1
Method: Least Squares
Date: 05/24/06 Time: 14:09
Sample (adjusted): 4 499
Included observations: 496 after adjustments
Convergence achieved after 4 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.121389	0.206434	-0.588028	0.5568
Y2	0.970262	0.042988	22.57072	0.0000
D(Y2(1))	0.062622	0.045515	0.057618	0.9541
D(Y2)	0.007644	0.054670	0.139816	0.8889
D(Y2(-1))	0.044936	0.045376	0.990307	0.3225
AR(1)	0.714675	0.031811	22.46608	0.0000

R-squared	0.929317	Mean dependent var	-4.402307
Adjusted R-squared	0.928595	S.D. dependent var	1.856414
S.E. of regression	0.496064	Akaike info criterion	1.447801
Sum squared resid	120.5792	Schwarz criterion	1.498687
Log likelihood	-353.0547	F-statistic	1288.465
Durbin-Watson stat	2.034233	Prob(F-statistic)	0.000000

Inverted AR Roots: .71



Test for Cointegration

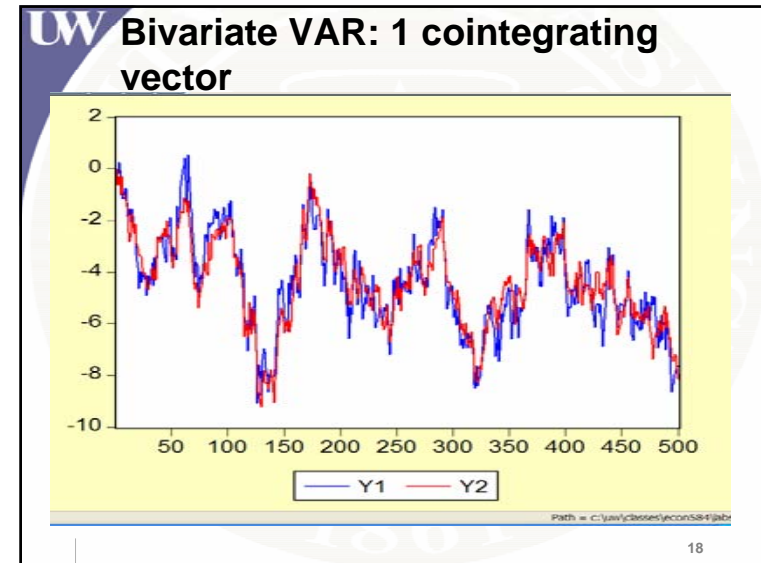
$$y_{1t} = y_{2t} + u_t, \quad u_t = 0.75u_{t-1} + \varepsilon_{1t}$$

$$y_{2t} = y_{2t-1} + \varepsilon_{2t}$$

$\varepsilon_{1t} \sim iid N(0, 0.5^2)$
 $\varepsilon_{2t} \sim iid N(0, 0.5^2)$
 $\beta = (1, -1)'$

Note: No drift and residuals have mean zero

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Johansen LR Tests based on Levels VAR(1) and No Deterministic Terms

Johansen Cointegration Test

Cointegration Test Specification

Deterministic trend assumption of test

Assume no deterministic trend in data:

1) No intercept or trend in CE or test VAR

2) Intercept (no trend) in CE - no intercept in VAR

Allow for linear deterministic trend in data:

3) Intercept (no trend) in CE and test VAR

4) Intercept and trend in CE - no trend in VAR

Allow for quadratic deterministic trend in data:

5) Intercept and trend in CE - linear trend in VAR

Summary:

6) Summarize all 5 sets of assumptions

Exog variables*

Lag intervals

0 0

Lag spec for differenced endogenous

Critical Values

MHM

Size 0.05

Osterwald-Lenum

OK Cancel

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Johansen LR Tests

Date: 05/31/06 Time: 13:36
Sample (adjusted): 2 500
Included observations: 499 after adjustments
Trend assumption: No deterministic trend
Series: Y1 Y2
Lags interval (in first differences): No lags

Trace test finds 1 cointegrating vector

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.145974	79.02705	12.32090	0.0001
At most 1	0.000578	0.288257	4.129906	0.6524

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.145974	78.73880	11.22480	0.0001
At most 1	0.000578	0.288257	4.129906	0.6524

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

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Johansen Test with Misspecified Trend

No lagged differences

Johansen Cointegration Test

Cointegration Test Specification

Deterministic trend assumption of test:

Assume no deterministic trend in data:

1) No intercept or trend in CE or test VAR

2) Intercept (no trend) in CE - no intercept in VAR

Allow for linear deterministic trend in data:

3) Intercept (no trend) in CE and test VAR

4) Intercept and trend in CE - no trend in VAR

Allow for quadratic deterministic trend in data:

5) Intercept and trend in CE - linear trend in VAR

Summary:

6) Summarize all 5 sets of assumptions

* Critical values may not be valid with exogenous variables; do not include C or Trend.

Exog variables*

Lag intervals: 0 0

Lag spec for differenced endogenous

Critical Values:

MacKinnon-Haug-Michelis (1999) p-values

Size: 0.05

Osterwald-Lenum

OK Cancel

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Misspecifying trend leads to wrong conclusion!

Test finds 2 cointegrating vectors!

Date: 05/31/06 Time: 13:33
 Sample (adjusted): 2 500
 Included observations: 499 after adjustments
 Trend assumption: No deterministic trend (restricted constant)
 Series: Y1 Y2
 Lags interval (in first differences): No lags

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.145989	90.23008	20.26184	0.0000
At most 1 *	0.022748	11.48237	9.164546	0.0179

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level
 * denotes rejection of the hypothesis at the 0.05 level
 **MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.145989	78.74771	15.89210	0.0000
At most 1 *	0.022748	11.48237	9.164546	0.0179

Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.05 level
 * denotes rejection of the hypothesis at the 0.05 level
 **MacKinnon-Haug-Michelis (1999) p-values

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Test for Cointegration

$$y_{4t} = 0.5y_{2t} + 0.5y_{3t} + u_t, u_t = 0.75u_{t-1} + \varepsilon_{1t}$$

$$\varepsilon_{1t} \sim iid N(0, 0.5^2)$$

$$y_{2t} = y_{2t-1} + \varepsilon_{2t}, \varepsilon_{2t} \sim iid N(0, 0.5^2)$$

$$y_{3t} = y_{3t-1} + \varepsilon_{3t}, \varepsilon_{3t} \sim iid N(0, 0.5^2)$$

$$Y_t = (y_{4t}, y_{2t}, y_{3t})'$$

$$\beta = (1, -0.5, -0.5)'$$

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