Econ 584 Final Exam
Spring 2004
Due: Friday, June 11 at 5 pm in my office or in my mail box.

June 8, 2004

Question 1

1. Give state space representations of the form

$$\mathbf{y}_t = \mathbf{Z}_t\boldsymbol{\alpha}_t + \mathbf{d}_t + \boldsymbol{\varepsilon}_t,\ \ \boldsymbol{\varepsilon}_t \sim iid\ N(\mathbf{0}, \mathbf{H}_t)$$
$$\boldsymbol{\alpha}_t = \mathbf{T}_t\boldsymbol{\alpha}_{t-1} + \mathbf{c}_t + \mathbf{R}_t\boldsymbol{\eta}_t,\ \ \boldsymbol{\eta}_t \sim iid\ N(\mathbf{0}, \mathbf{Q}_t)$$
$$E[\boldsymbol{\varepsilon}_t\boldsymbol{\eta}_t'] = \mathbf{0}$$
$$\boldsymbol{\alpha}_0 \sim N(\mathbf{a}_0, \mathbf{P}_0)$$

for the following models. Make sure to describe the distribution of the initial state vector $\boldsymbol{\alpha}_0$.

a. AR(p) model

$$\begin{aligned} y_t &= c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + u_t \\ & \quad u_t \sim iid\ N(0, \sigma^2) \end{aligned}$$

where the roots of $\phi(z) = 1 - \phi_1 z - \cdots - \phi_p z^p = 0$ lie outside the complex unit circle.

b. Time varying parameter regression model

$$\begin{aligned} y_t &= x_{1t}\beta_{1t} + x_{2t}\beta_{2t} + u_t,\ \ u_t \sim iid\ N(0, \sigma^2) \\ \beta_{it} &= \beta_{it-1} + v_{it},\ \ v_{it} \sim iid\ N(0, \sigma_i^2) \end{aligned}$$

c. Unobserved components model

$$\begin{aligned} y_t &= \tau_t + c_t \\ \tau_t &= \delta + \tau_{t-1} + v_t,\ \ v_t \sim iid\ N(0, \sigma_v^2) \\ c_t &= \phi_1 c_{t-1} + \phi_2 c_{t-2} + u_t,\ \ u_t \sim iid\ N(0, \sigma_u^2) \end{aligned}$$

where the roots of $\phi(z) = 1 - \phi_1 z - \phi_2 z^2 = 0$ lie outside the complex unit circle.

d. Bivariate cointegrated VAR(2)

$$\begin{aligned}\Delta \mathbf{Y}_t &= \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{Y}_{t-1} + \boldsymbol{\Gamma}\Delta\mathbf{Y}_{t-1} + \boldsymbol{\varepsilon}_t,\ \boldsymbol{\varepsilon}_t \sim iid\ N(\mathbf{0}, \boldsymbol{\Sigma}) \\ \boldsymbol{\beta}'\mathbf{Y}_t &= (1 + \boldsymbol{\beta}'\boldsymbol{\alpha})\boldsymbol{\beta}'\mathbf{Y}_{t-1} + \boldsymbol{\beta}'\boldsymbol{\Gamma}\Delta\mathbf{Y}_{t-1} + \boldsymbol{\beta}'\boldsymbol{\varepsilon}_t\end{aligned}$$

Hint: Define the state vector as $\boldsymbol{\alpha}_t = (\Delta\mathbf{Y}_t', \boldsymbol{\beta}'\mathbf{Y}_t)'$.

2. Briefly discuss how you would estimate a model put in state space form using the method of maximum likelihood.
3. After a state space model has been estimated, it is often desirable to compute and plot the filtered and smoothed estimates of the state vector $\boldsymbol{\alpha}_t$. What is the main difference between the filtered and smoothed estimates of $\boldsymbol{\alpha}_t$? Note: You *do not* have to give the algorithms for computing these estimates.
4. For the AR(p) model, briefly discuss how you can use the state space representation to forecast future values of y_t.
5. For the AR(p) model, briefly discuss how you can use the state space representation to compute the impulse response function $\frac{\partial y_{t+s}}{\partial u_t}$ for $s = 1, 2, \ldots$.

Question 2

Consider the linear regression model

$$\begin{aligned}y_t &= \mathbf{x}_t'\boldsymbol{\beta} + u_t,\ t = 1, \ldots, T \\ \mathbf{x}_t &\text{ is stationary and ergodic} \\ u_t &\sim WN(0, \sigma^2)\end{aligned}$$

Suppose it is thought that the $k \times 1$ vector $\boldsymbol{\beta}$ is subject to a single structural break at an unknown time $1 < T_B < T$.

a. Describe how you would test the null hypothesis that $\boldsymbol{\beta}$ is constant against the alternative hypothesis that $\boldsymbol{\beta}$ has under gone a single structural change at an unknown time T_B. Be sure to describe the asymptotic null distribution of your test (note: you can be somewhat general here; e.g. describe where you can get critical values for your test)

b. Suppose you reject the null hypothesis that $\boldsymbol{\beta}$ is constant in favor of the alternative that $\boldsymbol{\beta}$ has under gone a single structural break at time T_B. How would you estimate the break date T_B? How would you estimate the remaining parameters of the linear regression subject to the structural break?

Now suppose that you suspect that $\boldsymbol{\beta}$ is subject to at most M structural breaks at an unknown times $1 < T_1 < T_2, \ldots < T_M < T$.

c. Describe how you would test for the existence of structural change. That is, describe how you would test the null hypothesis of no structural change against the alternative of at most M structural breaks at an unknown times $1 < T_1 < T_2, \ldots < T_M < T$.

d. Suppose you reject the null hypothesis of no structural change against the alternative of at most M structural breaks at an unknown times $1 < T_1 < T_2, \ldots < T_M < T$. Briefly describe how you would determine the number of structural changes?

Question 3.
Consider the following dynamic simultaneous equations model

$$\begin{aligned} y_{1t} &= \beta_{12} y_{2t} + \gamma_{11} y_{1t-1} + \gamma_{12} y_{2t-1} + \varepsilon_{1t} \\ y_{2t} &= \beta_{21} y_{1t} + \gamma_{21} y_{1t-1} + \gamma_{22} y_{2t-1} + \varepsilon_{2t} \end{aligned}$$

In matrix form the model is written as

$$\mathbf{B}\mathbf{y}_t = \mathbf{\Gamma}\mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t \tag{1}$$

where $\mathbf{y}_t = (y_{1t}, y_{2t})'$, $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t})'$,

$$\mathbf{B} = \begin{pmatrix} 1 & -\beta_{12} \\ -\beta_{21} & 1 \end{pmatrix}, \ \mathbf{\Gamma} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix},$$

and is often referred to as a *structural* VAR model. It is assumed that the error term satisfies $\boldsymbol{\varepsilon}_t \sim iid\ (\mathbf{0}, \mathbf{\Sigma})$ and $\mathbf{\Sigma}$ has elements σ_{ij} $(j = 1, 2)$.

a. Determine the reduced form of the VAR model

$$\mathbf{y}_t = \mathbf{A}\mathbf{y}_{t-1} + \mathbf{e}_t \tag{2}$$

and use it to show that the parameters of the structural VAR (1) are not identified without further restrictions.

b. What kind of restrictions are usually put on the parameters of the structural VAR (1) in order to achieve identification?

c. Under what conditions is the reduced form VAR (2) covariance stationary? Under what conditions is it cointegrated with a single cointegrating vector?

d. A common method used to identify the parameters of the structural VAR (1) is the "Choleski factorization" of the reduced form covariance matrix (the "Sims" method). Briefly describe this method and discuss its limitations.

e. Assume that y_t is covariance stationary and that Σ is a diagonal matrix. Derive the orthogonal moving average representation of y_t with respect to the structural errors ε_t.

f. Briefly describe (but do not derive) the impulse response functions and the forecast error variance decompositions. Why are these quantities of interest to economists?

g. Suppose theory suggests that y_{1t} and y_{2t} should be cointegrated. What is the economic interpretation of cointegration?

h. Continuing with part g, briefly describe how you would test for cointegration. If cointegration is found, how would you estimate the cointegrating vector?