Econ 584 Lab 4

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1 Part I: Analytic Exercises

Question 1
Consider the VAR model

$$\begin{aligned}\mathbf{y}_t &= \mathbf{A}\mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t, \\ \boldsymbol{\varepsilon}_t &\sim \text{iid } (0, \boldsymbol{\Sigma})\end{aligned}$$

where

$$\mathbf{A} = \begin{pmatrix} \frac{1}{2} & 0 \\ 1 & \frac{1}{4} \end{pmatrix}.$$

1. Find the eigenvalues of $\mathbf{A}$
2. Find the roots of the characteristic polynomial

$$\det(\mathbf{I}_2 - \mathbf{A}z) = 0.$$

and show that the roots of the characteristic polynomial are the inverses of the eigenvalues of $\mathbf{A}$.

Question 2
Consider a bivariate VAR(p) model

$$\begin{aligned}\mathbf{A}(L)\mathbf{y}_t &= \boldsymbol{\varepsilon}_t,\ \mathbf{A}(L) = \mathbf{I} - \mathbf{A}_1\mathbf{L} - \cdots - \mathbf{A}_p\mathbf{L}^p \\ \boldsymbol{\varepsilon}_t &\sim \text{iid } (0, \boldsymbol{\Sigma})\end{aligned}$$

with Wold (moving average) representation

$$\mathbf{y}_t = \boldsymbol{\Psi}(L)\boldsymbol{\varepsilon}_t$$

where $\boldsymbol{\Psi}(L) = \sum_{k=0}^{\infty} \boldsymbol{\Psi}_k L^k$ and $\boldsymbol{\Psi}_0 = \mathbf{I}_2$.

1. Find the moving average coefficients $\boldsymbol{\Psi}_k$ for a VAR(1) model.

2. Show that the moving average coefficients for a VAR(2) model can be found recursively by

$$\mathbf{\Psi}_0 = \mathbf{I}_2,\ \mathbf{\Psi}_1 = \mathbf{A}_1$$

and

$$\mathbf{\Psi}_k = \mathbf{A}_1\mathbf{\Psi}_{k-1} + \mathbf{A}_2\mathbf{\Psi}_{k-2},\ k > 1$$

Question 3

Consider the bivariate cointegrated VECM

$$\begin{aligned} \mathbf{\Delta y}_t &= \mathbf{c} + \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t, \\ \boldsymbol{\varepsilon}_t &\sim \text{iid } (0, \mathbf{\Sigma}) \end{aligned}$$

where $\boldsymbol{\alpha} = (\alpha_1, 0)'$ and $\boldsymbol{\beta} = (1, -\beta_2)'$. Equation by equation, the system is given by

$$\begin{aligned} \Delta y_{1t} &= c_1 + \alpha_1(y_{1t-1} - \beta_2 y_{2t-1}) + \varepsilon_{1t} \\ \Delta y_{2t} &= c_2 + \varepsilon_{2t} \end{aligned}$$

1. From the cointegrated VECM representation above, derive the VECM representation

$$\mathbf{\Delta y}_t = \mathbf{c} + \mathbf{\Pi y}_{t-1} + \boldsymbol{\varepsilon}_t$$

and the VAR(1) representation

$$\mathbf{y}_t = \mathbf{c} + \mathbf{A y}_{t-1} + \boldsymbol{\varepsilon}_t.$$

That is, determine the elements of the matrices $\mathbf{\Pi}$ and $\mathbf{A}$.

2. Show that $\boldsymbol{\beta}'\mathbf{y}_t$ follows an AR(1) process. Show that the AR(1) process is stable provided that $-2 < \alpha_1 < 0$. What can you say about the system when $\alpha_1 = 0$?

2 Part II: Empirical Exercises

Question 1. Using the output and unemployment data in the Excel file `BQ.xls` on the class webpage, specify and estimate a SVAR model of the form

$$\begin{aligned} \mathbf{B y}_t &= \boldsymbol{\gamma}_0 + \sum_{j=1}^{p} \mathbf{\Gamma}_j \mathbf{y}_{t-j} + \boldsymbol{\varepsilon}_t \\ E[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'] &= \mathbf{D} = \text{diagonal} \\ \mathbf{y}_t &= (\Delta y_{1t}, y_{2t})' \end{aligned}$$

where y_1 represents log output and y_2 represents unemployment. Specify ε_1 as a permanent "supply" shock and ε_2 as a transitory "demand" shock. To identify the parameters of the SVAR, impose the Blanchard-Quah restriction

$$\lim_{s\to\infty} \frac{\partial y_{1t+s}}{\partial \varepsilon_{2t}} = \theta_{12}(1) = \sum_{s=0}^{\infty} \theta_{12}^{(s)} = 0$$

that transitory shocks have no long-run effect on the level of output.

1. Determine the lag length of the reduced form VAR using the AIC information criteria. Use a maximum lag of 8. Report the VAR estimates and briefly comment on the fit of the VAR.

2. Read the Eviews online manual entry on Structural (Identified) VARs. The subsection on Long Run Restrictions shows how to impose the restriction that $\theta_{12}(1) = 0$. Note: Eviews uses $\mathbf{C}$ to denote $\Theta(1)$. After imposing the restriction $\theta_{12}(1) = 0$, estimate the SVAR and report the resulting estimates.

3. Compute the impulse response functions and forecast error variance decompositions from the SVAR model using a maximum horizon of 40 quarters. Briefly comment on what you find.

Question 2: Hayashi, chapter 10 Empirical Exercises (a) - (d) (estimation of cointegrated Money Demand function). *Hint*: for the DOLS estimates, you can use the Newey-West HAC standard errors instead of going through the procedure described on pages 656-657.

Question 3: Here you will use the same data as in Question 2 but use Eviews to investigate cointegration using Johansen's maximum likelihood procedure. You will find it helpful to read the Eviews online help for Cointegration Test.

1. Let $\mathbf{Y}_t = (m_t - p_t, R_t, y_t)'$. Use the AIC model selection criteria to determine the lag length p for a VAR(p) model for Y_t :

$$\mathbf{Y}_t = \mathbf{c} + \mathbf{A}_1 \mathbf{Y}_{t-1} + \cdots + \mathbf{A}_p \mathbf{Y}_{t-p} + \boldsymbol{\varepsilon}_t$$

   Use $p_{\max} = 4$.

2. Based on your estimate for p, estimate a VECM($p-1$) and compute the Johansen trace and maximum eigenvalue statistics to determine the number of cointegrating vectors. Compute the test statistics using a VECM with an unrestricted constant as well as restricted trend. What do you find? Do your results agree with those in Question 2?

3. Regardless of the outcome of the cointegration tests, impose one cointegrating vector and estimate it using Johansen's maximum likelihood procedure. Normalize the cointegrating vector on $m_t - p_t$. Compare your results to these found in Question 2.

4. Using a likelihood ratio statistic, test the hypothesis that income elasticity is equal to unity.