Econ 582 Lecture 2

Note Title 4/3/2013

AR(1) $\Rightarrow$ $Y_t - \mu = \phi(Y_{t-1} - \mu) + \epsilon_t$

multiply both sides by $Y_{t-j} - \mu$

$$(Y_t - \mu)(Y_{t-j} - \mu) = \phi(Y_{t-1} - \mu)(Y_{t-j} - \mu) + \epsilon_t (Y_{t-j} - \mu)$$

Take Expectations

$$\gamma_j = E\left[(Y_t - \mu)(Y_{t-j} - \mu)\right] = \phi \underbrace{E\left[(Y_{t-1} - \mu)(Y_{t-j} - \mu)\right]}_{\gamma_{j-1}} + E\left[\epsilon_t (Y_{t-j} - \mu)\right]$$

ϵ_t b/c Y_{t-j} only depends on ϵ_{t-j} and earlier

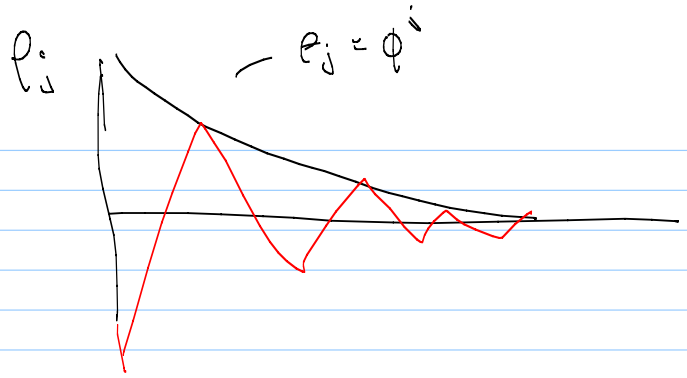
$$\Rightarrow \quad \gamma_j = \phi \gamma_{j-1}$$

$$\Rightarrow \mathrm{Cov}(Y_t, Y_{t-j}) = \phi \, \mathrm{Cov}(Y_t, Y_{t-j+1}) \; !$$

By recursive substitution

$$\Rightarrow \gamma_j = \phi^j \cdot \gamma_0 = \phi^j \cdot \frac{\sigma^2}{1-\phi^2}$$

$$\rho_j = \frac{\gamma_j}{\gamma_0} = \phi^j = \mathrm{CORR}(Y_t, Y_{t-j})$$



$0 < \phi < 1$

$-1 < \phi < 0$

$\rho_j = \phi^j$

Note: For AR(1) $\psi_j = \phi^j = \rho_j$

$\Rightarrow$ ACF = IRF!