Fin 483 Midterm

Suggested Solutions

I. Return calculations 5 pts each

1. Simple 1-month return

$$R_t = \frac{135.754 - 129.975}{129.975} = \boxed{4.45\%} \quad (0.0445)$$

Derived annual return.

$$R_A = (1 + R_t)^{12} - 1$$

$$= (1.0445)^{12} - 1 = \boxed{68.54\%} \quad (0.6854)$$

Actual annual return

$$R_t(12) = \frac{114.3 - 129.975}{129.975} = \boxed{-12.06\%} \quad (-0.1206)$$

Much different from derived annual return!

2. Simple annual real return

$$\frac{\left(\frac{114.3}{105}\right) - \left(\frac{129.975}{100}\right)}{\left(\frac{129.975}{100}\right)} = \boxed{-16.25\%} \quad (-0.1625)$$

3. CC 1-month return

$$r_t = \ln(1 + R_t) = \ln(1.0445) =$$

$\boxed{4.35\%}$ (0.0435)

Derived annual return

$$r_A = 12 * r_t = 12(0.0435) = \boxed{52.20\%} \quad (0.5220)$$

3 cont'd Actual 1 year cc return

$r_c(12) = \ln(1 + R_c(12)) = \ln(1 - 0.1206)$

$= -12.85\%$ (-0.1285)

Again, the actual cc return is much different than the derived cc return.

4. CC Annual real return

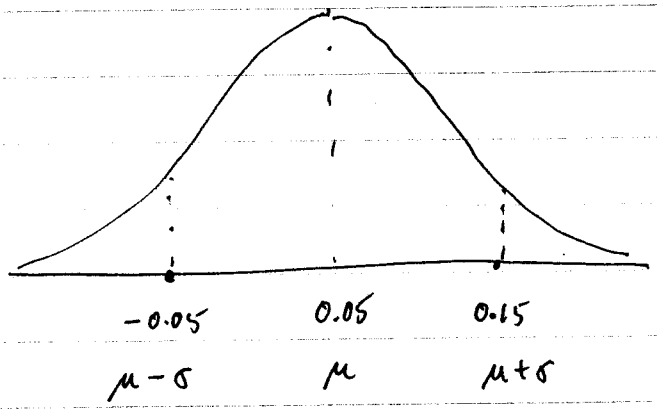
$r_c(12) - \pi$ where $\pi = \ln(CPI_t) - \ln(CPI_{t-1})$

$\pi = 4.88\%$

$\Rightarrow$ real cc return : $-12.88\% - 4.88\% = -17.73\%$

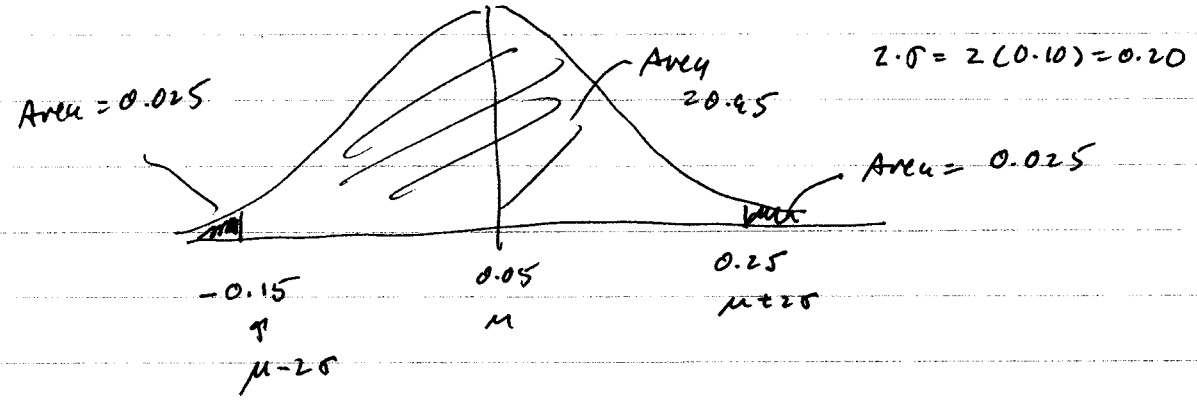
II. Random Variables and Probability 4 pts each.

1.



2. Skewness = 0, kurtosis = 3 (excess kurtosis = 0)

3. Roughly 95% probability lies b/w $\mu - 2\sigma$, $\mu + 2\sigma$



$\Rightarrow$ 2.5% quantile is $\underline{\underline{-0.15}}$, approximately

4. Use NORMDIST(0.5, 0.05, 0.10, TRUE)

$-$ NORMDIST(-1.5, 0.05, 0.10, TRUE)

5. $W = 1000 + 1000 \cdot X$

$E[W] = 1000 + 1000 \cdot E[X] = 1000 + 1000(0.05)$

$= \underline{\underline{1050}}$

$Var(W) = (1000)^2 Var(X) = (1000)^2 + (0.10)^2$

$= \underline{\underline{10,000}}$

$SD(W) = (1000)(0.10) = \underline{\underline{100}}$

Value-at-Risk

6. 1% VaR with cc compounding

$$(\$100{,}000) * (e^{-0.0025} - 1) = -\$249.7$$

5% VaR with cc compounding

$$(\$100{,}000) * (e^{-0.0017} - 1) = -\$169.9$$

7. $R_m = \sum_{t=0}^{29} R_t$ where R_t = daily cc return

8. $E[R_m] = 30 \cdot E[R_t] = 30(0.0004)$

$= 0.012$ or 1.2%

$Var(R_m) = 30 \cdot Var(R_t) = 30 * (0.00125)^2$

$= 0.000046875$

$= 0.004479\%$

$SD(R_m) = \sqrt{30}\ SD(R_t) = \sqrt{30}(0.00125)$

$= 0.006847$ or 0.68%

III. Descriptive Statistics — 4pts each.

1. Risk-return comparison

	$\hat{\mu}$	$\hat{\sigma}$
SP500	0.84%	4.06%
WAMU	1.56%	8.80%

WAMU has a higher estimated expected return than SP500 but also has a higher estimated return SD. We might say that WAMU is riskier than SP500 due to its higher SD value.

2. Both SP500 & WAMU have negative skewness values with SP500 having more skewness. The excess kurtosis for SP500 is 1.602 while the value for WAMU is only 0.444 If data were from a normal distn then skew = 0 and excess kurt = 0. Hence SP500 looks more nonnormal than WAMU.

The boxplots show reasonably symmetric distributions. SP500 has one negative moderate outlier which is probably creating the negative skewness. WAMU has both positive and negative moderate outliers. Interestingly, these outliers do not seem to influence the excess kurtosis. In general, the boxplots show that WAMU has more moderate outliers than one would expect from a normal distribution.

3. The scatterplot reveals a weak positive linear relationship. This is confirmed by the positive covariance value and correlation of 0.30.

4. CER Model Estimates

	$\hat{\mu}_i$	$\hat{\sigma}_i^2$	$\hat{\sigma}_i$	$\hat{\sigma}_{ij}$	$\hat{\rho}_{ij}$
SP500	0.84%	0.[illegible]%	4.06%	0.0011	0.3003
WAMU	1.56%	0.77%	8.80%		

5. SE calculations

	$SE(\hat{\mu})$	$SE(\hat{\sigma}_i)$	$SE(\hat{\rho}_{ij})$
Formula	$\hat{\sigma}_i/\sqrt{120}$	$\hat{\sigma}_i/\sqrt{2 \cdot 120}$	$(1-\hat{\rho}_{ij}^2)/\sqrt{120}$
SP500	0.37%	0.26%	0.083
WAMU	0.80%	0.57%	

The $SE(\hat{\mu}_i)$ values are about half the size of the $\hat{\mu}_i$ values indicating a ~~not very~~ fairly imprecise estimator.

The $SE(\hat{\sigma}_i)$ values much smaller than the $\hat{\sigma}_i$ values indicating that σ_i is estimated quite well.
The $SE(\hat{\rho})$ value is about 1/4 the value of $\hat{\rho}$, indicating

a moderately precise estimator

6. 95% C.I. for μ : $\hat{\mu} \pm 2 \cdot SE(\hat{\mu})$

	Lower	Upper
SP500	0.10%	1.58%
WAMU	−0.05%	3.17%

The 95% C.I. for SP500 is narrower than the 95% C.I. for WAMU. Notice that the 95% C.I. for WAMU contains both negative & positive values. This indicates an imprecise estimator of μ because our decisions would be very different for negative μ than for positive μ.

7. The CER model assumes that μ & σ are constant over time. The rolling estimates clearly indicate that μ and σ for the SP500 change over time. The μ values are low initially, rise during the late 90s than drop to negative values during 2001. Similarly, the σ values start off around 3% in the early 90s then jump to about 5% ~~in the early 90s~~ at the end of the sample.

IV. CER model

1. Assumptions

$$R_t = \mu + \epsilon_t$$

$$\epsilon_t \sim \text{iid } N(0, \sigma^2)$$

- Constant μ & σ values
- ϵ_t uncorrelated over time
- ϵ_t is normally distributed

2. ϵ_t is interpreted as the unexpected return

$$\epsilon_t = R_t - \mu = R_t - E[R_t]$$

or random news component between periods $t-1$ & t.

3. $\hat{\sigma}$ is an unbiased estimator of σ if $E[\hat{\sigma}] = \sigma$.
In a Monte Carlo set up, this means that the average value of $\hat{\sigma}$ computed over an infinity many Monte Carlo samples will be equal to σ.
To see if $\hat{\sigma}$ is an unbiased estimate do

- Generate a large number of simulated samples from CER model with $\mu = 0.01$ and $\sigma = 0.05$ - say $N = 10{,}000$ samples of size $T = 50$.
- For each sample, compute $\hat{\sigma}^i = \sqrt{\frac{1}{49}\sum_{t=1}^{50}(R_t^* - \hat{\mu})^2}$ $i = 1, \ldots, 10{,}000$

- Evaluate the average of the Monte Carlo $\hat{\sigma}$ estimates

$$\bar{\hat{\sigma}} = \frac{1}{10{,}000} \sum_{i=1}^{10{,}000} \hat{\sigma}_i$$

and compare $\bar{\hat{\sigma}}$ to $\sigma = 0.05$. If $\bar{\hat{\sigma}} \simeq 0.05$ then $\hat{\sigma}$ is an unbiased estimate.

4 The approximate formula for $SE(\hat{\sigma})$ is

$$SE(\hat{\sigma}) \approx \frac{\hat{\sigma}}{\sqrt{2 \cdot T}}, \quad T = \text{sample size}.$$

A monte carlo calculation of $SE(\hat{\sigma})$ would use the sample standard deviation of the 10,000 simulated returns

$$SE_{MC}(\hat{\sigma}) = \sqrt{\frac{1}{10000-1} \sum_{i=1}^{N} (\hat{\sigma}_i - \bar{\hat{\sigma}})^2}$$

where $\bar{\hat{\sigma}}$ is the average of the Monte Carlo estimates of σ.