University of Washington | Fall 2004
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Economics 483

**Midterm Exam**

This is a closed book and closed note exam. However, you are allowed one page of notes (double-sided). Answer all questions and write all answers in a blue book or on separate sheets of paper. Time limit is 1 hours and 20 minutes. Total points = 105.

I. Return Calculations (30 pts, 5 points each)

1. Consider a one month investment in two Northwest stocks: Amazon and Costco. Suppose you buy Amazon and Costco at the end of September at $P_{A,t-1} = \$38.23$, $P_{C,t-1} = \$41.11$ and then sell at the end of the October for $P_{A,t} = \$41.29$, $P_{C,t} = \$41.74$. (Note: these are actual closing prices for 2004 taken from Yahoo!)

```
> p.A = c(38.23, 41.29)
> p.C = c(41.11, 41.74)
```

a. What are the simple monthly returns for the two stocks?

```
> R.A = (p.A[2] - p.A[1])/p.A[1]
> R.A
[1] 0.080042
> R.C = (p.C[2] - p.C[1])/p.C[1]
> R.C
[1] 0.015325
```

b. What are the continuously compounded returns for the two stocks?

```
> r.A = log(1 + R.A)
> r.A
[1] 0.077
> r.C = log(1 + R.C)
> r.C
[1] 0.015209
```

c. Suppose Costco paid a $0.10 per share cash dividend at the end of October. What is the monthly simple total return on Costco? What is the monthly dividend yield?

```
> div.C = 0.10
> R.total.C = (p.C[2] + div.C - p.C[1])/p.C[1]
> R.total.C
[1] 0.017757
> div.yield.C = div.C/p.C[1]
> div.yield.C
[1] 0.0024325
```

d. Suppose the monthly returns on Amazon and Costco from question (a) above are the same every month for 1 year. Compute the simple annual returns as well as the continuously compounded annual returns for the two stocks.

```
> R.A.annual = (1 + R.A)^12 - 1
> R.C.annual = (1 + R.C)^12 - 1
> R.A.annual
[1] 1.5193
> R.C.annual
[1] 0.20022
> r.A.annual = log(1 + R.A.annual)
> r.C.annual = log(1 + R.C.annual)
> r.A.annual
[1] 0.924
> r.C.annual
[1] 0.1825
```

e. At the end of September, 2004, suppose you have $10,000 to invest in Amazon and Costco over the next month. If you invest $8000 in Amazon and $2000 in Costco, what are your portfolio shares, $x_A$ and $x_C$.

```
> x.A = 8000/10000
> x.C = 2000/10000
> x.A
[1] 0.8
> x.C
[1] 0.2
```

f. Continuing with the previous question, compute the monthly simple return and the monthly continuously compounded return on the portfolio. Assume that Costco does not pay a dividend.

```
> R.port = x.A*R.A + x.C*R.C
> R.port
[1] 0.067098
> r.port = log(1 + R.port)
> r.port
[1] 0.064943
```

II. Probability Theory (35 points, 5 points each)

1. Consider an investment in Starbucks stock over the next year. Let $R$ denote the monthly *simple* return and assume that $R \sim N(0.02,(0.20)^2)$. That is, $E[R]=0.02$ and $\text{var}(R)=(0.20)^2$. Let $W_0$ = \$1,000 denote the initial investment (at the beginning of the month), and let $W_1 = W_0(1 + R)$ denote the investment value at the end of the month.

```
> mu.R = 0.02
> sd.R = 0.20
> W0 = 1000
```

a) Compute $E[W_1]$, $\text{var}(W_1)$ and $SD(W_1)$.

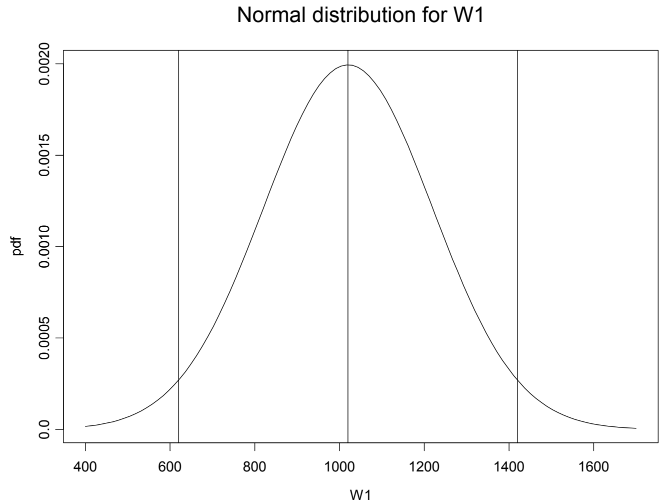
```
> mu.W1 = 1000*(1 + mu.R)
> var.W1 = 1000*1000*sd.R^2
> sd.W1 = 1000*sd.R
> mu.W1
[1] 1020
> var.W1
[1] 40000
> sd.W1
[1] 200
```

b) What is the probability distribution of $W_1$? Sketch this distribution, indicating the location of $E[W_1]$ and $E[W_1] \pm 2 \cdot SD(W_1)$.

$W_1 \sim N(1020,(200)^2)$

Note:

```
> mu.W1 - 2*sd.W1
[1] 620
> mu.W1 + 2*sd.W1
[1] 1420
```




c) Approximately, what is $\Pr(W_1 < \$620)$. Hint: How much of the area under the probability curve for $W_1$ is between $E[W_1] \pm 2 \cdot SD(W_1)$?

Since $E[W_1] - 2 \cdot SD(W_1) = 620$, and roughly 95% of the area is between $E[W_1] \pm 2 \cdot SD(W_1)$ it follows that $\Pr(W_1 < \$620) \approx 0.05/2 = 0.025$. The exact answer is

```
> pnorm(620,mean=mu.W1, sd=sd.W1)
[1] 0.02275
```

d) Compute the 5% quantile of the distribution for $W_1$. (Hint: the 5% quantile for a standard normal random variable is -1.645.) Compute how much you would lose over the month if $W_1$ was equal to the 5% quantile.

The 5% quantile of the distribution for $W_1$ is

```
> q.05.W1 = mu.W1 + sd.W1*qnorm(0.05)
> q.05.W1
[1] 691.03
```

If $W_1$ was equal to the 5% quantile you would lose

```
> W0 - q.05.W1
[1] 308.97
```

e) Compute the 5% quantile of the distribution for $R$. Using this quantile, compute the monthly 5% value-at-risk ($VaR_{.05}$) of the $1,000 investment.

The 5% quantile of the distribution for *R* is

```
> q.05 = mu.R + sd.R*qnorm(0.05)
> q.05
[1] -0.30897
```

The monthly 5% value-at-risk ($VaR_{.05}$) of the $1,000 investment is

```
> VaR.05 = q.05*W0
> VaR.05
[1] -308.97
```

2. Let $\{R_t\}_{t=-\infty}^{\infty} = \{\ldots, R_1, \ldots, R_T, \ldots\}$ denote a stochastic process (time series) for returns.

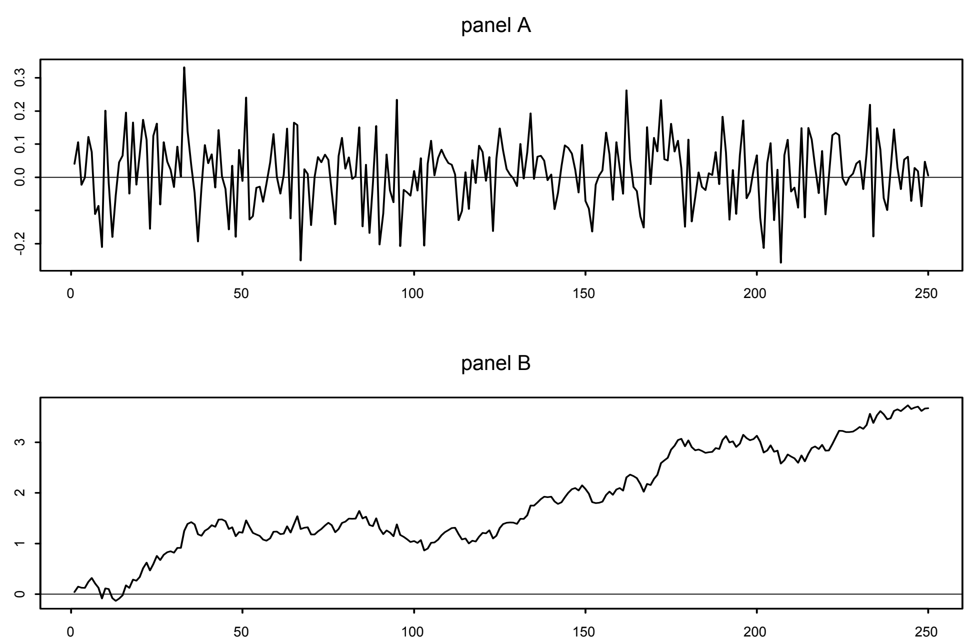
a) What conditions are required for $\{R_t\}_{t=-\infty}^{\infty}$ to be covariance (weakly) stationary?

The conditions for covariance stationarity are

- $E[R_t] = \mu$ independent of *t*
- $\mathrm{var}(R_t) = \sigma^2$ independent of *t*
- $\mathrm{cov}(R_t, R_{t-j}) = \gamma_j$ depends only on *j* but not on *t*.

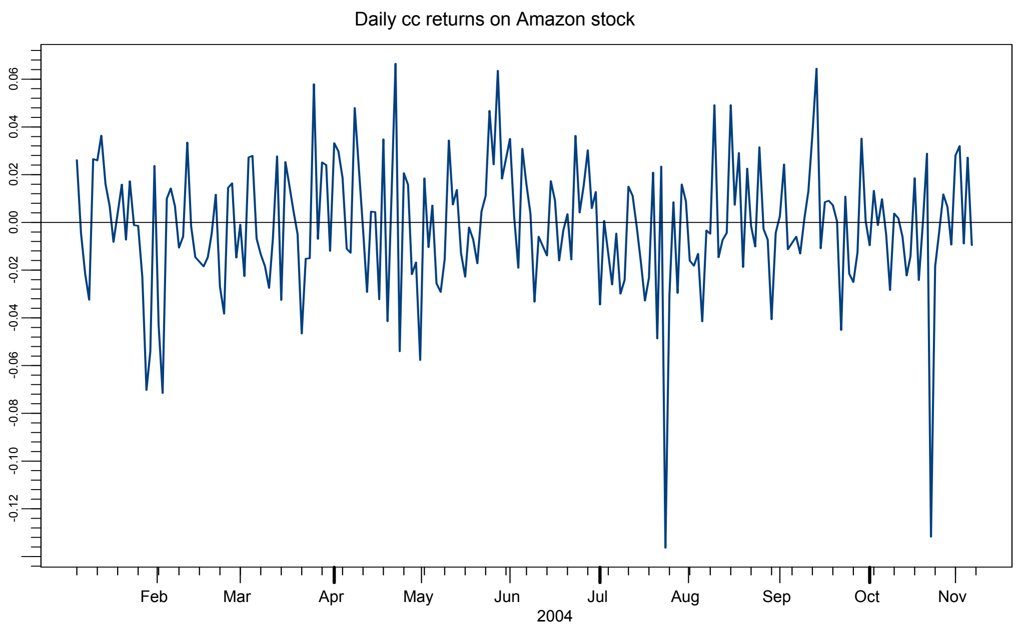
b) In the figure below, which panel represents a realization of a covariance stationary time series?

Panel A is the covariance stationary time series.



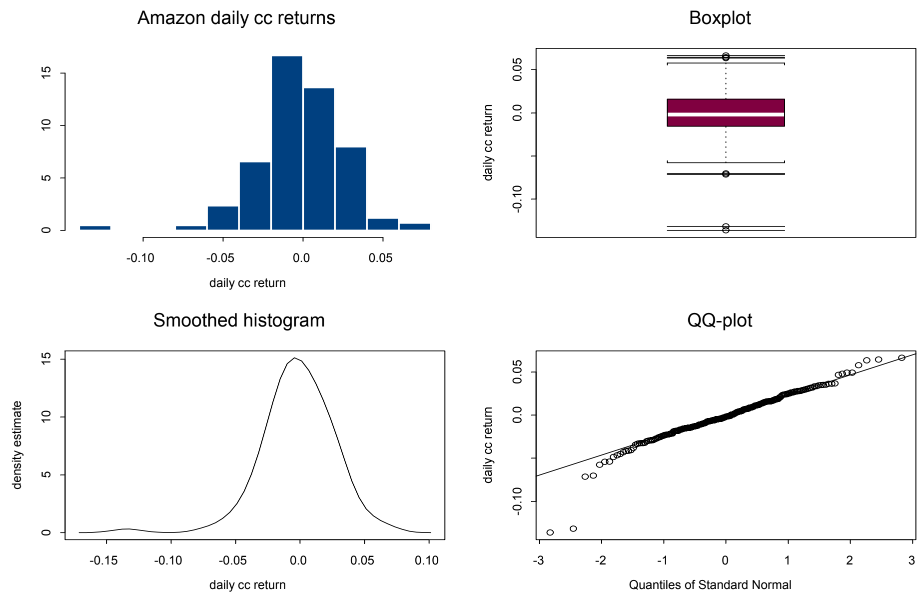
III. Descriptive Statistics (20 points, 5 points each)

1. Consider the *daily* continuously compounded (cc) returns on Amazon stock computed using *daily* closing prices over the period January 5, 2004 – November 5, 2003.




a. Do the daily cc returns appear to be a realization from a covariance stationary stochastic process? Briefly justify your answer.

The daily cc returns look very much like a covariance stationary time series. The mean appears to be constant (no trending behavior). The volatility of the series for the most part looks constant. There are a few large negative outliers but these do not appear to greatly inflate the volatility. From the graph it is hard to determine the properties of the autocorrelations. However, the plot looks remarkably like a computer simulation of white noise (which is a covariance stationary process).




b. The figure above shows various graphical diagnostics regarding the empirical distribution of the daily cc returns on Amazon. Based on these diagnostics, do you think that the normal distribution is a good model for the underlying probability distribution of the daily cc returns on Amazon? Briefly justify your answer by commenting on each of the four plots.

- The histogram shows a slight asymmetry (long left tail) due to the two large negative returns. The normal distribution is symmetric, and this is evidence against symmetry
- The asymmetry in the empirical distribution is more clearly revealed in the smoothed histogram
- The boxplot also reveals the asymmetry of the distribution. The boxplot also shows four outliers, which is evidence of a distribution with fatter tails than the normal distribution
- The qq-plot shows that the returns seem to match the normal distribution in the middle of the distribution but not in the tails. The large drop from linearity for negative returns shows that the data are asymmetric.

Summary: except for the two large negative outliers, the daily returns are well characterized by a normal distribution.

c. Summary descriptive statistics, computed from S-PLUS, for the daily cc returns are given below. Which of these summary statistics indicate evidence for, or against, the normal distribution model for the daily cc returns.

```
> summaryStats(amzn.ret)

Sample Quantiles:
     min       1Q    median      3Q     max
 -0.1363 -0.01559 -0.002018 0.01577 0.06642

Sample Moments:
      mean     std skewness kurtosis
 -0.001645 0.02746  -0.9524    7.143

Number of Observations:  213
```

- The normal distribution has zero skewness and kurtosis equal to three. The sample skewness is negative (due to the two large negative returns) indicating asymmetry in the distribution, and the sample kurtosis is much larger than 3 indicating fatter tails than the normal distributions.

d. The empirical 1% and 5% quantiles from the daily cc returns are given below.

```
> quantile(amzn.ret,probs=c(0.01,0.05))
        1%       5%
 -0.071331 -0.04219
```

Using these quantiles, compute the daily 1% and 5% value-at-risk (*VaR*) based on an investment of $100,000.

For continuously compounded returns, the 1% and 5% VaR values are computed as follows

```
> q.01 = quantile(amzn.ret,probs=0.01)
> q.05 = quantile(amzn.ret,probs=0.05)
> VaR.01 = (exp(q.01) - 1)*100000
> VaR.01
      1%
 -6884.6
> VaR.05 = (exp(q.05) - 1)*100000
> VaR.05
      5%
 -4131.2
```

IV. Constant Expected Return Model (20 points, 5 points each)

1. Consider the constant expected return model

$$R_{it} = \mu_i + \varepsilon_{it},\ \varepsilon_{it} \sim iid\ N(0, \sigma_i^2)$$
$$\text{cov}(R_{it}, R_{jt}) = \sigma_{ij},\ corr(R_{it}, R_{jt}) = \rho_{ij}$$

for the monthly continuously compounded returns on Boeing and Microsoft (same data as lab 5) over the period July 1992 through October 2000. For this period there are 100 monthly observations.

a) Based on the S-PLUS output below, give the "plug-in principle" estimates for $\mu_i, \sigma_i^2, \sigma_i, \sigma_{ij}$ and $\rho_{ij}$ for the two assets.

```
        muhat.vals sigma2hat.vals sigmahat.vals
rboeing   0.012436      0.0057945      0.076121
  rmsft   0.027564      0.0114112      0.106823

   covhat.vals rhohat.vals
rboeing,rmsft -0.000067409  -0.0082898
```

This is the "Who's buried in Grant's tomb question". The CER model estimates are just the sample statistics above

| | $\hat{\mu}$ | $\hat{\sigma}^2$ | $\hat{\sigma}$ | $\hat{\sigma}_{b,m}$ | $\hat{\rho}_{b,m}$ |
|---|---|---|---|---|---|
| Boeing | 0.0124 | 0.0058 | 0.0761 | -0.00007 | -0.008 |
| Microsoft | 0.0276 | 0.0114 | 0.1068 | | |

b) Using the above output, compute estimated standard errors for $\hat{\mu}_i,\ \hat{\sigma}_i,\ (i = boeing, microsoft)$ and $\hat{\rho}_{msft,boeing}$. Briefly comment on the precision of the estimates.

```
        muhat.vals sigmahat.vals
rboeing   0.012436      0.076121
  rmsft   0.027564      0.106823

> se.muhat = sigmahat.vals/sqrt(100)
> se.muhat
   rboeing     rmsft
 0.0076121 0.010682

> se.sigmahat = sigmahat.vals/sqrt(2*100)
> se.sigmahat
   rboeing      rmsft
 0.0053826 0.0075536
```

```
> se.rhohat = (1 - rhohat^2)/sqrt(100)
> se.rhohat
[1] 0.099993
```

For Boeing and Microsoft, the estimated SE values for $\hat{\mu}$ are around 0.01, which are similar in size to the values of $\hat{\mu}$. This indicates that the means are not estimated very precisely. In contrast, the estimated SE values for $\hat{\sigma}$ are around 0.005, whereas the values of $\hat{\sigma}$ are about 0.1. Hence, the return standard deviations are estimated much more precisely than the means. Finally, the estimated SE value for $\hat{\rho}$ is about 0.1 whereas $\hat{\rho}$ is roughly zero. Since the SE is fairly small it is quite likely that the true correlation is close to zero.

c) For Microsoft, compute 95% confidence intervals for $\mu$ and $\sigma$. Also, compute a 95% confidence interval for $\rho$. Briefly comment on the precision of the estimates.

The 95% confidence for $\mu$ is

```
> muhat.lower = muhat.vals["rmsft"] - 2*se.muhat["rmsft"]
> muhat.upper = muhat.vals["rmsft"] + 2*se.muhat["rmsft"]
> cbind(muhat.lower,muhat.upper)
      muhat.lower muhat.upper
rmsft   0.0061992    0.048929
```

This interval is quite wide – ranging from 0.6% per day to almost 5% per day. That is a big variation for a daily return!

The 95% confidence for $\sigma$ is

```
> sigmahat.lower = sigmahat.vals["rmsft"] -
2*se.sigmahat["rmsft"]
> sigmahat.upper = sigmahat.vals["rmsft"] +
2*se.sigmahat["rmsft"]
> cbind(sigmahat.lower,sigmahat.upper)
      sigmahat.lower sigmahat.upper
rmsft       0.091716        0.12193
```

This interval is fairly narrow – ranging from 9% to about 12% - and indicates that the standard deviation is estimated fairly precisely.

The 95% confidence for $\rho$ is

```
> rhohat.lower = rhohat.vals - 2*se.rhohat
> rhohat.upper = rhohat.vals + 2*se.rhohat
> cbind(rhohat.lower,rhohat.upper)
              rhohat.lower rhohat.upper
rboeing,rmsft     -0.20828       0.1917
```

Notice that the 95% confidence interval for $\rho$ contains both negative and positive values so the direction of linear association between the returns on Boeing and Microsoft is uncertain. However, whatever the direction the magnitude is most likely small.

d) Briefly describe how you could compute an estimated standard error for the estimated 5% monthly value-at-risk, based on a $100,000 investment, computed using the formula

$$V\hat{a}R_{.05} = (e^{\hat{q}_{.05}} - 1)\cdot 100{,}000,\ \hat{q}_{.05} = \hat{\mu} + \hat{\sigma}\cdot(-1.646)$$

The VaR formula is a complicated nonlinear function of $\hat{\mu}$ and $\hat{\sigma}$ making it a difficult problem to derive an analytic formula for the standard error. However, the bootstrap can be used to estimate the SE without any problem. The bootstrap works by sampling with replacement from the original data B times and computing an estimate of VaR for each bootstrap sample. This gives B bootstrap estimates of the VaR. The bootstrap estimate of the SE is simply the sample standard deviation of the bootstrap values of the VaR.

e) Consider a portfolio of Boeing and Microsoft stock with 50% of wealth invested in each asset (that is $x_{boeing} = x_{microsoft} = 0.5$). Using the CER model estimates, compute an estimate of the portfolio expected return, portfolio variance and portfolio standard deviation. That is, compute $\hat{\mu}_p, \hat{\sigma}_p^2$ and $\hat{\sigma}_{\mathrm{p}}$.

```
> mu.p = x.b*muhat.vals["rboeing"]+x.m*muhat.vals["rmsft"]
> mu.p
     0.02
> sig2.p = (x.b^2)*sig2hat.vals["rboeing"] +
(x.m^2)*sig2hat.vals["rmsft"] + 2*x.b*x.m*covhat
> sig2.p
 0.0042677
> sqrt(sig2.p)
 0.065328
```