University of Washington
Department of Economics

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Economics 483

Midterm Exam

This is a closed book and closed note exam. However, you are allowed one page of notes (double-sided). Answer all questions and write all answers in a blue book or on separate sheets of paper. Time limit is 1 hours and 20 minutes. Total points = 80.

I. Return Calculations (20 pts)

Consider a one year investment in two Northwest stocks: Amazon and Costco. Suppose you buy one share of Amazon and Costco at the beginning of the year at $P_{A,t-1} = \$27,\ P_{C,t-1} = \$15$ and then sell the shares at the end of the year for $P_{A,t} = \$32,\ P_{C,t} = \$12$.

1. What are the simple annual returns on the investments in the two stocks?

2. What are the continuously compounded returns on the investments in the two stocks?

3. Suppose Costco pays a \$3 per share dividend during the year. What is the annual simple total return on Costco? What is the annual dividend yield?

4. If you consider the investment in the two stocks a portfolio, what is the simple annual return on the portfolio? For this calculation, assume that Costco does not pay a dividend. (Hint: what is the initial investment and what are the investment shares?)

5. If you consider the investment in the two stocks a portfolio, what is the continuously compounded annual return on the portfolio? For this calculation, assume that Costco does not pay a dividend.

II. Normal Distribution (20 pts)

Let *X* be a normally distributed random variable with mean $\mu = 0.01$ and variance $\sigma^2 = (0.50)^2$.

1. Sketch the pdf (probability curve) of *X*. On your sketch indicate the location of $\mu,\ \mu + 2\sigma,\ \mu - 2\sigma, \mu + 3\sigma,\ \mu - 3\sigma$.

2. What is the approximate 0.5% quantile of this normal distribution? (Hint: What is the area under the normal curve between $\mu \pm 3\sigma$?).

3. Based on your sketch of the pdf, give an approximate guess for $\Pr(X < -1)$.

4. Could this normal distribution characterize the distribution of the simple return on an asset? Justify your answer.

5. Could this normal distribution characterize the distribution of the continuously compounded return on an asset? Justify your answer.

III. Descriptive Statistics and the CER Model (20 pts)

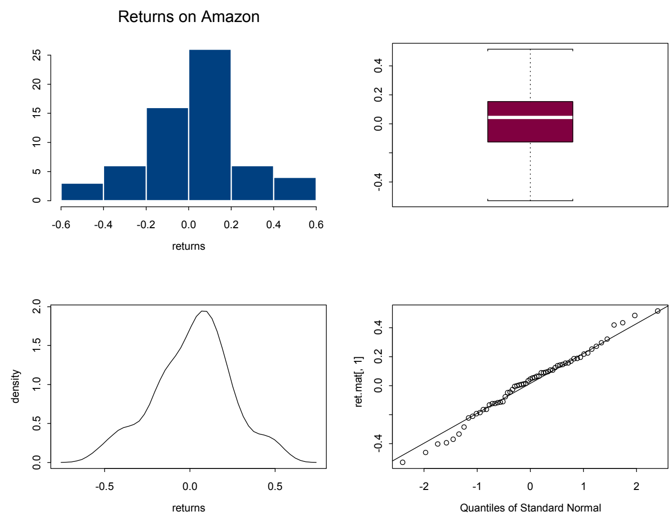
Consider the monthly continuously compounded returns on Amazon stock and the Dow Jones industrial average computed using end of month closing prices over the period October 1998 – October 2003. Descriptive statistics for these returns are given below as well as some diagnostic plots.

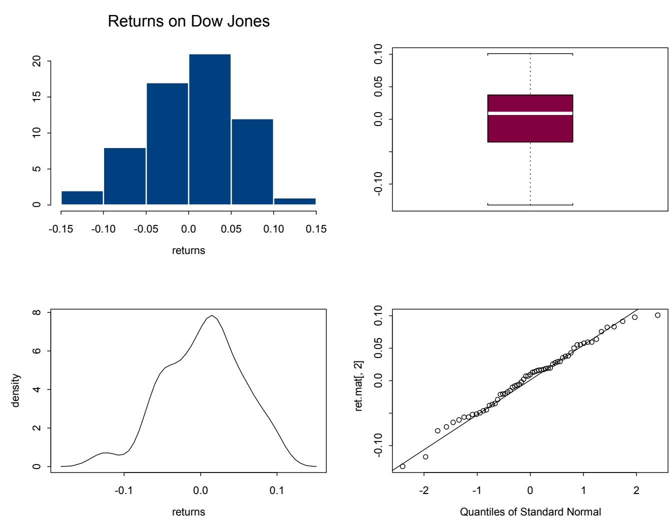
```
     ***  Summary Statistics for data in:  ret.mat ***

                amazon       DowJones
     Min:  -0.52987752  -0.132081448
 1st Qu.:  -0.12382137  -0.035134133
    Mean:   0.01801885   0.003653492
  Median:   0.04454562   0.009135012
 3rd Qu.:   0.15455020   0.037313865
     Max:   0.51469314   0.100779438
 Total N:  61.00000000  61.000000000
   NA's :   0.00000000   0.000000000
Std Dev.:   0.22594294   0.051094648
Skewness:  -0.20516274  -0.277010210
Kurtosis:   0.07940281  -0.077905528

> var(ret.mat)
              amazon    DowJones
  amazon 0.051050212 0.005415075
DowJones 0.005415075 0.002610663

> cor(ret.mat)
           amazon DowJones
  amazon 1.000000 0.469062
DowJones 0.469062 1.000000
```





Based on the descriptive statistics and graphs, answer the following questions.

1. Do the return distributions of the two assets look like they could be normal distributions? Use the univariate statistics in the table below and the diagnostic plots to justify your answers. Which asset appears to be more risky? Why?

2. For the constant expected return model

$$R_{it} = \mu_i + \varepsilon_{it},\ \varepsilon_{it} \sim iid\ N(0, \sigma_i^2)$$
$$\operatorname{cov}(R_{it}, R_{jt}) = \sigma_{ij},\ corr(R_{it}, R_{jt}) = \rho_{ij}$$

give the “plug-in principle” estimates for $\mu_i, \sigma_i^2, \sigma_i, \sigma_{ij}$ and ρ_{ij} for the two assets.

3. The bootstrap is used to compute estimates of $SE(\hat{\mu}_i), SE(\hat{\sigma}_i)$. These results are given below.

Amazon return data:

```
bootstrap(data = ret.mat[, "amazon"], statistic = mean)

Number of Replications: 1000

Summary Statistics:
     Observed       Bias    Mean      SE
mean  0.01802 -0.0002416 0.01778 0.02954

bootstrap(data = ret.mat[, "amazon"], statistic = stdev)

Number of Replications: 1000

Summary Statistics:
      Observed      Bias   Mean      SE
stdev   0.2259 -0.003005 0.2229 0.02054
```

Dow Jones data:

```
bootstrap(data = ret.mat[, "DowJones"], statistic = mean)

Number of Replications: 1000

Summary Statistics:
     Observed        Bias     Mean       SE
mean 0.003653 -8.033e-006 0.003645 0.006227

bootstrap(data = ret.mat[, "DowJones"], statistic = stdev)

Number of Replications: 1000

Summary Statistics:
      Observed       Bias    Mean       SE
stdev  0.05109 -0.0006712 0.05042 0.004593
```

Using these results, comment on the precision of the CER model estimates of μ_i and σ_i.

4. For each asset, use the analytical formulas to compute estimated standard error values for μ_i and σ_i. That is compute $\hat{SE}(\hat{\mu}_i), \hat{SE}(\hat{\sigma}_i)$. Compare these standard errors to the bootstrap standard errors from question 3 above. Are they close?

5. Consider an investment of $100,000 in the Dow Jones average for one month. Use the CER model estimates to compute an estimate of the 5% Value-at-Risk on this investment. (Hint: the 5% quantile from a standard normal distribution is -1.645.)

V. Portfolio Theory (20 points)

Consider investments in a 2 risky assets (assets A and B) and a risk-free asset (T-bill). Assume the following information:

$$r_f = 0.02,\ \mu_A = 0.05,\ \mu_B = 0.07$$
$$\sigma_A = 0.10,\ \sigma_B = 0.20$$
$$\sigma_{AB} = 0,\ \rho_{AB} = 0$$

1. Consider an equally weighted portfolio of the two risky assets ($x_A = x_B = 0.5$). What is the expected return and standard deviation of this portfolio?

2. Draw a graph, with μ_p on the vertical axis and σ_p on the horizontal axis, showing the portfolio frontier for portfolios of asset A and the risk free asset. What is the slope of the frontier?

3. Consider a portfolio of asset B and the risk free asset. Let x_B denote the share of wealth invested in asset B, and let $x_f = 1 - x_B$ denote the share of wealth invested in the risk-free asset. Recall, the return on this portfolio is given by

$$R_p = r_f + x_B(R_B - r_f)$$

Find the value of x_B such that the expected return on the portfolio is equal to 0.10.

4. Continuing with question 3, find the value of x_B such that the portfolio standard deviation is equal to 0.10.

5. Consider a portfolio of the two risky assets (assets A and B). Determine the global minimum variance portfolio. That is, determine how much should be invested in each asset to minimize the variance of the portfolio. What is the expected return on this portfolio?