University of Washington   Fall 2003
Department of Economics   Eric Zivot

Economics 483

Final Exam

This is a closed book and closed note exam. However, you are allowed one page of handwritten notes. Answer all questions and write all answers in a blue book. Total points = 100.

I. Matrix Algebra and Portfolio Math (20 points)

Let R_i denote the continuously compounded return on asset i ($i = 1, \ldots, N$) with $E[R_i] = \mu_i$,

$\text{var}(R_i) = \sigma_i^2$ and $\text{cov}(R_i, R_j) = \sigma_{ij}$. Define the ($N \times 1$) vectors $\mathbf{R} = (R_1, \ldots, R_N)'$, $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_N)'$,

$\mathbf{m} = (m_1, \ldots, m_N)'$, $\mathbf{x} = (x_1, \ldots, x_N)'$, $\mathbf{t} = (t_1, \ldots, t_N)'$, $\mathbf{1} = (1, \ldots, 1)'$ and the ($N \times N$) covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{12} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1N} & \sigma_{2N} & \cdots & \sigma_N^2 \end{pmatrix}.$$

Using simple matrix algebra, answer the following questions.

a. Write down the optimization problem used to determine the global minimum variance portfolio assuming short sales are allowed. Let $\mathbf{m}$ denote the vector of portfolio weights in the global minimum variance portfolio.

b. Write down the optimization problem used to determine the global minimum variance portfolio assuming short sales are *not* allowed. Again, let $\mathbf{m}$ denote the vector of portfolio weights in the global minimum variance portfolio.

c. Write down the optimization problem used to determine an efficient portfolio with target return equal to μ_0 assuming short sales are allowed. Let $\mathbf{x}$ denote the vector of portfolio weights in the efficient portfolio.

d. Write down the optimization problem used to determine the tangency portfolio, assuming short sales are allowed and the risk free rate is give by r_f. Let $\mathbf{t}$ denote the vector of portfolio weights in the tangency portfolio.

II. Stability of parameters in the CER model (20 points)

Recall the constant expected return model

$$R_{it} = \mu_i + \varepsilon_{it}, \ t = 1, \ldots, T$$
$$\varepsilon_{it} \sim iid \ N(0, \sigma_i^2)$$
$$\text{cov}(\varepsilon_{it}, \varepsilon_{jt}) = \sigma_{ij}$$

A key assumption of this model is that the parameters μ_i, σ_i^2, σ_{ij} are constant over time. In this question you will evaluate this constant parameter assumption for returns on the Vanguard short-term bond index (vbisx) and the Vanguard European equity index (veurx)

a. The following graphs show 24-month rolling means and standard deviations for the returns on the two mutual funds. Using these graphs, do you think that the assumption that μ_i, σ_i^2 are constant over time holds for the two funds? Briefly explain your answer.

24 month rolling means and stdevs for Short-term bond index

24 month rolling means and stdevs for European equity index
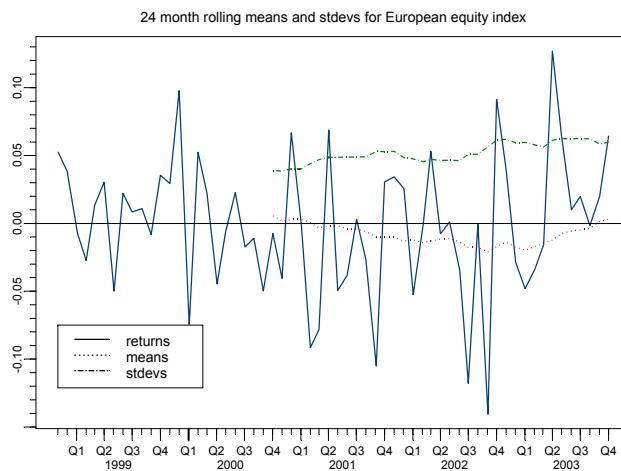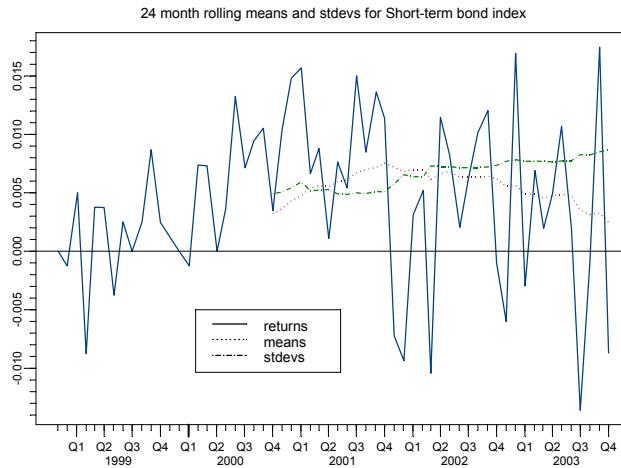
b. The following graphs show 24-month rolling correlations between the returns on the short-term bond index and the European equity index. Using these graphs, do you think that the assumption that σ_{ij} is constant over time holds for the two funds? Briefly explain your answer.

Returns on short-term bonds and European equity

24 month rolling correlations b/w short-term bonds and European equity
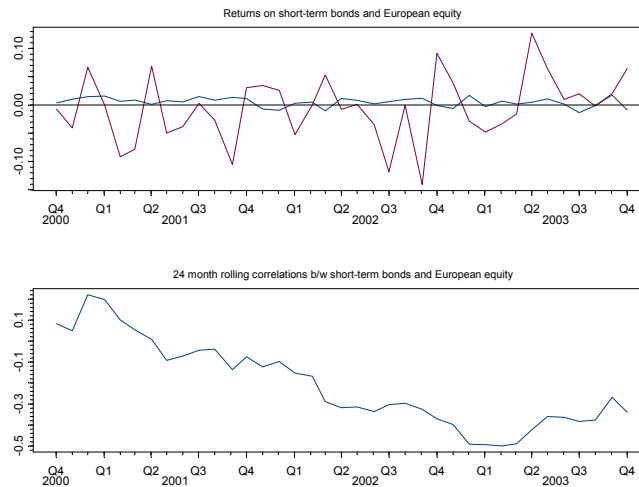
III. Empirical Analysis of the single index model (40 points)

The following represents S-PLUS linear regression output from estimating the single index model for the Vanguard short-term bond index (vbisx) and the Vanguard European Equity index (veurx) using monthly continuously compounded return data over the period November 1998 – October 2003. In the regressions, the market index is the Vanguard extended market index (vexmx).

```
> vbisx.fit = lm(vbisx~vexmx)
> summary(vbisx.fit,cor=F)

Call: lm(formula = vbisx ~ vexmx)
Residuals:
     Min        1Q    Median       3Q      Max
 -0.01654 -0.004081 -0.000257 0.004462 0.01301

Coefficients:
             Value Std. Error t value Pr(>|t|)
(Intercept)  0.0042  0.0009     4.7468  0.0000
      vexmx -0.0282  0.0126    -2.2290  0.0297

Residual standard error: 0.00688 on 58 degrees of freedom
Multiple R-Squared: 0.0789
F-statistic: 4.969 on 1 and 58 degrees of freedom, the p-value is 0.0297
```
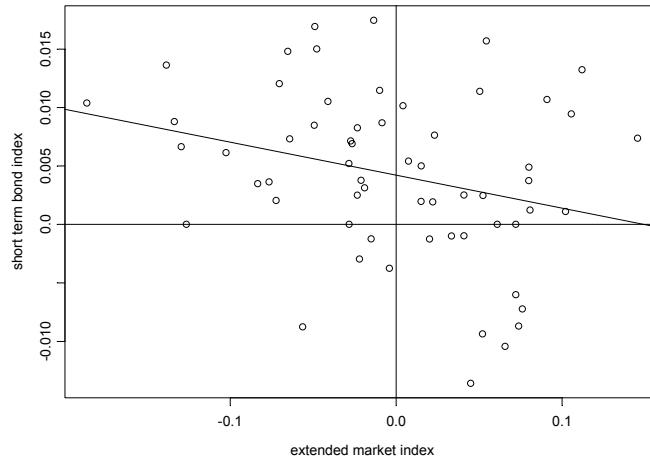
SI regression for short term bonds


```
> veurx.fit = lm(veurx~vexmx)
> summary(veurx.fit,cor=F)

Call: lm(formula = veurx ~ vexmx)
Residuals:
     Min      1Q     Median      3Q Max
 -0.1047 -0.02203 -0.0009752 0.02241 0.1

Coefficients:
              Value Std. Error t value Pr(>|t|)
(Intercept) -0.0004   0.0049   -0.0875  0.9305
      vexmx  0.5039   0.0699    7.2054  0.0000

Residual standard error: 0.03805 on 58 degrees of freedom
Multiple R-Squared: 0.4723
F-statistic: 51.92 on 1 and 58 degrees of freedom, the p-value is 1.324e
-009
```
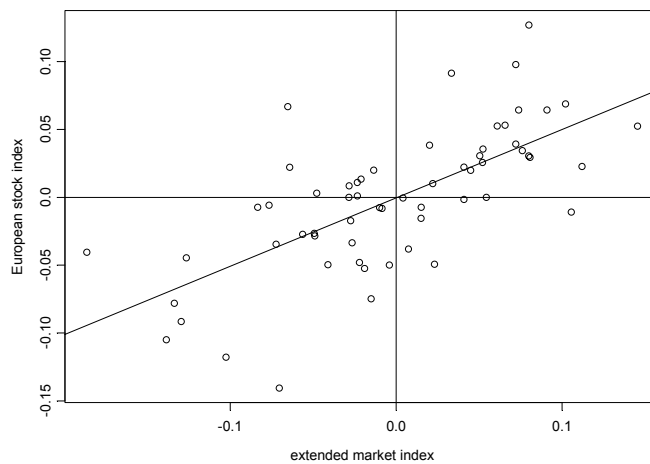
SI regression for European stock index

a. For the short term bond index and the European equity index, what are the estimated values of α and β and what are the estimated standard errors for these estimates?

b. Based on the estimated values of β, what can you say about the risk characteristics of the short-term bond index and the European equity index relative to the extended market index?

c. Is β for the European equity index estimated more precisely than β for the short-term bond index? Why or why not?

d. For each regression, what is the proportion of market or systematic risk and what is the proportion of firm specific or unsystematic risk? For each regression, what does the Residual Standard Error represent?

e. Why should the European equity index have a greater proportion of systematic risk and a larger standard error than the short-term bond index?

f. For the short-term bond index and the European equity index, test the null hypothesis that $\beta = 0$ against the alternative hypothesis that $\beta \neq 0$ using a 5% significance level. What do you conclude?

g. For Short term bond index and the European equity index, test the null hypothesis that $\beta = 1$ against the alternative hypothesis that $\beta \neq 1$ using a 5% significance level. What do you conclude?

h. The following graphs show the 24-month rolling estimates of β for the SI models for the short-term bond index and the European equity index. Using these graphs, what can you say about the stability of β over time?

24 month rolling betas for short-term bond index

24 month rolling betas for European equity index
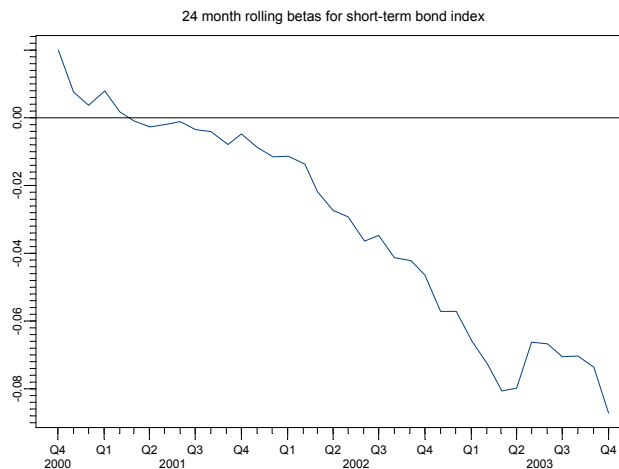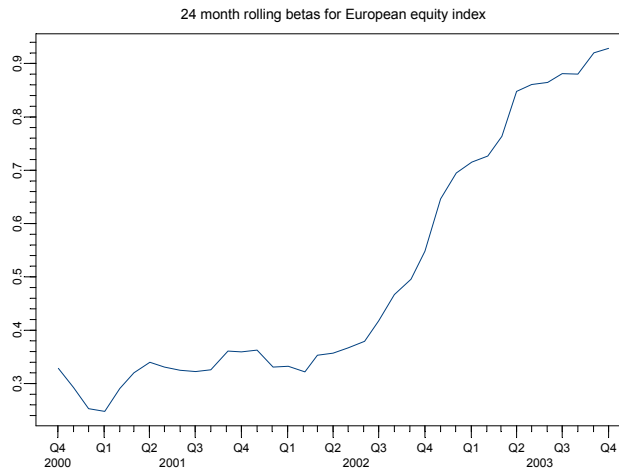
IV. CAPM (20 points)

a. What are the main assumptions underlying the CAPM?

b. What is the security market line (SML) pricing relationship? Draw a graph showing this relationship, and indicate the intercept and slope.

c. Suppose you have a history of returns on N assets for T months and a market index, and you estimate the excess returns single index model regression

$$R_{it} - r_{ft} = \alpha_i^* + \beta_i(R_{Mt} - r_{ft}) + \varepsilon_{it}$$

where r_f denotes the risk-free rate and R_{Mt} denotes the return on a value-weighted market index. How would you test the null hypothesis that the CAPM holds for every asset?

d. The prediction test of the CAPM plots estimates of average excess returns $\hat{\mu}_i - r_f$ against beta estimates $\hat{\beta}_i$. Based on these estimates one may estimate the simple linear regression equation

$$\hat{\mu}_i - r_f = \gamma_0 + \gamma_1\hat{\beta}_i + error_i, \quad i = 1,\dots,N$$

If the CAPM were true, what should the estimated values for γ_0 and γ_1 be?