

Economics 483

Final Exam

This is a closed book and closed note exam. However, you are allowed one page of handwritten notes. Answer all questions and write all answers in a blue book. Total points = 100.

I. Portfolio Theory (20 points)

1. Consider the problem of allocating wealth between a collection of N risky assets and a risk-free asset (T-bill) under the assumption that investors only care about maximizing portfolio expected return and minimizing portfolio variance. Use the graph below to answer the following questions.

- a. Mark on the graph the set of efficient portfolios for the risky assets only (transfer the graph to your blue book). Briefly describe how you would compute this set using Excel. (10 pts)
- b. Mark on the graph the set of efficient portfolios that include risky assets and a single risk-free asset (transfer the graph to your blue book). Briefly describe how you would compute this set using Excel. (10 pts)

II. CAPM (20 points)

1. Consider the CAPM regression

$$R_t - r_f = \alpha + \beta(R_{Mt} - r_f) + \varepsilon_t, \quad t = 1, \dots, T$$
$$\varepsilon_t \sim iid N(0, \sigma_\varepsilon^2) \text{ and } R_{Mt} \text{ is independent of } \varepsilon_t \text{ for all } t$$

where R_t denotes the return on an asset or portfolio, R_{Mt} denotes the return on the market portfolio proxy and r_f denotes the risk-free T-bill rate. Let μ and μ_M denote the expected returns on the asset and the market, respectively, and let σ^2 and σ_M^2 denote the variances of the asset and the market, respectively. Finally, let σ_{RM} denote the covariance between the asset and the market.

a. What is the interpretation of α and β in the CAPM regression? What restriction does the CAPM place on the value of α ? (4 pts)

b. What is the interpretation of ε_t in the CAPM regression? (2 pts)

c. Using the CAPM regression compute $E[R_t]$ and $\text{var}(R_t)$. (2 pts)

d. Using the expression for $\text{var}(R_t)$, what is the proportion of the variance of the asset due to the variability in the market return and what is the proportion unexplained by variability in the market? (2 pts)

2. The following output is based on estimating the CAPM regression for IBM and an equally weighted portfolio of 15 stocks using monthly return data over the period January 1978 to December 1982:

$$R_{\text{IBM}} - r_f = -0.0002 + 0.3390*(R_M - r_f), R^2 = 0.2008, \text{var}(\varepsilon_{\text{IBM}}) = (0.0524)^2 \\ (0.0068) \quad (0.0888)$$

$$R_{\text{port}} - r_f = 0.0006 + 0.6316*(R_M - r_f), R^2 = 0.6280, \text{var}(\varepsilon_{\text{port}}) = (0.0335)^2 \\ (0.0030) \quad (0.0447)$$

a. For IBM and the portfolio of 15 stocks, what are the estimated values of α and β and what are the estimated standard errors for these estimates? (2 pts)

b. Is the beta for the portfolio estimated more precisely than the beta for IBM? Why or why not? (2 pts)

c. For each regression, what is the proportion of market or systematic risk and what is the proportion of firm specific or unsystematic risk? Why should the portfolio have a greater proportion of systematic risk and smaller value of $\text{SD}(\varepsilon)$ than IBM? (2 pts)

d. Based on the regression estimates, does the CAPM appear to hold for IBM and the portfolio? Justify your answer. (4 pts)

III. Return Calculations (20 points)

1. Consider a portfolio of 3 risky stocks denoted by A, B and C (say Apple, Boeing and Coca Cola). Let R_A , R_B and R_C denote the monthly returns on these stock and it is assumed that these returns are jointly normally distributed with means μ_i ($i = A, B, C$), variances σ_i^2 ($i = A, B, C$) and covariances σ_{ij} ($i = A, B, C$ and $i \neq j$). Consider forming a portfolio of these stocks where x_i = share of wealth invested in asset i such that $x_A + x_B + x_C = 1$.

a. What is the expected return on the portfolio? (2 pts)

b. What is the variance of the portfolio return? (2 pts)

c. What is the probability distribution for the portfolio return? (2 pts)

2. Throughout the course we have made the assumption that the continuously compounded returns on risky assets (e.g. stocks) are normally distributed. Based on the data analysis we have done in the labs and in class, is this a believable assumption? Briefly justify your answer. (8 pts)

3. Consider the following monthly data for Microsoft stock over the period December 1995 through December 1996:

End of Month Price Data for Microsoft Stock	
December, 1995	43.12
January, 1996	43.87
February, 1996	47.06
March, 1996	47.75
April, 1996	51.37
May, 1996	57.56
June, 1996	59.19
July, 1996	61.16
August, 1996	60.31
September, 1996	61.25
October, 1996	66.06
November, 1996	68.69
December, 1996	78.87

a. Using the data in the table, what is the continuously compounded monthly return between December, 1995 and January 1996? (2 pts)

b. Assuming that the continuously compounded monthly return you computed in part (a) is the same for 12 months, what is the continuously compounded annual return? (2 pts)

c. Using the data in the table, compute the actual (annual) continuously compounded return between December 1995 and December 1996. Compare with your result in part (b). (2 pts)

IV. Arbitrage (15 points)

a. What is an *arbitrage opportunity*? (5 pts)

b. Give a simple example of an arbitrage opportunity. (5 pts)