University of Washington
Department of Economics

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Eric Zivot

Economics 483

Final Exam
Suggested Solutions

This is a closed book and closed note exam. However, you are allowed one page (double sided) of notes. Answer all questions and write all answers in a blue book. Each question is worth 5 points, and total points = 120.

I. Matrix Algebra and Portfolio Math (20 points)

Let R_i denote the continuously compounded return on asset i (i = 1, …, N) with $E[R_i] = \mu_i$, var(R_i) = σ_i^2 and cov(R_i, R_j) = σ_{ij}. Define the ($N \times 1$) vectors $\boldsymbol{R} = \left(R_1, \ldots, R_N\right)'$, $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_N)'$, $\boldsymbol{m} = \left(m_1, \ldots, m_N\right)'$, $\boldsymbol{x} = \left(x_1, \ldots, x_N\right)'$, $\boldsymbol{t} = \left(t_1, \ldots, t_N\right)'$, $\mathbf{1} = \left(1, \ldots, 1\right)'$ and the ($N \times N$) covariance matrix

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{12} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1N} & \sigma_{2N} & \cdots & \sigma_N^2 \end{pmatrix}.$$

Using simple matrix algebra, answer the following questions.

a. Write down the optimization problem used to determine the global minimum variance portfolio assuming short sales are allowed. Let $\boldsymbol{m}$ denote the vector of portfolio weights in the global minimum variance portfolio.

$$\min_{m} \ \boldsymbol{m'\Sigma m} \text{ s.t. } \boldsymbol{m'1} = 1$$

b. Give expressions for the expected return and variance on the global minimum variance portfolio.

$$\mu_{g\min} = m'\mu, \ \sigma^2_{g\min} = m'\Sigma m$$

c. Write down the optimization problem used to determine an efficient portfolio with target return equal to μ_0 assuming short sales are allowed. Let $\boldsymbol{x}$ denote the vector of portfolio weights in this efficient portfolio.

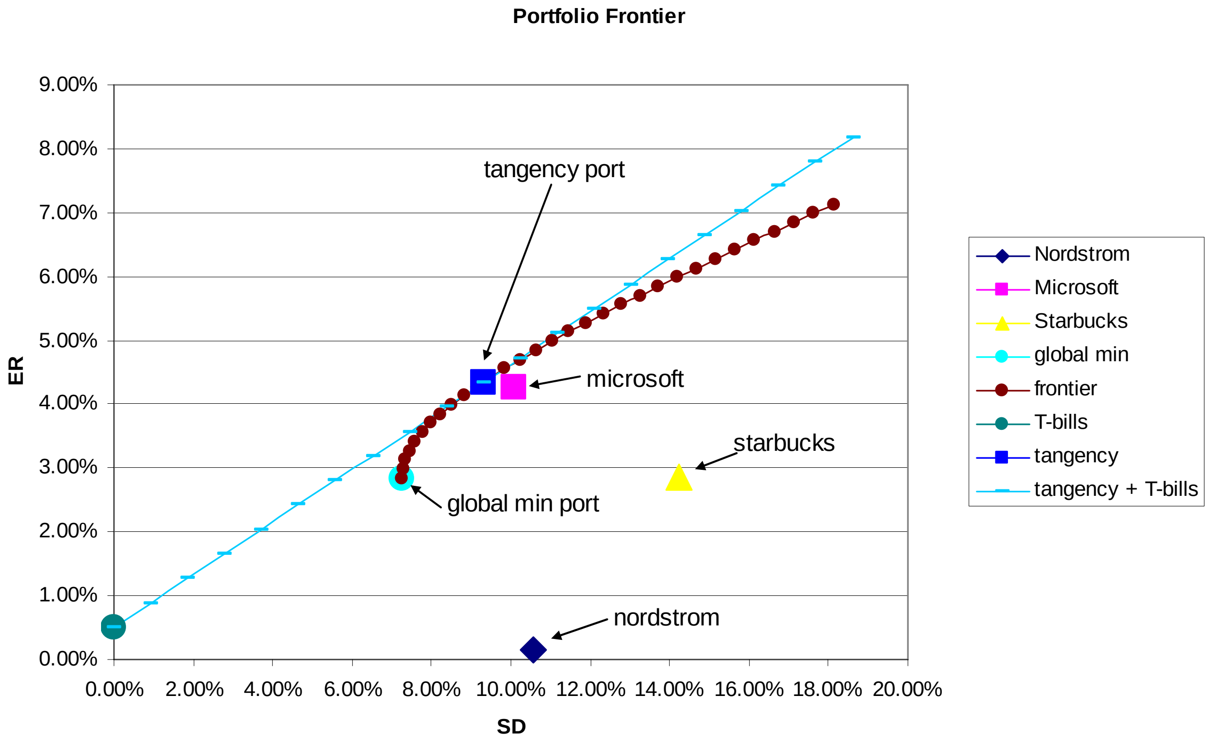
$$\min_{x} \ \boldsymbol{x'\Sigma x} \text{ s.t. } \boldsymbol{x'1} = 1 \text{ and } \boldsymbol{x'\mu} = \mu_0$$

d. Write down the optimization problem used to determine the tangency portfolio, assuming short sales are allowed and the risk free rate is give by r_f. Let $\boldsymbol{t}$ denote the vector of portfolio weights in the tangency portfolio.

$$\max_{t} \ \frac{\boldsymbol{t'\mu} - r_f}{\left(\boldsymbol{t'\Sigma t}\right)^{1/2}} \text{ s.t. } \boldsymbol{t'1} = 1$$

Efficient Portfolios (40 points)

The graph below shows the efficient frontier with three risky assets (Microsoft, Nordstrom and Starbucks), as well as the efficient frontier with T-bills and three risky assets.



Expected return and standard deviation values for specific assets are summarized in the table below (the mean for Microsoft is omitted on purpose – you will be asked to deduce it)

Asset	**Mean (E[R])**	**Standard deviation (SD(R))**	**Weight in tangency portfolio**
Microsoft		10.06%	75%
Starbucks	2.85%	14.22%	41%
Nordstrom	0.15%	10.55%	-16%
T-Bills	0.5%	0%	
Tangency Portfolio	4.34%	9.3%	

a. What are the equations that give the expected return and variance of efficient combinations of T-bills and the tangency portfolio?

$$\mu_p^e = r_f + x_{\tan}(\mu_{\tan} - r_f)$$

$$\left(\sigma_p^e\right)^2 = x_{\tan}^2 \sigma_{\tan}^2$$

b. How much should be invested in T-bills and the tangency portfolio to create an efficient portfolio with the same expected return as Starbucks? Transfer the above graph to your blue book and indicate the location of this efficient portfolio on the graph.

$$x_{\tan} = \frac{\mu_{sbux} - r_f}{\mu_{\tan} - r_f} = \frac{.0285 - 0.005}{.0434 - 0.005} = 0.612$$
$$x_f = 1 - x_{\tan} = 1 - 0.612 = 0.388$$

c. In the above portfolio, what are the shares of wealth invested in Microsoft, Nordstrom and Starbucks?

Microsoft	Starbucks	Nordstrom
0.612*0.75=0.459	0.612*0.41=0.251	0.612*(-0.16)=-0.098

d. What is the standard deviation of the efficient portfolio with the same expected return as Starbucks?

$$\sigma_p^e = x_{\tan}\sigma_{\tan} = (0.612)(0.093) = 0.057$$

e. How much should be invested in T-bills and the tangency portfolio to create an efficient portfolio with the same standard deviation as Starbucks? Transfer the above graph to your blue book and indicate the location of this efficient portfolio on the graph.

$$x_{\tan} = \frac{\sigma_{sbux}}{\sigma_{\tan}} = \frac{0.1422}{0.093} = 1.529$$
$$x_f = 1 - x_{\tan} = 1 - 1.529 = -0.529$$

f. In the above portfolio, what are the shares of wealth invested in Microsoft, Nordstrom and Starbucks?

Microsoft	Nordstrom	Starbucks
1.529*0.75=1.147	1.529*0.41=0.627	1.529*(-0.16)=-0.245

g. What is the expected return of the efficient portfolio with the same standard deviation as Starbucks?

$$\mu_p^e = r_f + x_{\tan}(\mu_{\tan} - r_f) = 0.005 + (1.529)(0.0434 - 0.005) = 0.064$$

h. Suppose the beta of Microsoft with respect to the tangency portfolio is 0.982. That is,

$$\beta_{msft,\tan} = \frac{\mathrm{cov}(R_{msft}, R_{\tan})}{\mathrm{var}(R_{\tan})} = 0.982$$

If the risk free rate is 0.5% per month, what is the monthly expected return on Microsoft?

$$\mu_{msft} = r_f + \beta_{msft,\tan}(\mu_{\tan} - r_f) = 0.005 + 0.982(0.0434 - 0.005) = 0.0427$$

III. Empirical Analysis of the single index model (25 points)

The following represents S-PLUS linear regression output from estimating the single index model for IBM and an equally weighted portfolio of 16 stocks using monthly continuously compounded return data over the 5 year period January 1983 – December 1987. In the regressions, the market index is a value weighted index of all stocks on the NYSE and the American stock exchange.

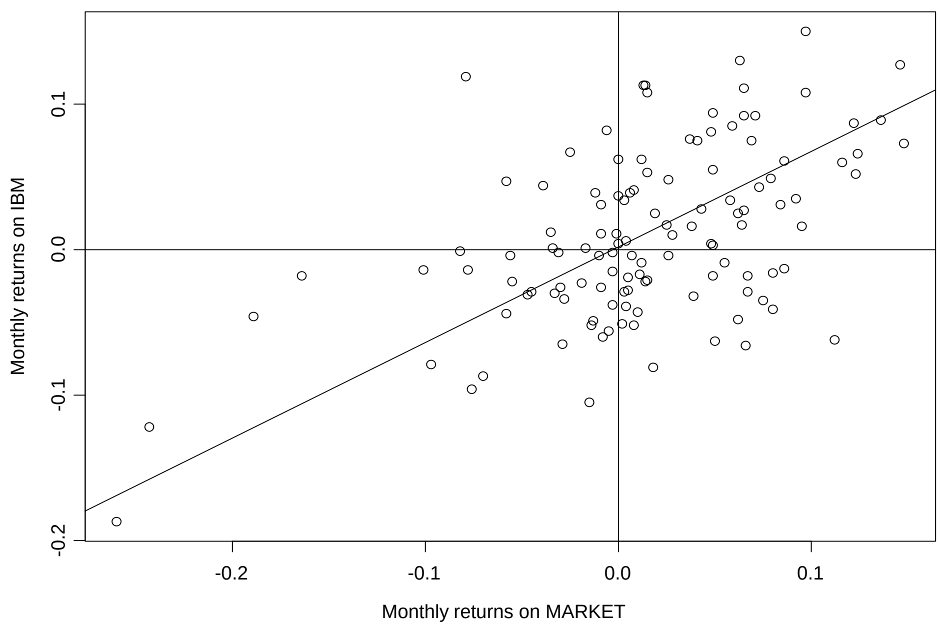
```
> summary(ibm.fit)

Call: lm(formula = IBM ~ MARKET, data = returns.df, subset =
61:120)
Residuals:
    Min      1Q   Median    3Q   Max
 -0.111 -0.0298 -0.00196 0.035 0.103

Coefficients:
            Value Std. Error t value Pr(>|t|)
(Intercept) 0.002 0.006      0.306   0.761
     MARKET 0.657 0.103      6.388   0.000

Residual standard error: 0.0472 on 58 degrees of freedom
Multiple R-Squared: 0.413
```

SI regression for IBM

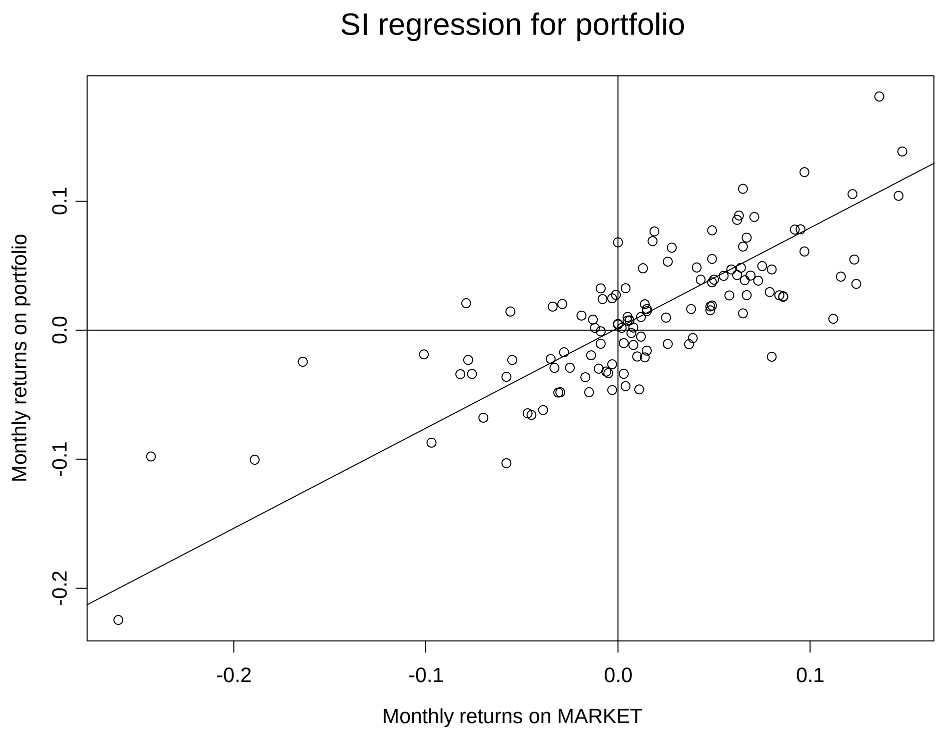



```
> summary(port.fit)

Call: lm(formula = port ~ MARKET, data = returns.df, subset =
61:120)
Residuals:
     Min      1Q    Median     3Q    Max
 -0.0845 -0.0207 -0.000799 0.0245 0.0662

Coefficients:
             Value Std. Error t value Pr(>|t|)
(Intercept)  0.002  0.004      0.451   0.654
     MARKET  0.777  0.068     11.475   0.000

Residual standard error: 0.0311 on 58 degrees of freedom
Multiple R-Squared: 0.694
```



a. For IBM and the equally weighted portfolio, what are the estimated values of α and β, what are the estimated standard errors for these estimates, and what are the estimates of σ_{ε}?

$$\hat{R}_{IBM} = \underset{(0.006)}{0.002} + \underset{(0.103)}{0.657} * R_M, \ \hat{\sigma}_{\varepsilon} = 0.0472$$

$$\hat{R}_{port} = \underset{(0.004)}{0.002} + \underset{(0.068)}{0.777} * R_M, \ \hat{\sigma}_{\varepsilon} = 0.0311$$

b. Based on the estimated values of β, what can you say about the risk characteristics of IBM and the portfolio relative to the market index?

Both estimates of β are less than one, which indicates that adding either IBM or the portfolio to the market will reduce the volatility (SD) of the market. According, both IBM and the portfolio are less risky than the market as a whole.

c. For each regression, what is the proportion of market or systematic risk and what is the proportion of firm specific or unsystematic risk?

	R-square (% market risk)	*1-(R-square) (% non-market risk)*
IBM	0.413	0.587
Portfolio	0.694	0.306

d. Why should the portfolio have a greater proportion of systematic risk and a smaller value of σ_ε than IBM?

The portfolio is a moderately diversified portfolio of stocks, whereas IBM is an individual stock. The diversification effect concentrates the market risk of the portfolio and reduces the amount of non-market risk. This tends to increase the R-squared (% of market risk). A single asset typically does not have much market risk (recall Sharpe's rule of thumb that typical stocks have an R-squared of 30%). Hence the R-squared of IBM is expected to be lower than the R-squared of the portfolio. This is what we see in the data.

e. For IBM and the portfolio, test the null hypothesis that $\beta = 1$ against the alternative hypothesis that $\beta \neq 1$ using a 5% significance level. What do you conclude?

	$t_{\beta=1} = \frac{\hat{\beta}-1}{SE(\hat{\beta})}$	*Decision: reject if |t-stat| > 2*
IBM	$\frac{0.657-1}{0.103} = -3.3301$	*Reject* $H_0 : \beta = 1$ *at 5% level*
Portfolio	$\frac{0.777-1}{0.068} = -3.279$	*Reject* $H_0 : \beta = 1$ *at 5% level*

IV. CAPM (35 points)

a. What are the main assumptions underlying the CAPM?

- *Many identical investors who are price takers*
- *All investors plan to invest over the same time horizon*
- *No taxes or transactions costs*
- *Can borrow or lend at the risk free rate*
- *Investors only care about expected return and variance*
- *Market consists of all publicly traded goods*

b. What is the security market line (SML) pricing relationship? Draw a graph showing this relationship, and indicate the intercept and slope.

$$E[R_{it}] = r_f + \beta_i (E[R_{Mt}] - r_f)$$

c. Suppose you have a history of returns on N assets for T months and a market index, and you estimate the excess returns single index model regression

$$R_{it} - r_{ft} = \alpha_i^* + \beta_i (R_{Mt} - r_{ft}) + \varepsilon_{it}$$

where r_f denotes the risk-free rate and R_{Mt} denotes the return on a value-weighted market index. How would you test the null hypothesis that the CAPM holds for every asset?

Use a t-test to test the null hypothesis that $\alpha_i^* = 0$ *for all assets.*

d. The following is Splus output of the excess returns single index (CAPM) regression for IBM using monthly data over the five year period Jan 1983 through Dec 1987. Using a 5% significance level, test the hypothesis that the CAPM holds for IBM? What do you conclude?

```
> summary(capm.fit)

Call: lm(formula = IBM ~ MARKET, data = excessReturns.df, subset
= 61:120)
Residuals:
    Min      1Q   Median     3Q   Max
 -0.112 -0.0296 -0.00207 0.0354 0.103

Coefficients:
             Value Std. Error t value Pr(>|t|)
(Intercept)  0.000  0.006     -0.004   0.997
     MARKET  0.654  0.103      6.356   0.000

Residual standard error: 0.0472 on 58 degrees of freedom
Multiple R-Squared: 0.411
```

d. The prediction test of the CAPM plots estimates of average excess returns $\hat{\mu}_i - r_f$ against beta estimates $\hat{\beta}_i$. Based on these estimates one may estimate the simple linear regression equation

$$\hat{\mu}_i - r_f = \gamma_0 + \gamma_1 \hat{\beta}_i + error_i,\ i = 1, \ldots, N$$

If the CAPM were true, what should the estimated values for γ_0 and γ_1 be?

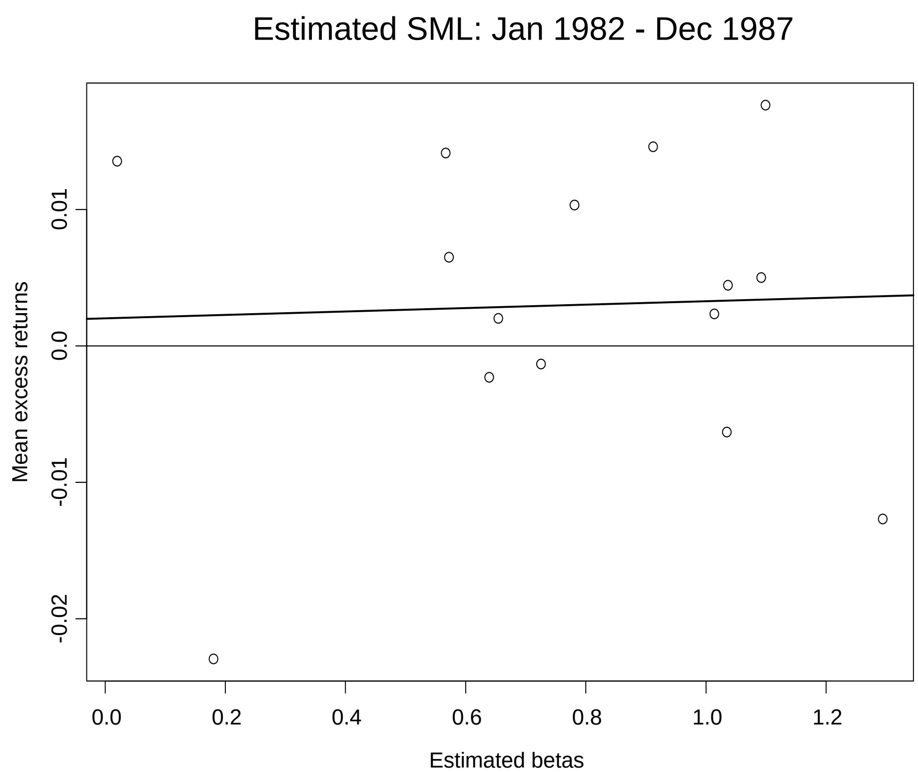
If the CAPM were true, then $\gamma_0 = 0$ and $\gamma_1 = \hat{\mu}_M - r_f$

e. The following graph plots estimates of average excess returns $\hat{\mu}_i - r_f$ against beta estimates $\hat{\beta}_i$ for 15 stocks based on monthly data over the five year period Jan 1983 through Dec 1987.

Superimposed on the graph is an estimate of the security market line (SML) based on the simple linear regression

$$\hat{\mu}_i - r_f = \gamma_0 + \gamma_1 \hat{\beta}_i + error_i,\ i = 1, \ldots, 15$$

The average excess return on the market index over this period was 0.0031. If the CAPM were true, what should the estimated values for γ_0 and γ_1 be?



If the CAPM were true then $\hat{\gamma}_0 \approx 0,$ and $\hat{\gamma}_1 \approx 0.0031$

f. The least squares estimates of the SML presented in the previous question are summarized below. Using these estimates, test the null hypothesis that the CAPM is true using a 5% significance level.

```
> summary(sml.fit)

Call: lm(formula = mu.hat ~ betas)
Residuals:
     Min      1Q  Median      3Q    Max
 -0.0252 -0.0047 0.00112 0.00936 0.0142
```

```
Coefficients:
             Value Std. Error t value Pr(>|t|)
(Intercept) 0.002 0.007      0.275   0.788
     Betas  0.001 0.009      0.143   0.888

Residual standard error: 0.0115 on 13 degrees of freedom
Multiple R-Squared: 0.00157
```

Null Hypothesis	$t_{\gamma_i=\gamma_i^0} = \frac{\hat{\gamma}_i - \gamma_i^0}{SE(\hat{\gamma}_i)}$	*Decision: reject if |t-stat| > 2*
$H_0 : \gamma_0 = 0$	$\frac{0.002 - 0}{0.007} = 0.286$	*Do not reject* $H_0 : \gamma_0 = 0$ *at 5% level*
$H_0 : \gamma_1 = 0.0031$	$\frac{0.001 - 0.0031}{0.009} = -0.233$	*Do not reject* $H_0 : \gamma_1 = 0.0031$ *at 5% level*