Diagnostics for Constant Parameters

Recall, in the CER model

$$R_t \sim \text{iid } N(\mu, \sigma^2),\ t = 1, \ldots, T$$
$$\mu \text{ is constant over time}$$
$$\sigma^2 \text{ is constant over time}$$

Q: Is the assumption of constant means and variances consistent with the observed data?

Q: How can you determine if μ and σ^2 are constant over time?

Rolling Means

Idea: compute estimate of μ over rolling windows of length $n < T$

$$\begin{aligned}\hat{\mu}_t(n) &= \frac{1}{n}\sum_{i=0}^{n-1} R_{t-i} \\ &= \frac{1}{n}(R_t + R_{t-1} + \cdots + R_{t-n+1}) \\ t &= n, n+1, \ldots, T\end{aligned}$$

Plot $\hat{\mu}_t(n)$ over time. If $\hat{\mu}_n(n) \approx \hat{\mu}_{n+1}(n) \approx \cdots \hat{\mu}_T(n)$ then the data support μ constant

Note: The S-PLUS function

`aggregateSeries`

can be used to easily compute rolling means.

Rolling Variances and Standard Deviations

Idea: Compute estimates of σ^2 and σ over rolling windows of length $n < T$

$$\begin{aligned} \hat{\sigma}_t^2(n) &= \frac{1}{n-1}\sum_{i=0}^{n-1}(R_{t-i} - \hat{\mu}_t(n))^2 \\ \hat{\sigma}_t(n) &= \sqrt{\hat{\sigma}_t^2(n)} \\ t &= n, n+1, \ldots, T \end{aligned}$$

Plot $\hat{\sigma}_t(n)$ over time. If $\hat{\sigma}_n(n) \approx \hat{\sigma}_{n+1}(n) \approx \cdots \approx \hat{\sigma}_T(n)$ then the data support σ constant

Rolling Covariances and Correlations

Idea: Compute estimates of σ_{jk} and ρ_{jk} over rolling windows of length $n < T$

$$
\begin{aligned}
\hat{\sigma}_{jk,t}(n) &= \frac{1}{n-1}\sum_{i=0}^{n-1}(R_{jt-i} - \hat{\mu}_j(n))(R_{kt-i} - \hat{\mu}_k(n)) \\
\hat{\rho}_{jk,t}(n) &= \frac{\hat{\sigma}_{jk,t}(n)}{\hat{\sigma}_{jt}(n)\hat{\sigma}_{kt}(n)} \\
t &= n, n+1, \ldots, T
\end{aligned}
$$

Plot $\hat{\rho}_{jk,t}(n)$ over time. If $\hat{\rho}_{jk,n}(n) \approx \hat{\rho}_{jk,n+1}(n) \approx \cdots \approx \hat{\rho}_{jk,T}(n)$ then the data support ρ_{ij} constant

Rolling Regression

Recall the Single Index Model

$$\begin{aligned} R_{it} &= \alpha_i + \beta_i R_{Mt} + \varepsilon_{it} \\ & \quad \alpha_i \text{ is constant over time} \\ & \quad \beta_i \text{ is constant over time} \end{aligned}$$

Idea: Compute least squares estimates of α_i and β_i from SI model over rolling windows of length $n < T$

$$\begin{aligned} R_{it}(n) &= \alpha_i(n) + \beta_i(n) R_{Mt}(n) + \varepsilon_{it}(n) \\ \hat{\beta}_{in}(n) &= \frac{\hat{\sigma}_{jM,t}(n)}{\hat{\sigma}^2_{Mt}(n)}, \\ \hat{\alpha}_{in}(n) &= \hat{\mu}_{in}(n) - \hat{\beta}_{in}(n)\hat{\mu}_{Mn}(n) \\ t &= n, n+1, \ldots, T \end{aligned}$$

Plot $\hat{\beta}_{in}(n)$ over time. If $\hat{\beta}_{in}(n) \approx \hat{\beta}_{in+1}(n) \approx \cdots \approx \hat{\beta}_{iT}(n)$ then the data support β constant

Rolling Efficient Portfolios

Idea: Using rolling estimates of μ and Σ compute rolling efficient portfolios

- global minimum variance portfolio
- tangency portfolio
- efficient frontier