

UW

Review of Probability Theory

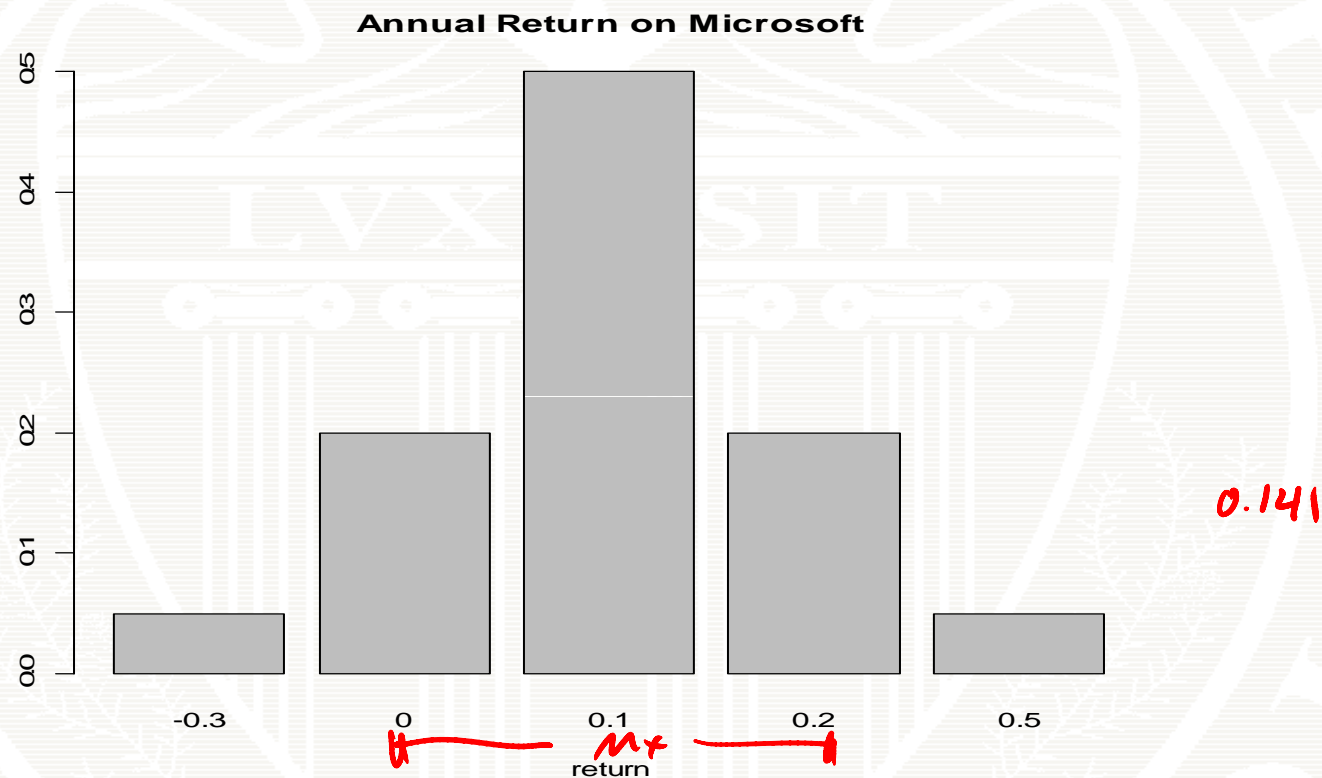
AMATH 462/ECON 424

Summer 2012

Eric Zivot

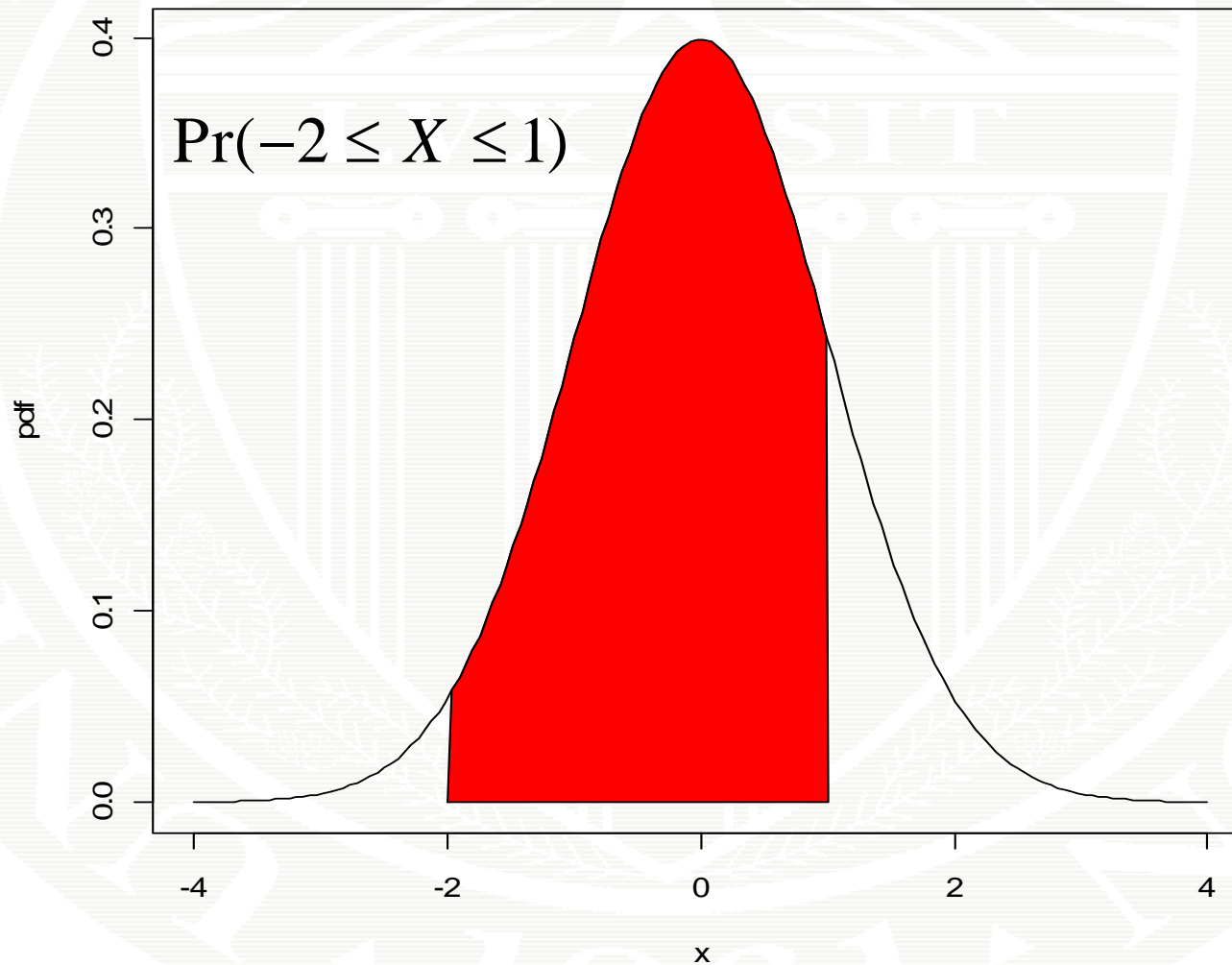
Updated: July 9, 2013

Discrete Distribution



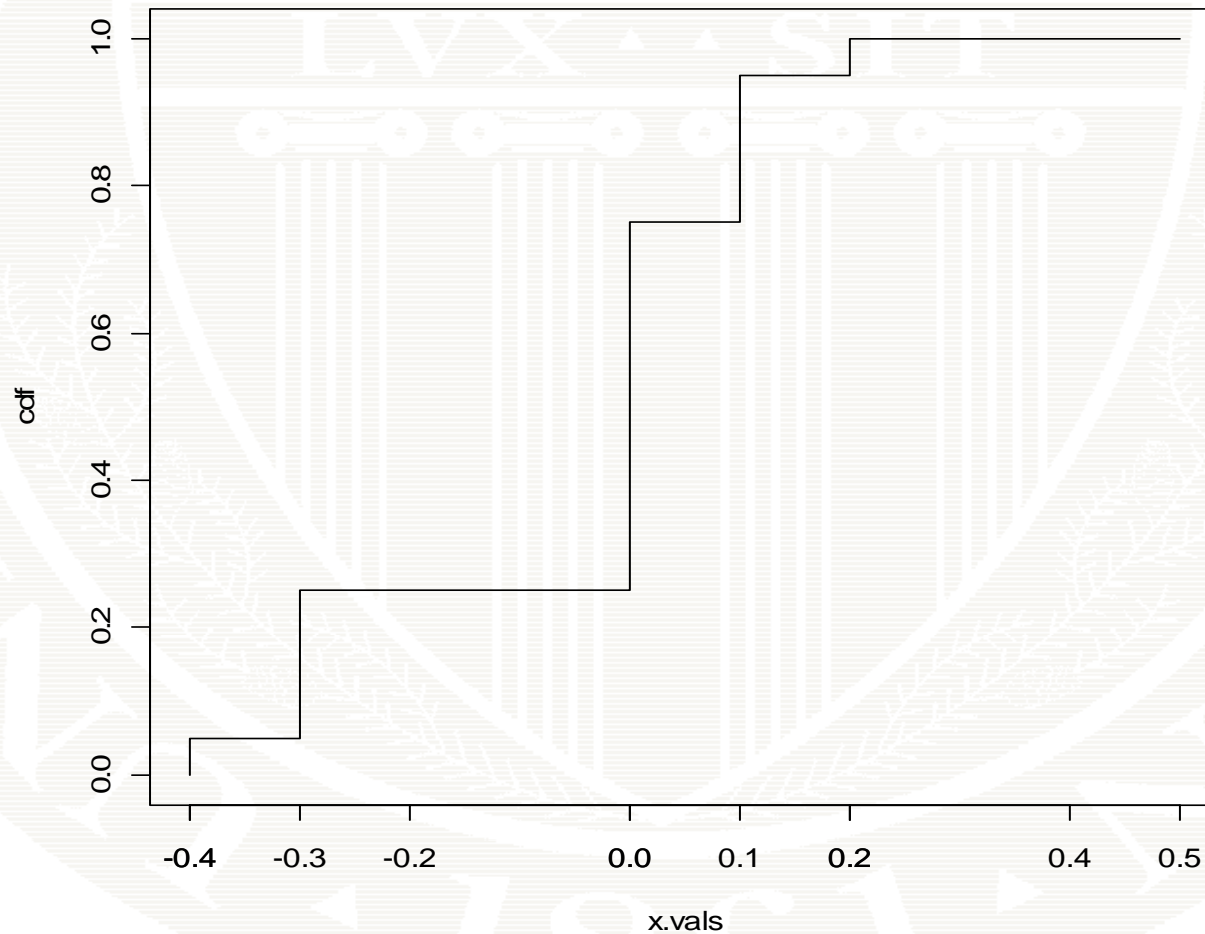
```
> r.msft = c(-0.3, 0, 0.1, 0.2, 0.5)
> prob.vals = c(0.05, 0.20, 0.50, 0.20, 0.05)
> barplot(prob.vals, names.arg = as.character(r.msft),
+         xlab="return")
> title("Annual Return on Microsoft")
```

Probability Curve for Continuous RV

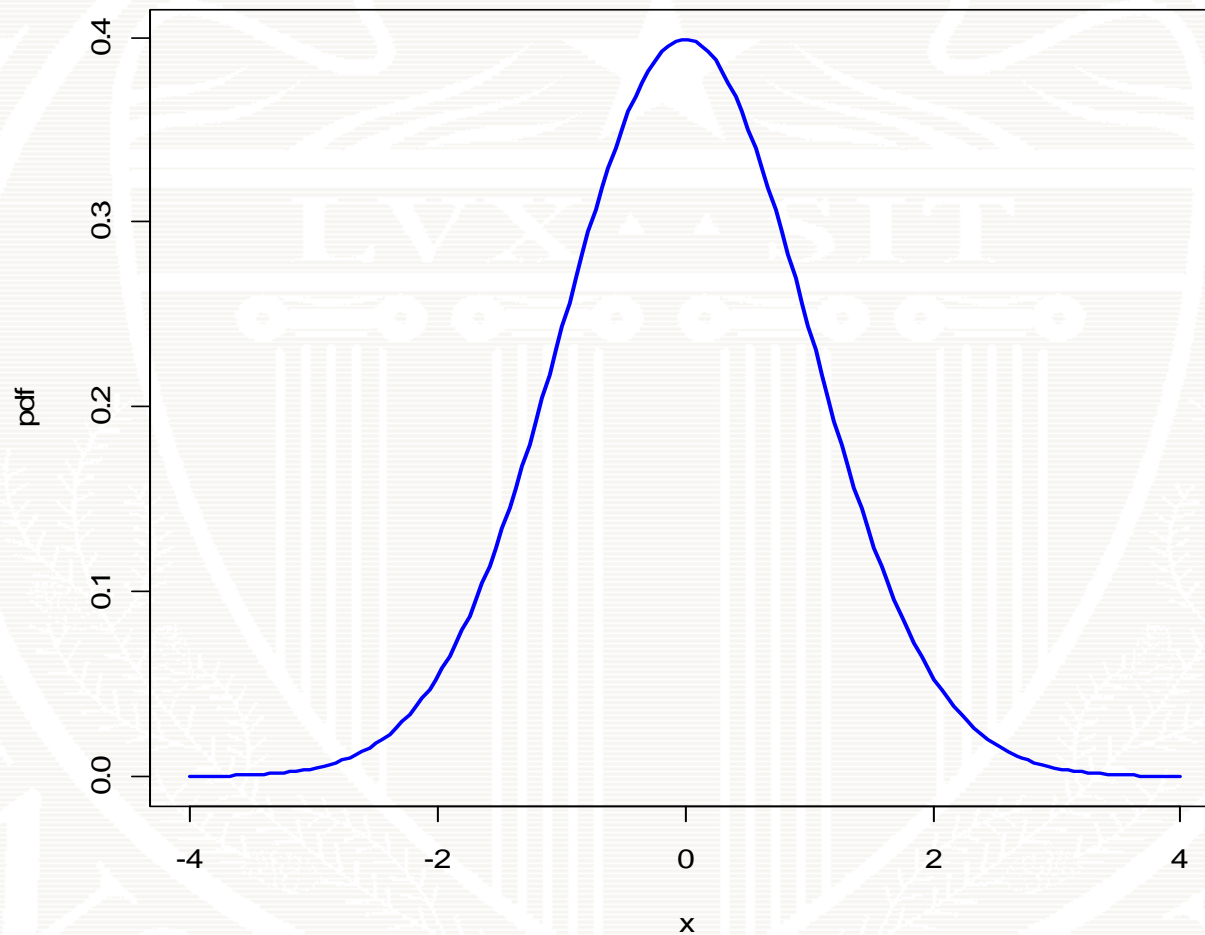


CDF of Discrete Distribution

$$F_X(x) = \Pr(X \leq x)$$

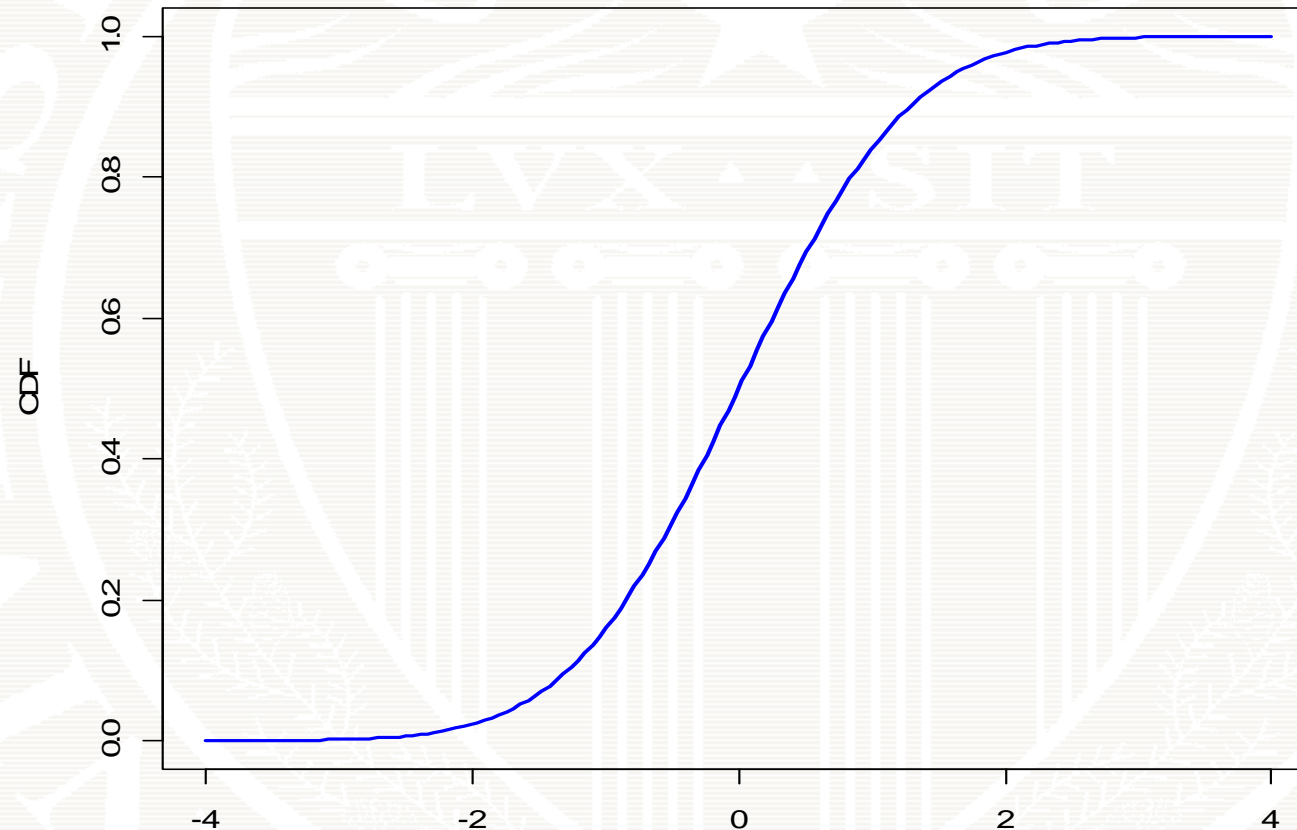


Standard Normal Distribution



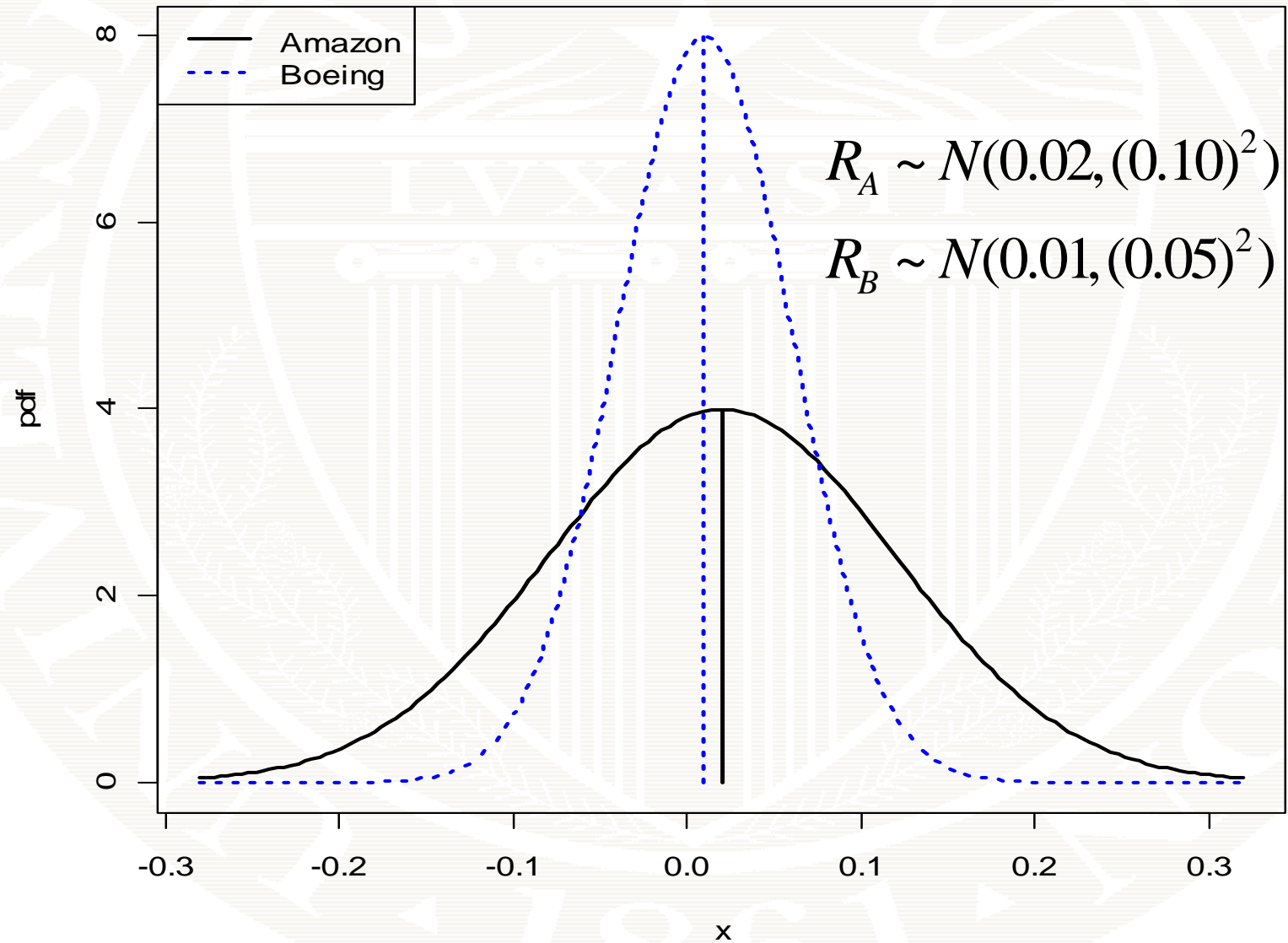
```
> x.vals = seq(-4, 4, length=150)
> plot(x.vals, dnorm(x.vals), type="l", lwd=2, col="blue",
+      xlab="x", ylab="pdf")
```

Standard Normal CDF

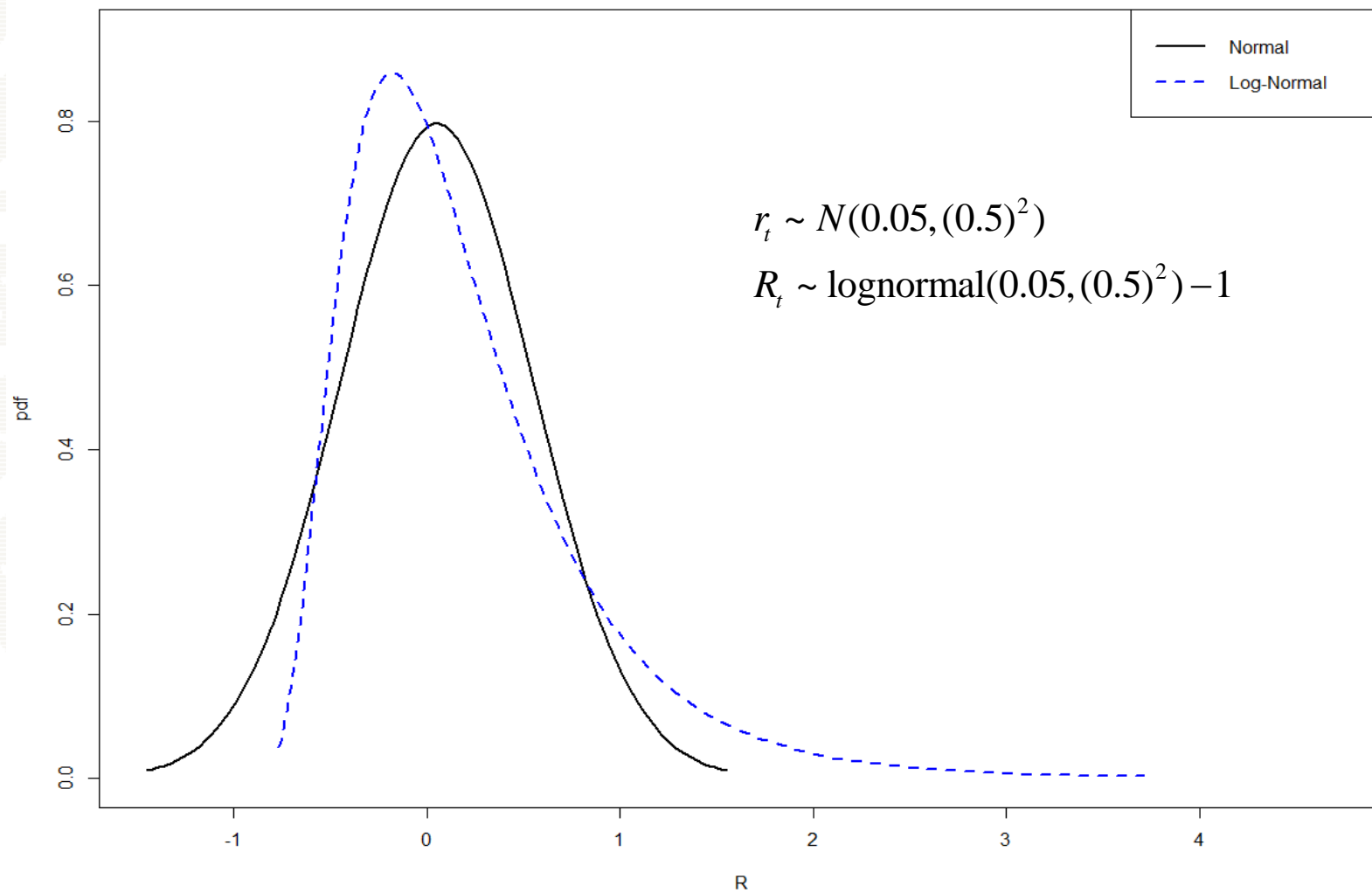


```
> x.vals = seq(-4, 4, length=150)
> plot(x.vals, pnorm(x.vals), type="l", lwd=2, col="blue",
+       xlab="x", ylab="CDF")
```

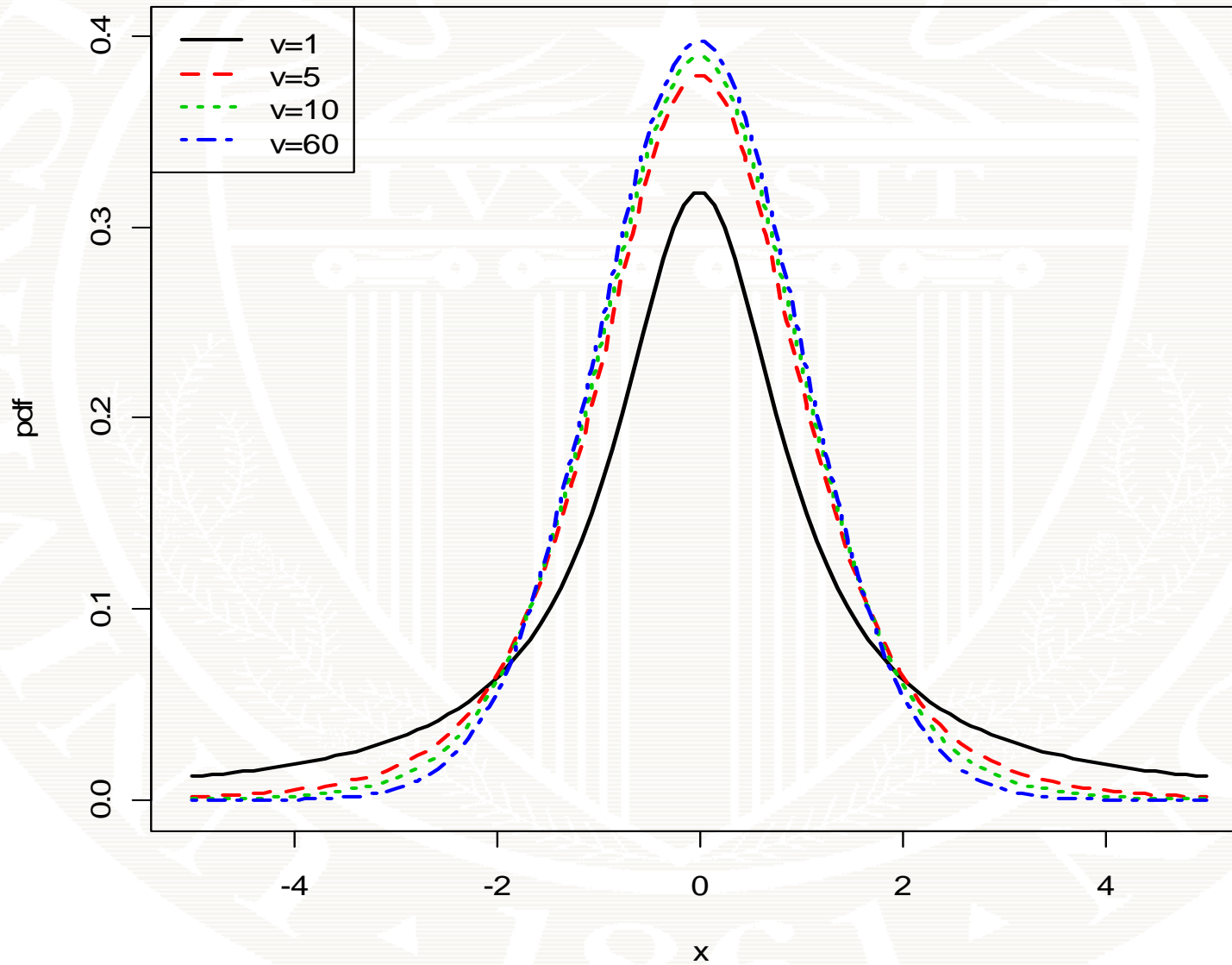
SD as a Measure of Risk



Normal vs. Log-Normal Distribution for Returns

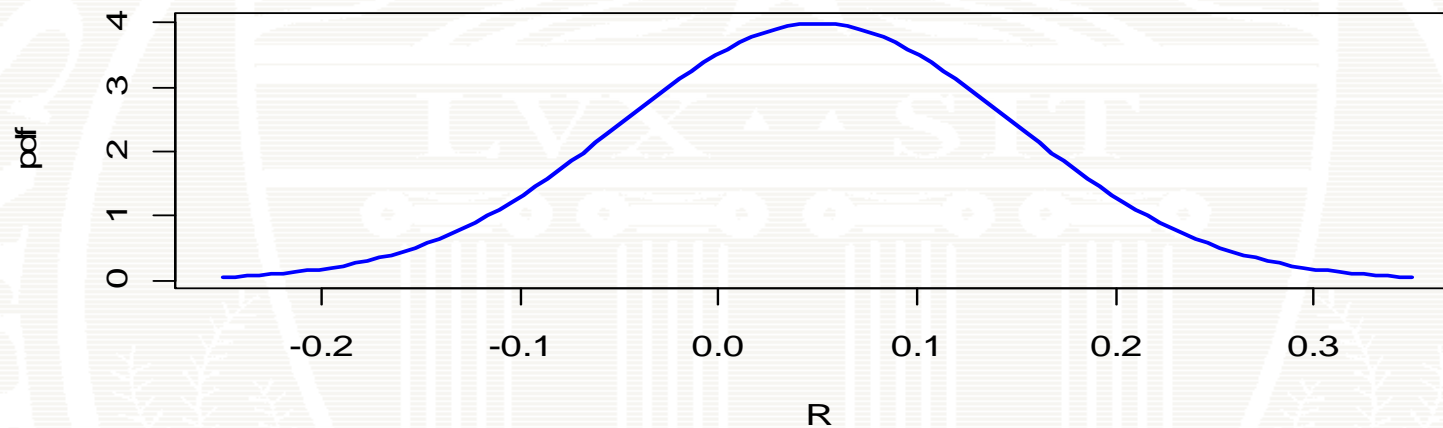


Student's t Distribution

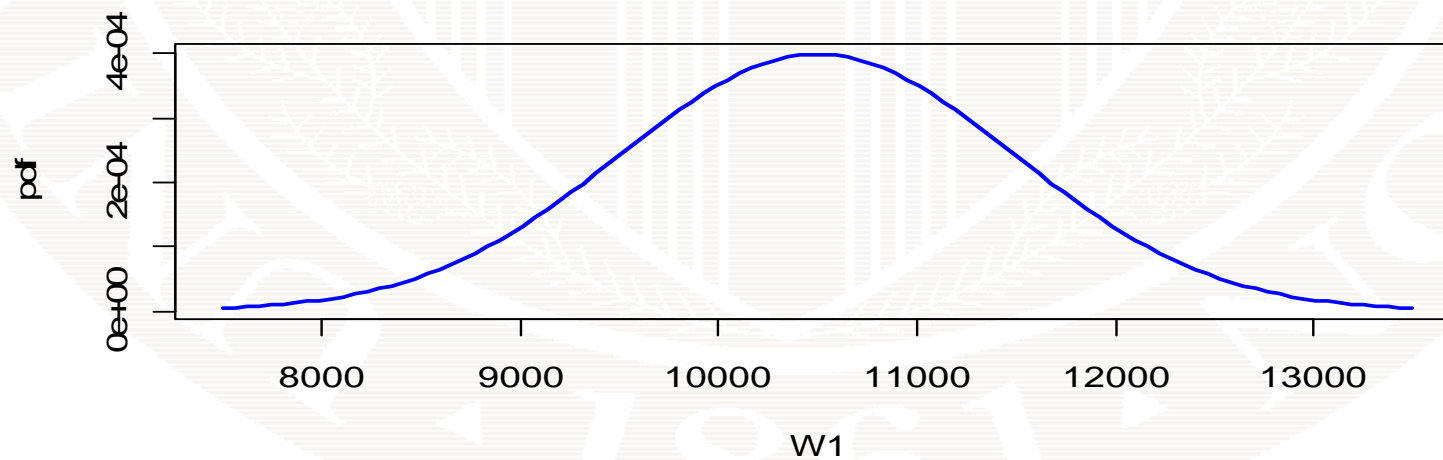


Return and Wealth Distributions

$$R(t) \sim N(0.05, (.10)^2)$$



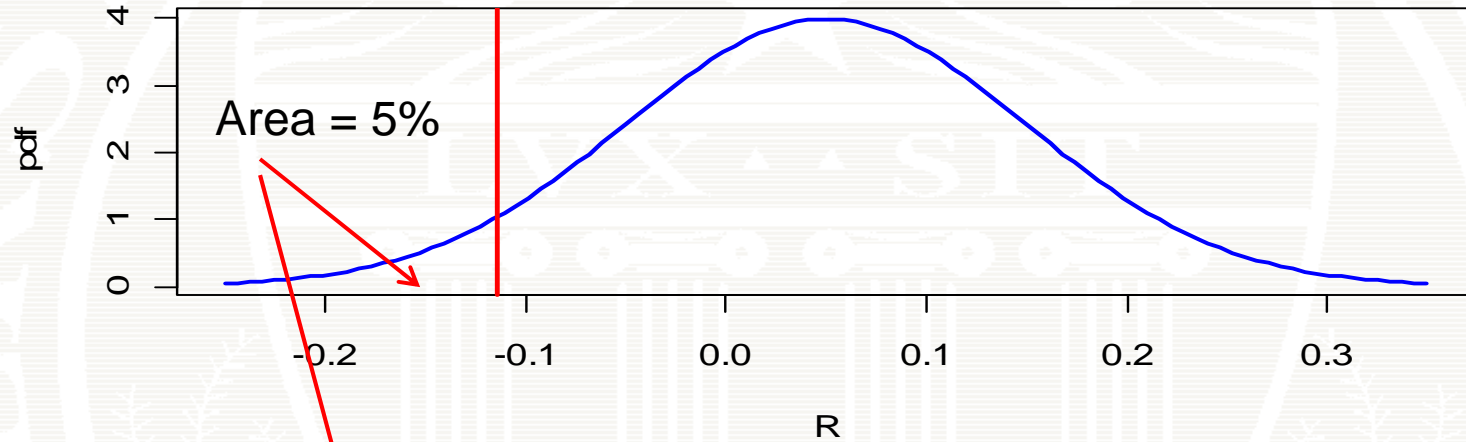
$$W1 \sim N(10,500, (1,000)^2)$$



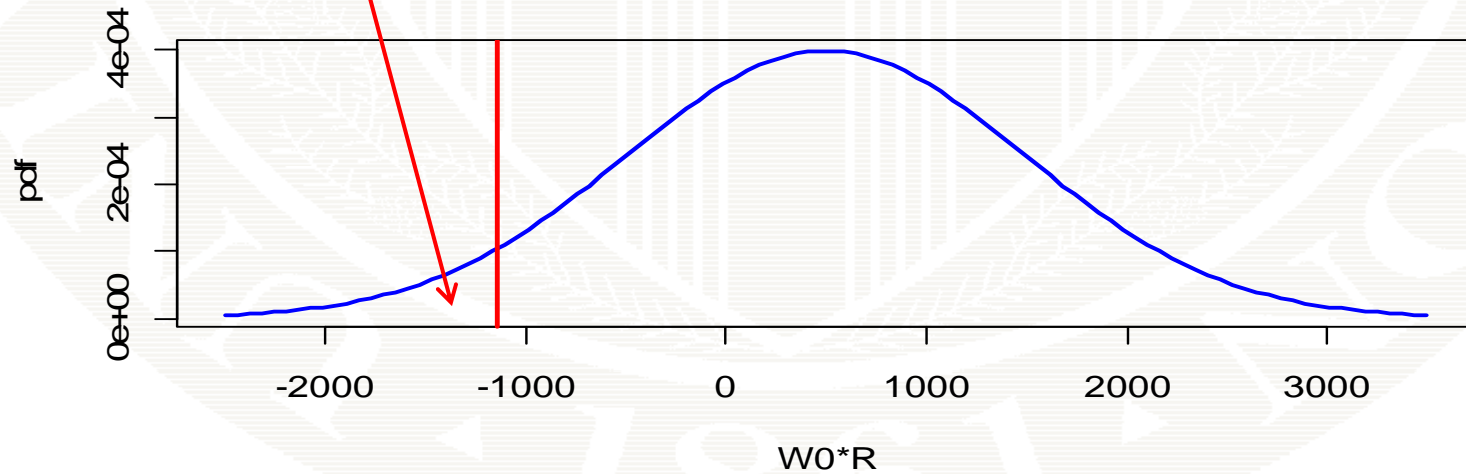
$$\Pr(R \leq q_{1-0.05}^R) = 0.05$$

5% Value-at-Risk

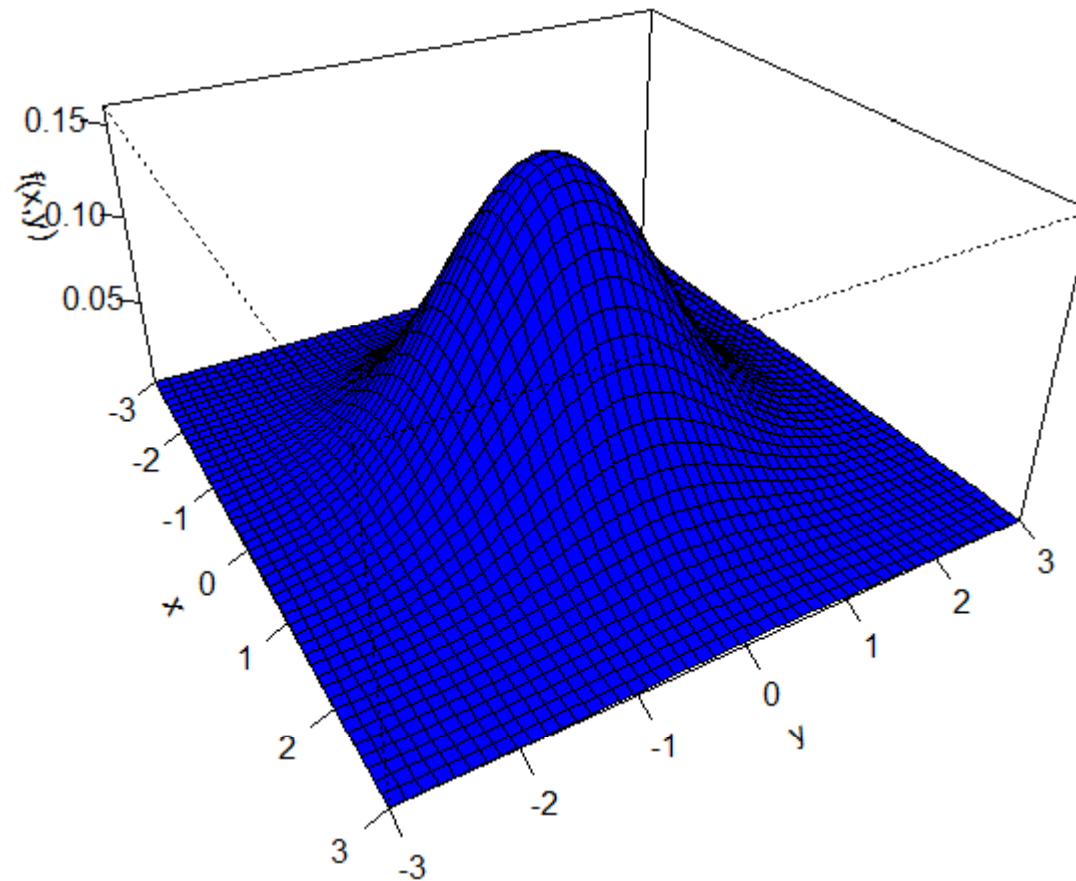
$$R(t) \sim N(0.05, (.10)^2)$$



$$R^*W_0 \sim N(500, (1,000)^2)$$

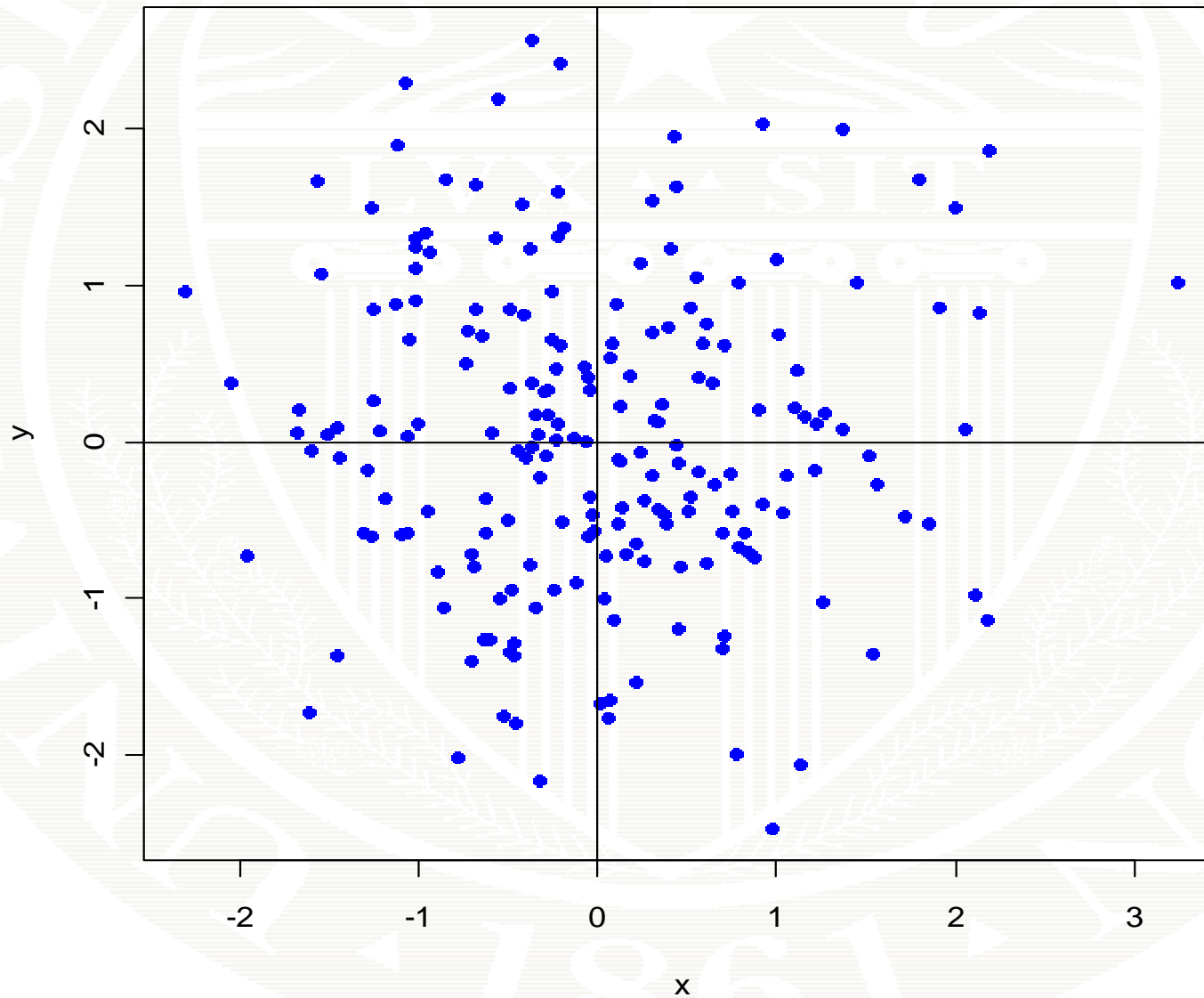


Bivariate Standard Normal

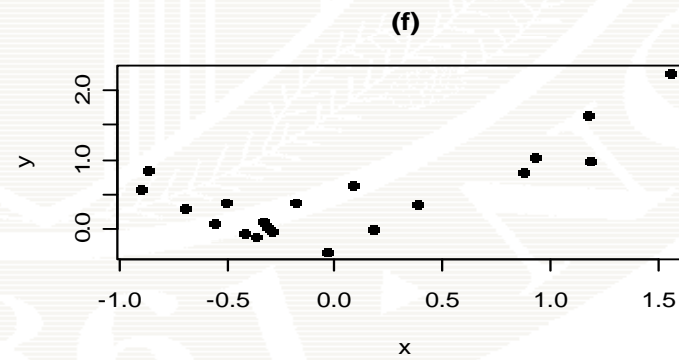
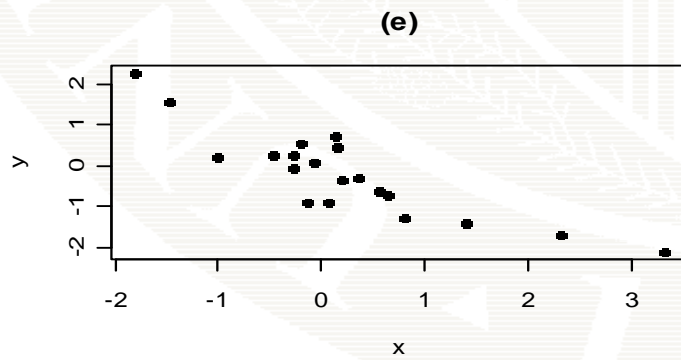
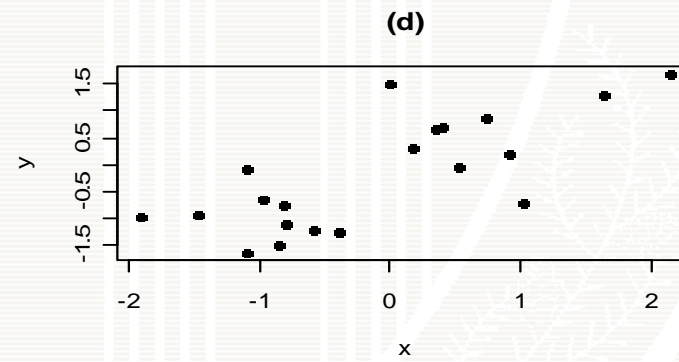
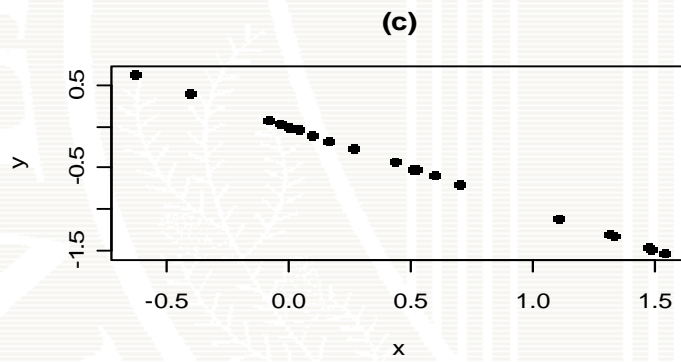
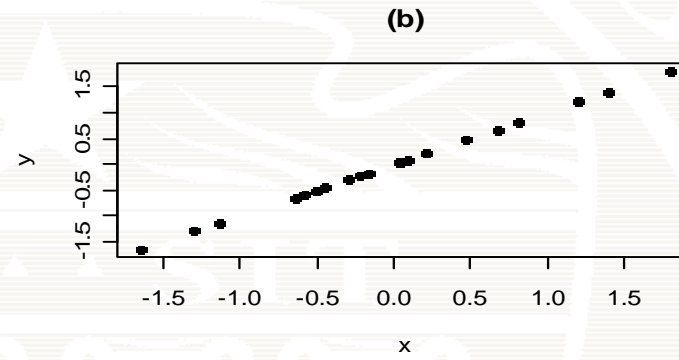
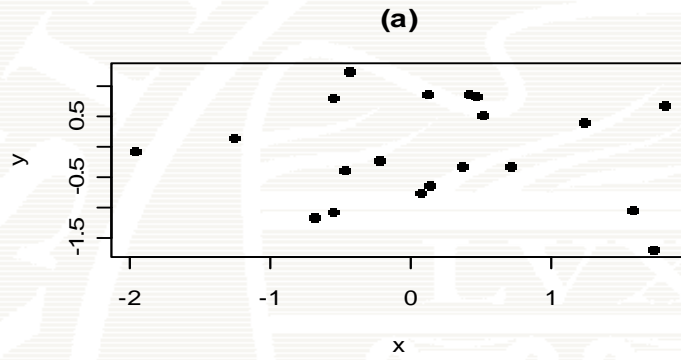


```
> pmvnorm(lower=c(-1, -1), upper=c(1, 1))  
[1] 0.4661
```

Simulated Data from Bivariate Standard Normal



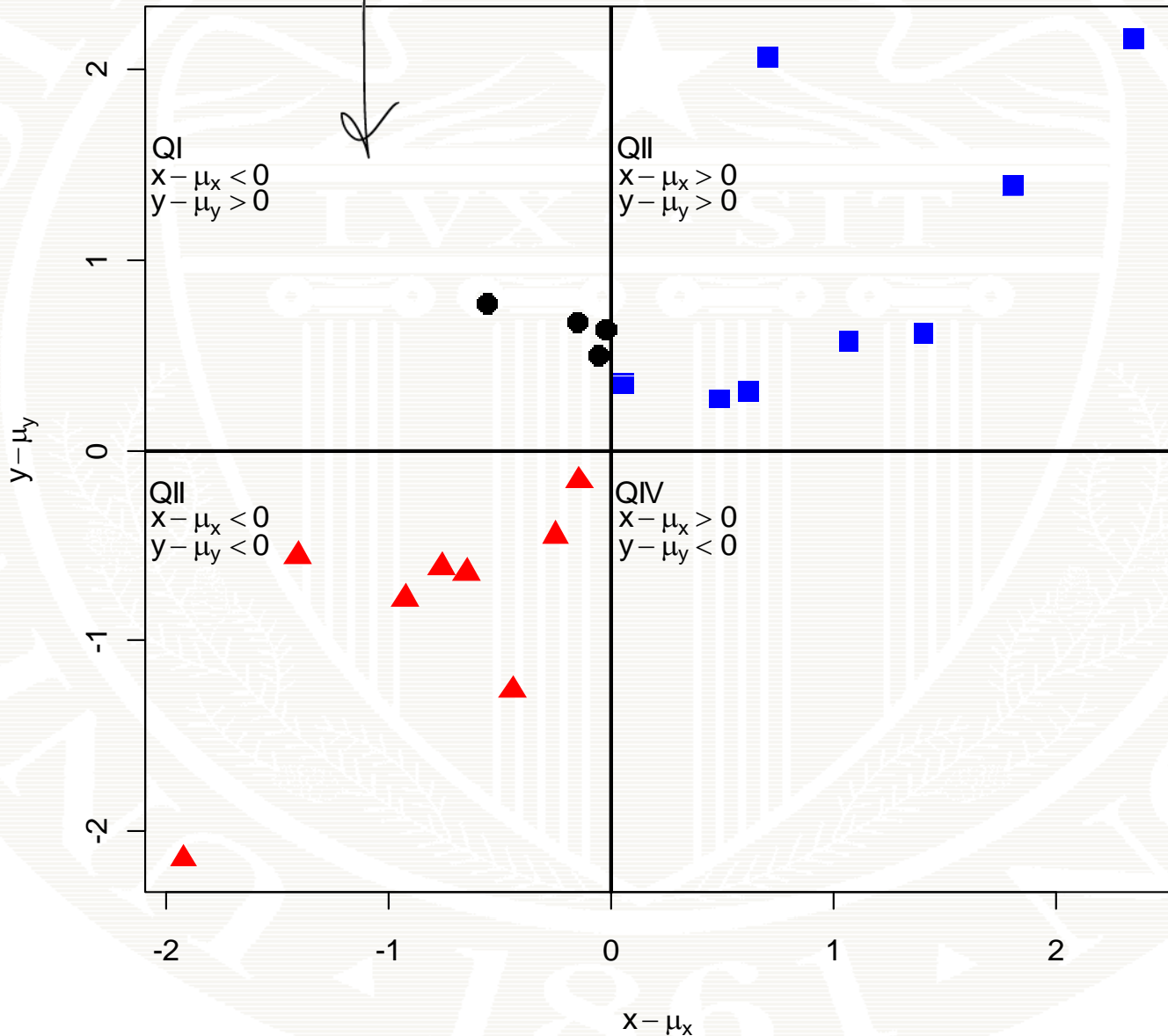
Probability Scatterplots



$$\begin{aligned} \text{Cov}(x, y) &= E\{(x - \mu_x)(y - \mu_y)\} \\ &= \sum_{x, y} (x - \mu_x)(y - \mu_y) \cdot p(x, y) \end{aligned}$$

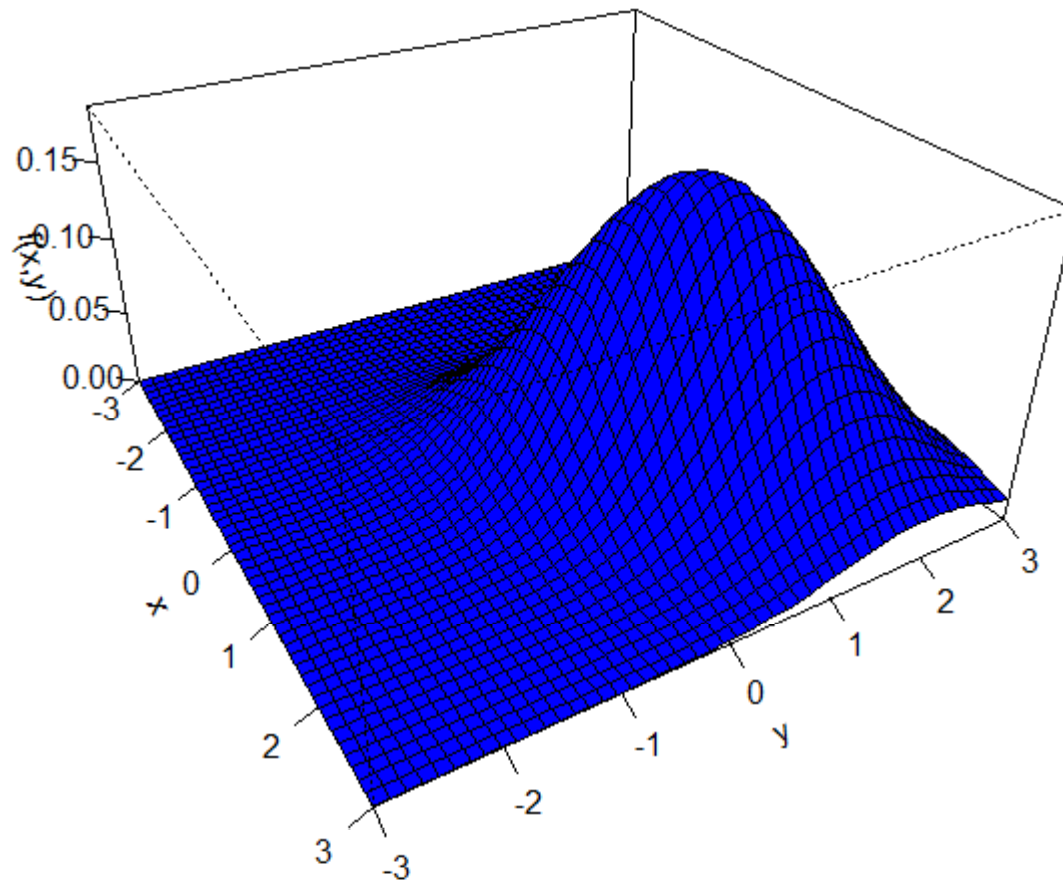
$$\begin{array}{c} (x - \mu_x)(y - \mu_y) < 0 \\ - \quad + \end{array}$$

$\text{Cov}(x, y) > 0$



Bivariate Normal

$$\mu_X = \mu_Y = 1, \sigma_X = \sigma_Y = 1, \rho_{XY} = 0.5$$



Simulated Data from Bivariate Normal

