Portfolio Performance Measurement

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1 Investment Styles

1.1 Passive Management

- Believe that markets are in equilibrium
  - Assets are correctly priced
- Hold securities for relatively long periods with small infrequent changes

- Hold surrogates for market portfolio known as index funds
  - Low cost diversified portfolios (e.g. Vanguard Index Funds)
  - motivated by portfolio theory and CAPM: efficient portfolios are combinations of T-Bills and a market index portfolio
- Do not try to create portfolios to "actively" beat the returns on index funds

1.2 Active Management

- Markets are not always in equilibrium
  - Some securities are "mis-priced"
- Buy under-priced (positive "alpha") assets and sell over-priced (negative "alpha") assets
- Active managers often "tweak" a benchmark (index) portfolio

Security	weight in benchmark	weight in active port	active position
MSFT	.05	.10	+.05
GM	.02	-.05	-.07
⋮	⋮	⋮	⋮

- Active management strategies
  - individual stock selection
  - sector selection (e.g. utility, technology)
  - asset class selection (stocks, bonds, real estate)
- Most mutual funds are actively managed.
  - management fees can vary substantially from fund to fund
  - fee is often a percentage of assets under management

2 Evaluating Investment Performance

Q: Is it worthwhile to "pay" for active management of portfolios?

Key Concepts

- Actively managed portfolios should be compared with passive (index) benchmarks of a similar risk class
- Superior past performance could be luck or could be skill
- Often very little historical data to evaluate managed portfolios
  - Statistical analysis is difficult

2.1 Risk Adjusted Measures of Performance

Observe returns on active portfolio and benchmark over some time horizon (e.g. 5 years of monthly data)

- Does the managed portfolio exhibit superior performance adjusted for risk?

- How to rank different actively managed portfolios?

Measures of risk

- Market risk (portfolio beta, β_p, from SI model or CAPM)
- Total risk (portfolio standard deviation, σ_p)

Ex Post (Historical) measures

$$\begin{aligned}
\hat{\mu}_p &= \frac{1}{T}\sum_{t=1}^{T} R_{p,t},\ \hat{r}_f = \frac{1}{T}\sum_{t=1}^{T} r_{f,t} \\
\hat{\sigma}_p &= \left(\frac{1}{T-1}\sum_{t=1}^{T}(R_{p,t}-\hat{\mu}_p)^2\right)^{1/2} \\
\hat{\beta}_p &= \frac{\widehat{\mathsf{cov}}(R_{p,t}, R_{M,t})}{\widehat{\mathsf{var}}(R_{M,t})}
\end{aligned}$$

Types of Performance Measures

- Average return difference adjusted for risk

  ave return on active portfolio -
  ave return on risk adjusted benchmark

- Risk adjusted reward/risk ratio

$$\frac{\text{average excess return}}{\text{risk measure}}$$

2.1.1 Performance Measures Based on Market Risk

Idea: Under CAPM, market risk is captured by β and expected returns are captured by the Security Market Line (SML)

$$\mu_{p,CAPM} = r_f + \beta_p(\mu_M - r_f)$$

Jensen's alpha

Risk-adjusted return difference

$$\hat{\alpha}_p^* = \hat{\mu}_p - \hat{\mu}_{p,CAPM}$$

Computation: use linear regression to estimate the excess returns SI model

$$R_{p,t} - r_f = \alpha_p^* + \beta_p(R_{Mt} - r_f) + \varepsilon_{pt},\ \varepsilon_{pt} \sim iid\ N(0, \sigma_\varepsilon^2)$$

Statistical evaluation:

$$H_0 : \alpha_p^* = 0 \text{ (no superior performance) vs. } H_1 : \alpha_p^* \neq 0$$

Information Ratio

$$\widehat{IR}_p = \frac{\hat{\alpha}_p^*}{\hat{\sigma}_\varepsilon}$$

Statistical evaluation: Use bootstrap to compute standard error and confidence interval

2.1.2 Performance Measures Based on Total Risk

Idea: Efficient portfolios are combination of T-bills and tangency portfolio. Under CAPM, the tangency portfolio is the market portfolio

Sharpe ratio

$$SR_p = \frac{\hat{\mu}_p - \hat{r}_f}{\hat{\sigma}_p}$$

$=$ excess return per unit portfolio risk

Statistical evaluation:

$H_0 : SR_p = SR_M$ (no superior performance) vs $H_1 : SR_p \neq SR_M$

Evaluate H_0 using bootstrap

R Package for Performance Evaluation

`PerformanceAnalytics`