Bootstrapping Estimates of the CER Model

Econ 424/Amath 462
Eric Zivot
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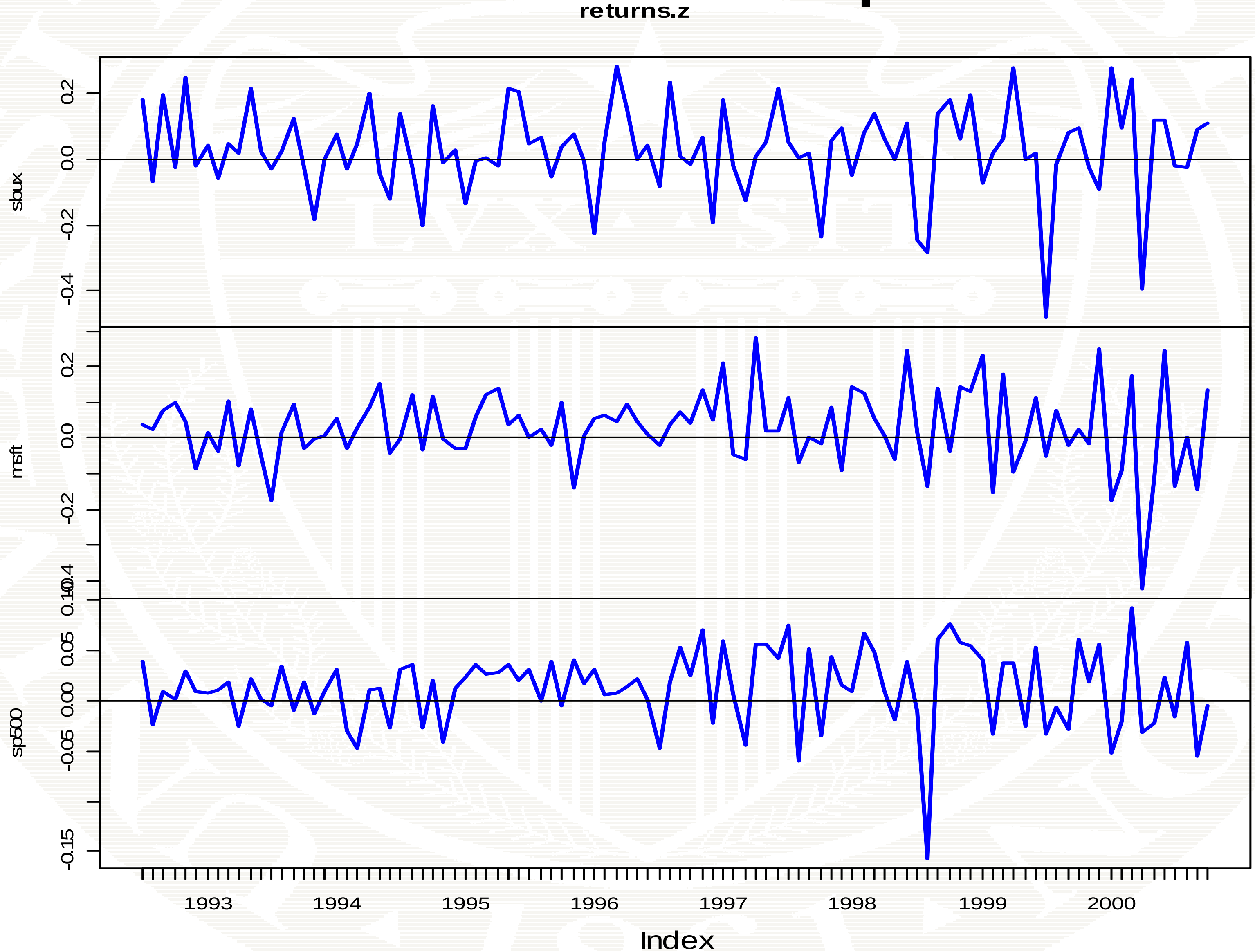
Data for Examples



T = 100 months

Estimated Standard Errors

```
> se.muhat = sigmahat.vals/sqrt(nobs)
> rbind(muhat.vals,se.muhat)
               sbux     msft     sp500
muhat.vals     0.0277   0.0275   0.01253
  se.muhat     0.0135   0.0106   0.00378
```

```
> se.sigma2hat = sigma2hat.vals/sqrt(nobs/2)
> rbind(sigma2hat.vals,se.sigma2hat)
                    sbux      msft      sp500
sigma2hat.vals      0.01845   0.01141   0.00143
  se.sigma2hat      0.00261   0.00161   0.00020
```

```
> se.sigmahat = sigmahat.vals/sqrt(2*nobs)
> rbind(sigmahat.vals,se.sigmahat)
                    sbux      msft      sp500
sigmahat.vals       0.1358    0.1068    0.0378
  se.sigmahat       0.0096    0.0075    0.0026
```

R function **sample()**

```
# random permutations of the index vector 1:5
> sample(5)
[1] 1 3 2 5 4

> sample(5)
[1] 4 2 3 5 1

# random sample of size 5 from MSFT return with
replacement
> sample(MSFT, 5, replace=TRUE)
[1] -0.02904  0.12130 -0.01890 -0.15332 -0.14627
```

Brute Force Bootstrap

Same idea as Monte Carlo Simulation but instead of generating random data from an assumed distribution, you generate random data by sampling with replacement from the observed data

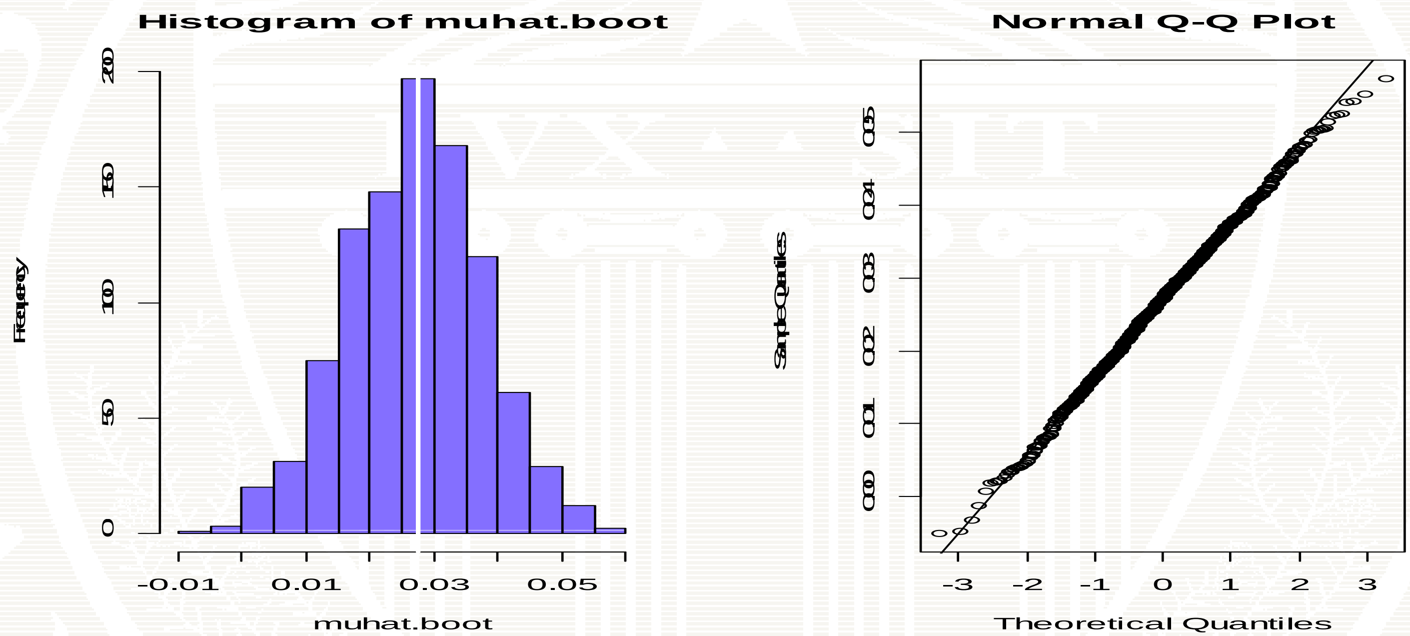
```
# bootstrap distribution for μ̂
> B = 999 # why use 999?
> muhat.boot = rep(0, B)
> nobs = length(MSFT)
> for (i in 1:B) {
+  boot.data = sample(MSFT, nobs, replace=TRUE)
+  muhat.boot[i] = mean(boot.data)
}
```

Brute Force Bootstrap

```
# bootstrap bias
> mean(muhat.boot) - muhat.MSFT
[1] -0.0005643

# bootstrap SE
> sd(muhat.boot)
[1] 0.01045

# analytic SE
> sigmahat.MSFT/sqrt(length(MSFT))
[1] 0.01068
```

Bootstrap SE is very close to analytic SE

Brute Force Bootstrap




```
par(mfrow=c(1,2))
  hist(muhat.boot, col="slateblue1")
  abline(v=muhat.MSFT, col="white", lwd=2)
  qqnorm(muhat.boot)
  qqline(muhat.boot)
par(mfrow=c(1,1))
```

R Package boot

- Implements a variety of bootstrapping functions
- Background material is book by Davidson and Hinkley, *Bootstrap Methods and Their Application*, Cambridge University Press, 1997.
- Main functions are:
  - **`boot()`** bootstrap user supplied function
  - **`boot.ci()`** compute bootstrap confidence interval

Example: Bootstrapping sample mean

```
# function for bootstrapping sample mean
mean.boot = function(x, idx) {
# arguments:
# x         data to be resampled
# idx       vector of scrambled indices created
#           by boot() function
# value:
# ans       mean value computed using resampled
#           data
     ans = mean(x[idx])
     ans
}
```

Example: Bootstrapping sample mean

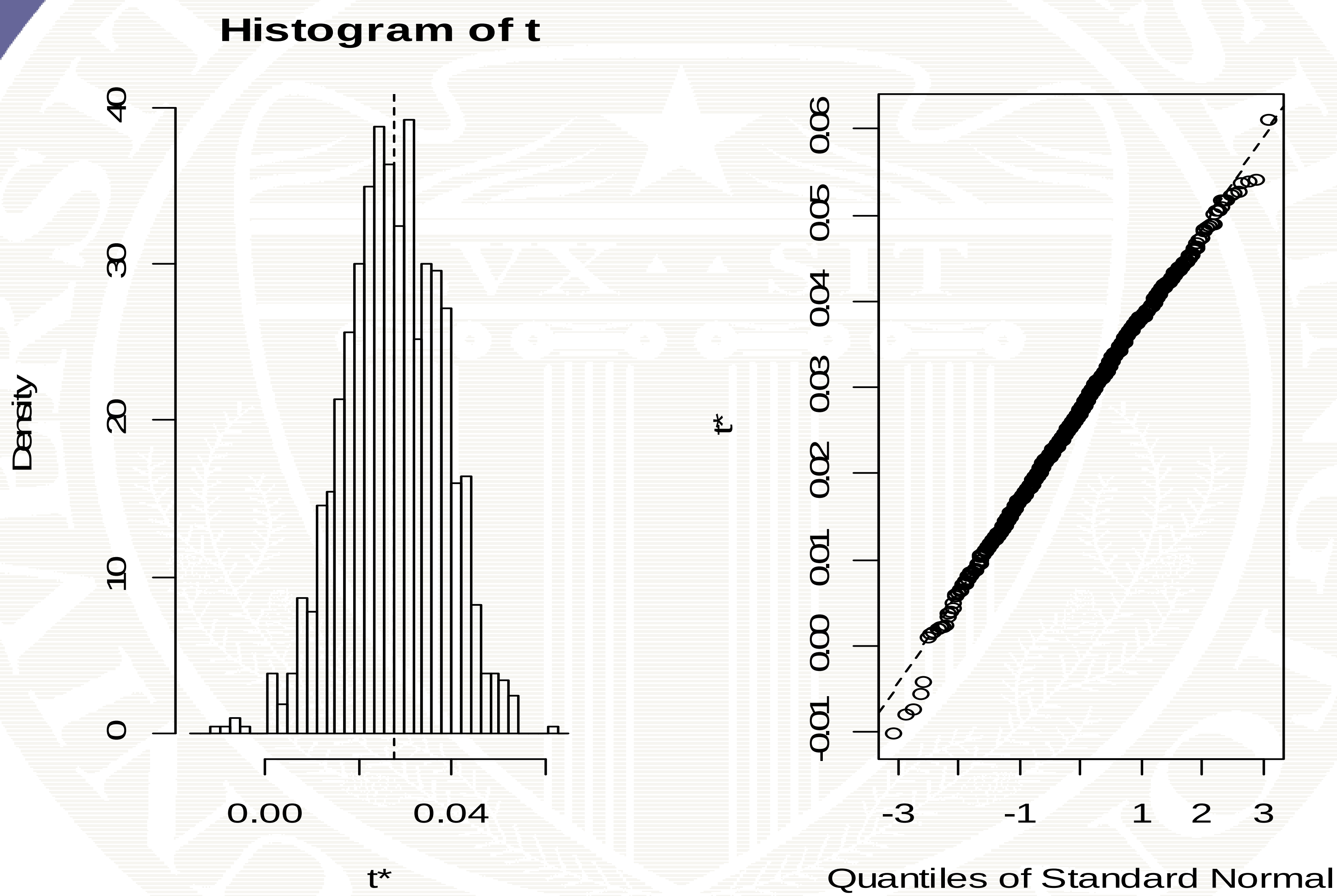
```
> MSFT.mean.boot = boot(MSFT, statistic = mean.boot, R=999)
> class(MSFT.mean.boot)
[1] "boot“

> MSFT.mean.boot

ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = MSFT, statistic = mean.boot, R = 999)


Bootstrap Statistics :
      original        bias     std. error
t1*   0.02756       -0.00013   0.01052
```

Number of bootstrap samples

Sample mean

Bootstrap estimate of bias

Bootstrap estimate of SE




```
> plot(MSFT.mean.boot)
```

Compare Bootstrap Statistics with Analytic Formulas

```
ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = MSFT, statistic = mean.boot, R = 999)


Bootstrap Statistics :
      original        bias     std. error
t1*   0.02756       -0.00013   0.01052

# compare boot SE with analytic SE
> se.muhat.MSFT = sigmahat.MSFT/sqrt(length(MSFT))
> se.muhat.MSFT

[1] 0.01068
```

Bootstrap Confidence Intervals

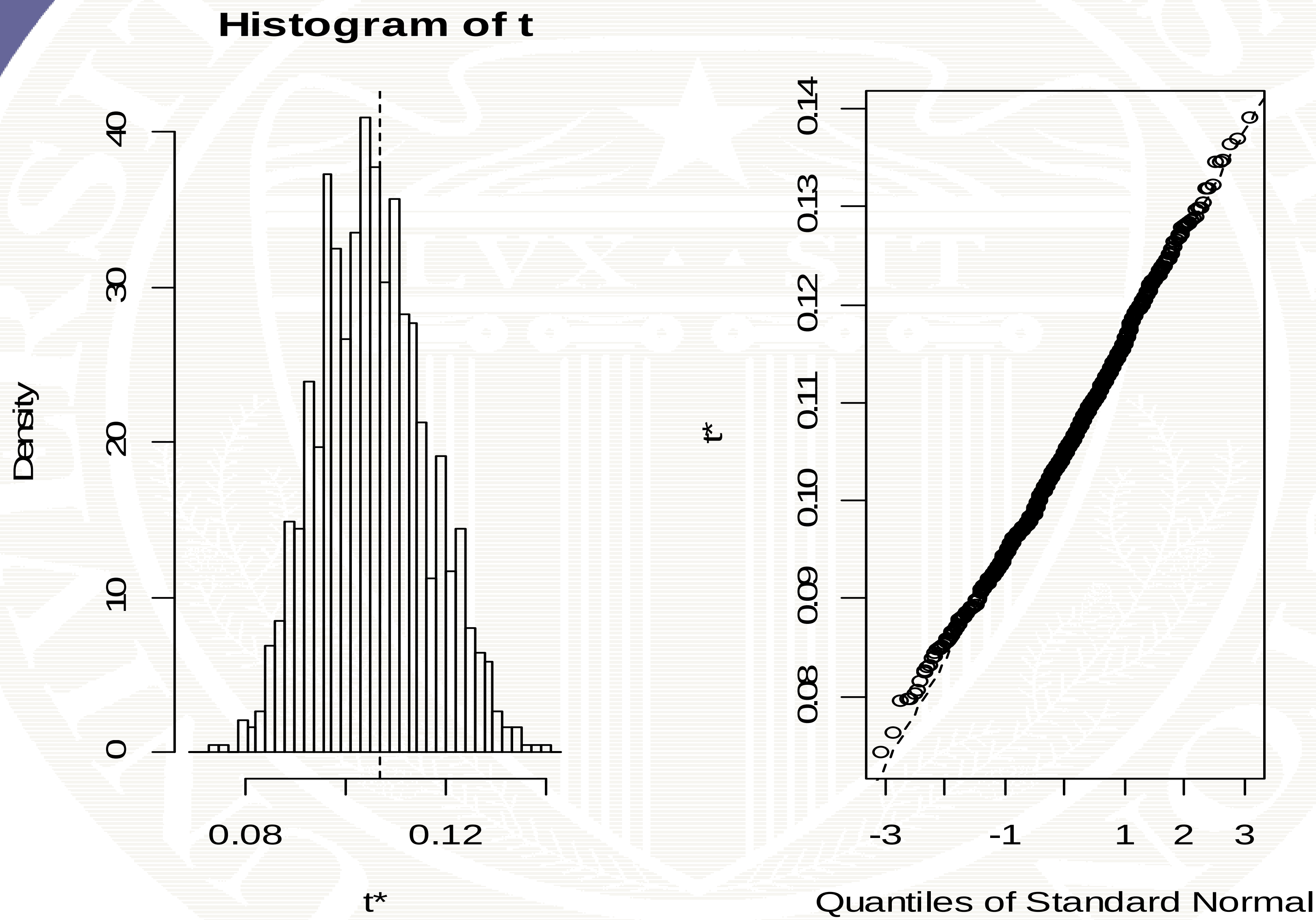
```
> boot.ci(MSFT.mean.boot, conf = 0.95, type =
+         c("norm","perc"))
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 999 bootstrap replicates

CALL :
boot.ci(boot.out = MSFT.mean.boot, conf = 0.95, type =
        c("norm", "perc"))

Intervals :
Level      Normal              Percentile
95%   ( 0.0071,  0.0483 )   ( 0.0065,  0.0471 )
Calculations and Intervals on Original Scale
```

Example: Bootstrapping Sample SD

```
# function for bootstrapping sample standard deviation
sd.boot = function(x, idx) {
# arguments:
# x            data to be resampled
# idx          vector of scrambled indices created by
#              boot() function
# value:
# ans          sd value computed using resampled data
     ans = sd(x[idx])
     ans
}
```

Example: Bootstrapping Sample SD

```
> MSFT.sd.boot = boot(MSFT, statistic = sd.boot, R=999)
> MSFT.sd.boot

ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = MSFT, statistic = sd.boot, R = 999)


Bootstrap Statistics :
     original     bias      std. error
t1*  0.1068      -0.00145   0.01078

# compare boot SE with analytic SE based on CLT
> se.sigmahat.MSFT = sigmahat.MSFT/sqrt(2*length(MSFT))
> se.sigmahat.MSFT

[1] 0.00755
```




```
> plot(MSFT.sd.boot)
```

Example: Boostrapping Normal VaR

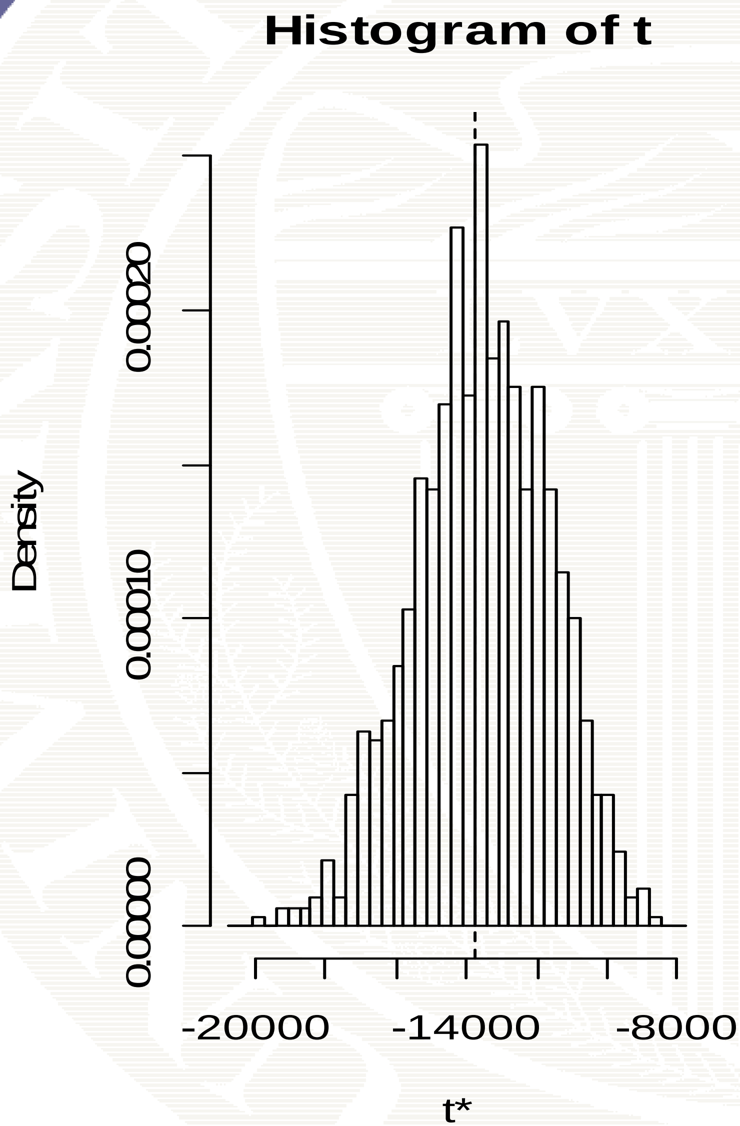
```
ValueAtRisk.boot = function(x, idx, p=0.05, w=100000) {
# x.mat       data to be resampled
# idx         vector of scrambled indices created by
#             boot() function
# p           probability value for VaR calculation
# w           value of initial investment
# value:
# ans         Value-at-Risk computed using resampled data

      q = mean(x[idx]) + sd(x[idx])*qnorm(p)
      VaR = (exp(q) - 1)*w
      VaR
}
```

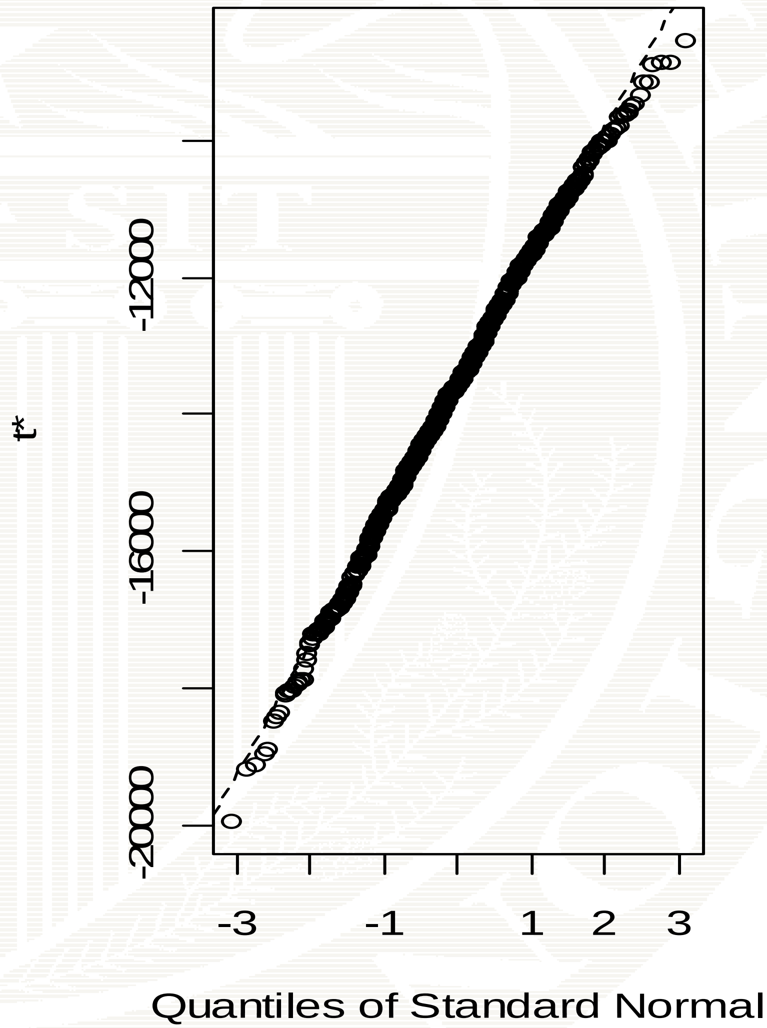
Example: Boostrapping Normal VaR

```
> MSFT.VaR.boot

ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = MSFT, statistic = ValueAtRisk.boot, R = 999)


Bootstrap Statistics :
     original   bias    std. error
t1* -13769.40 210.2801    1886.953
```

Sample VaR estimate

Bootstrap SE






```
> plot(MSFT.VaR.boot)
```

Example: Boostrapping Normal VaR

```
> boot.ci(MSFT.VaR.boot, conf=0.95, type=c("norm", "perc"))

BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 999 bootstrap replicates

CALL :
boot.ci(boot.out = MSFT.VaR.boot, conf = 0.95, type =
c("norm", "perc"))

Intervals :
Level      Normal                  Percentile
95%   (-17678, -10281 )   (-17212, -10009 )
```

$$\hat{\theta} \pm 2 \times SE_{boot}(\hat{\theta}) \qquad [q^*_{.025},\ q^*_{.975}]$$