University of Washington — Fall 2008
Department of Economics — Eric Zivot

Economics 424

Midterm Exam

This is a closed book and closed note exam. However, you are allowed one page of notes (double-sided). Answer all questions and write all answers in a blue book or on separate sheets of paper. Time limit is 1 hours and 50 minutes. Total points = 100.

I. Return Calculations (20 pts, 5 points each)

1. Consider a one year investment in two assets: Amazon stock and the S&P 500 index. Suppose you buy Amazon and S&P 500 at the end of September 2007 at $P_{A,t-1}$= 93.15, $P_{S,t-1} = 1526.75$ and then sell at the end of September 2008 for $P_{A,t}$= 72.76, $P_{S,t} = 1164.74$. (Note: these are actual closing prices taken from Yahoo!)

a. What are the simple annual returns for the two stocks?

```
> pa.1 = 93.15
> pa.2 = 72.76
> ps.1 = 1526.75
> ps.2 = 1164.74

# a) simple returns on Amazon and sp500
> ra = (pa.2 - pa.1)/pa.1
> rs = (ps.2 - ps.1)/ps.1
> ra
[1] -0.2188943
> rs
[1] -0.2371115
```

b. What are the continuously compounded annual returns for the two stocks?

```
> log(1 + ra)
[1] -0.2470447
> log(1 + rs)
[1] -0.2706434
```

c. The annual inflation rate between September 2007 and September 2008 was about 5%. Using this information, determine the simple and continuously compounded real annual returns on Amazon and S&P 500.

```
> inflat = 0.05
> # simple real returns
```

```
> ra.real = (1+ra)/(1+inflat) - 1
> rs.real = (1+rs)/(1+inflat) - 1
> ra.real
[1] -0.2560898
> rs.real
[1] -0.2734395

> # cc real returns
> log(1+ra.real)
[1] -0.2958349
> log(1+rs.real)
[1] -0.3194336
```

d. At the end of September, 2006, suppose you have $100,000 to invest in Amazon and S&P 500 over the next year. Suppose you sell short $60,000 in S&P 500 and use the proceeds to buy $160,000 in Amazon. Using the results from part a, compute the annual simple and continuously compounded return on the portfolio.

```
> xs = -60000/100000
> xa = 160000/100000
> xa
[1] 1.6
> xs
[1] -0.6
> rp = xa*ra + xs*rs
> rp
[1] -0.2079639

> # cc portfolio return
> log(1 + rp)
[1] -0.2331483
```

II. Probability Theory and Matrix Algebra (20 points, 5 points each)

1. Suppose you currently hold $2M (million) in Starbucks stock. That is, your initial wealth at the beginning of the month is $W_0 = \$2M$. Let R_{sbux} denote the monthly *simple* return on Starbucks stock, and assume that $R_{sbux} \sim N(0.03,(0.20)^2)$. Let $W_1 = W_0(1+R_{SBUX})$ be a random variable representing your wealth at the end of the month.

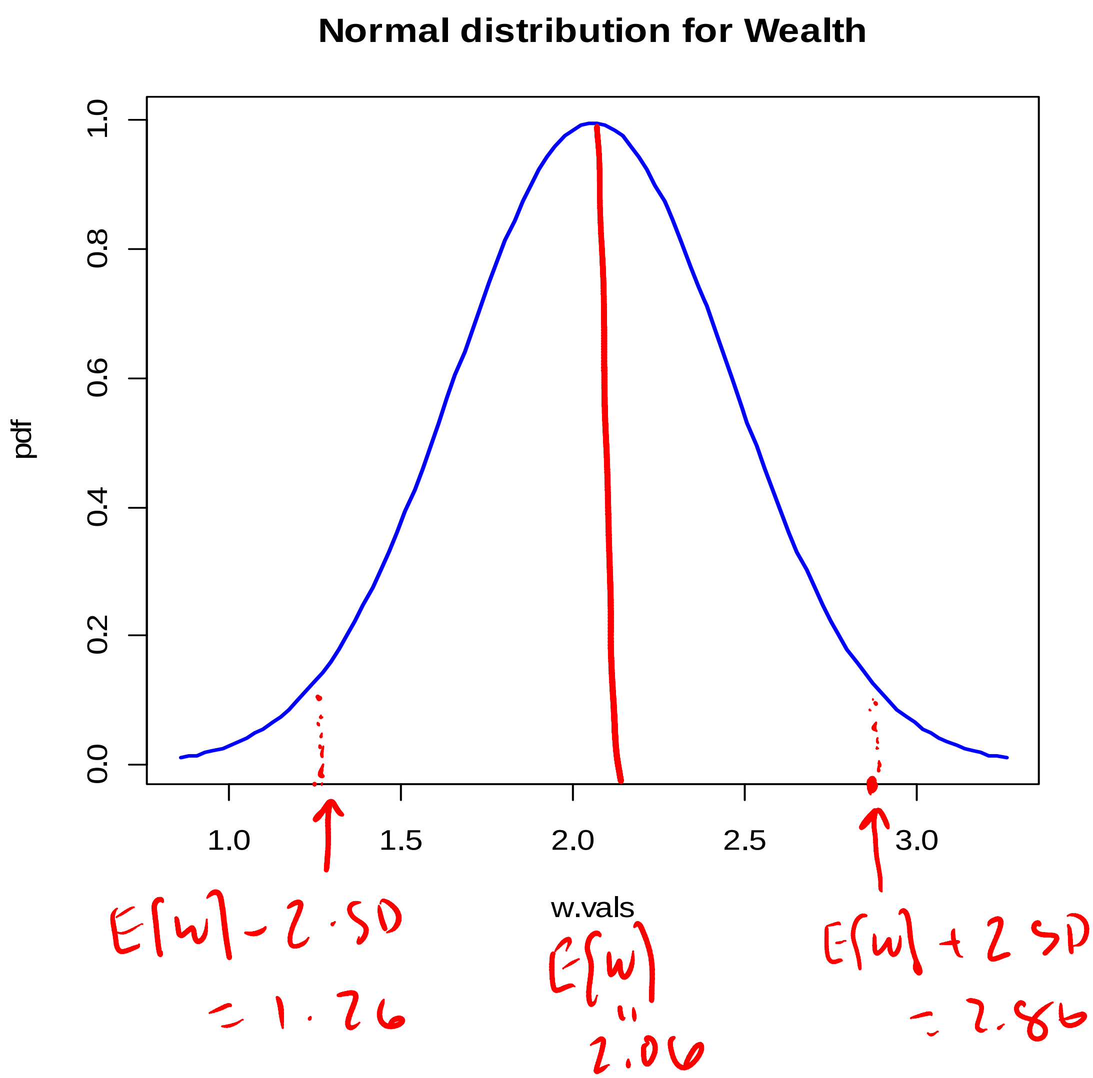
a) Compute $E[W_1]$, $\text{var}(W_1)$ and $SD(W_1)$

```
> w0 = 2
> e.rsbux = 0.03
> sd.rsbux = 0.20
> e.w = w0 + w0*e.rsbux
> e.w
[1] 2.06
```

```
> var.w = w0*(w0*sd.rsbux*sd.rsbux)
> var.w
[1] 0.16
> sd.w = w0*sd.rsbux
> sd.w
[1] 0.4
```

b) What is the probability distribution of W_1? Sketch the distribution, indicating the location of $E[W_1]$ and $E[W_1] \pm 2 \cdot SD(W_1)$.

```
Since R is normally distributed and W₁ is a linear
function of R, W₁ is also normally distributed with
mean $2.06M and SD $0.4M.
> e.w + 2*sd.w
[1] 2.86
> e.w - 2*sd.w
[1] 1.26
```



c) Briefly explain why the normal distribution may not be appropriate for describing the distribution of simple returns.

The normal distribution is defined from $-\infty$ to ∞. Simple returns cannot be smaller than -1. Also, multi-period simple returns are multiplicative (geometric average). That is, the 2 period return is a geometric average (multiplicative) of two 1 period returns. If the 1 period returns are normally distributed then the 2 period return will not be normal.

2. Let R_i denote the continuously compounded return on asset i (i = 1,2,3) with $E[R_i] = \mu_i$, var(R_i) = σ_i^2 and cov(R_i, R_j) = σ_{ij}. Define the 3 × 1 vectors

$$\mathbf{R} = \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix}, \boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}, \mathbf{1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

and the 3 × 3 covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{pmatrix}$$

The vectors **x** and **y** represent portfolio weights (i.e., shares of wealth invested in the three assets).

Using matrix algebra, give expressions for the returns, expected returns and variances for the two portfolios.

$$R_{p,x} = \mathbf{x}'\mathbf{R}, R_{p,y} = \mathbf{y}'\mathbf{R},$$
$$\mu_{p,x} = \mathbf{x}'\boldsymbol{\mu}, \mu_{p,x} = \mathbf{y}'\boldsymbol{\mu},$$
$$\sigma_{p,x}^2 = \mathbf{x}'\boldsymbol{\Sigma}\mathbf{x}, \sigma_{p,x}^2 = \mathbf{y}'\boldsymbol{\Sigma}\mathbf{y}$$

III. Time Series Concepts (15 points)

1. Let $\{Y_t\}$ represent a stochastic process. Under what conditions is $\{Y_t\}$ covariance stationary? (5 points)

$$E[Y_t] = \mu$$
$$\text{var}(Y_t) = \sigma^2$$
$$\text{cov}(Y_t, Y_{t-j}) = \gamma_j \text{ (depends on } j \text{ and not } t)$$

2. Consider the random walk model

$$Y_t = Y_{t-1} + \varepsilon_t = Y_0 + \sum_{j=1}^{t} \varepsilon_j,\ Y_0 = \text{constant}.$$

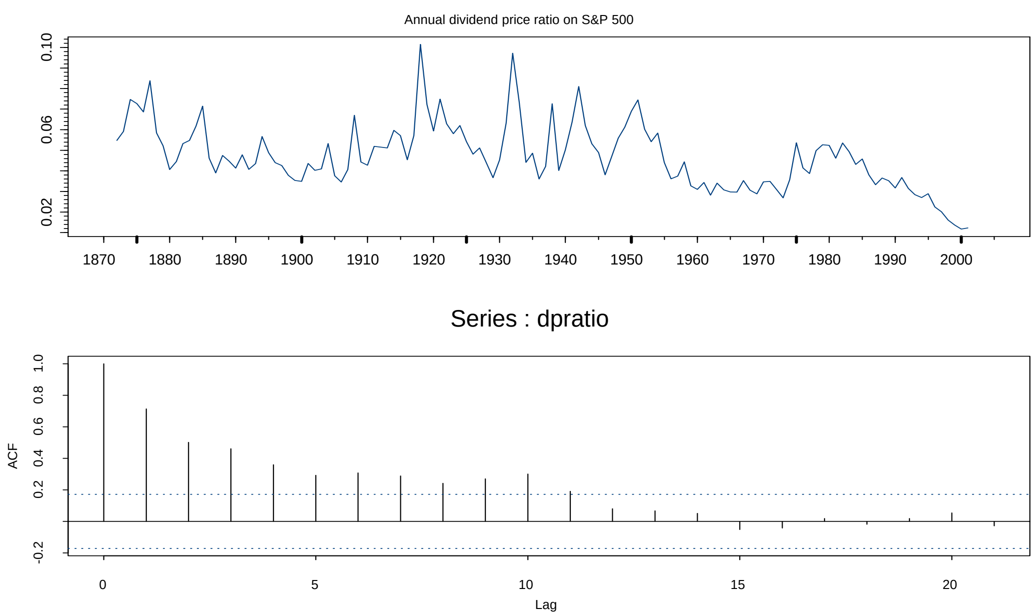
$$\varepsilon_t \sim iid\ N(0, \sigma^2)$$

Is $\{Y_t\}$ a covariance stationary stochastic process? Why or why not? (5 points)

`No. The random walk process is not stationary. The variance of the random walk process depends on time:`

$$\text{var}(Y_t) = \sum_{j=1}^{t} \text{var}(\varepsilon_j) = \sum_{j=1}^{t} \sigma^2 = t\sigma^2$$

3. The figure below shows annual observations on the dividend yield of the S&P 500 index over the period 1871 through 2000 along with the sample ACF. (5 points)



Assume the dividend yield is covariance stationary. Based on the shape of the sample autocorrelation function, would an MA(1) process or an AR(1) process best describe the data? Briefly justify your answer.

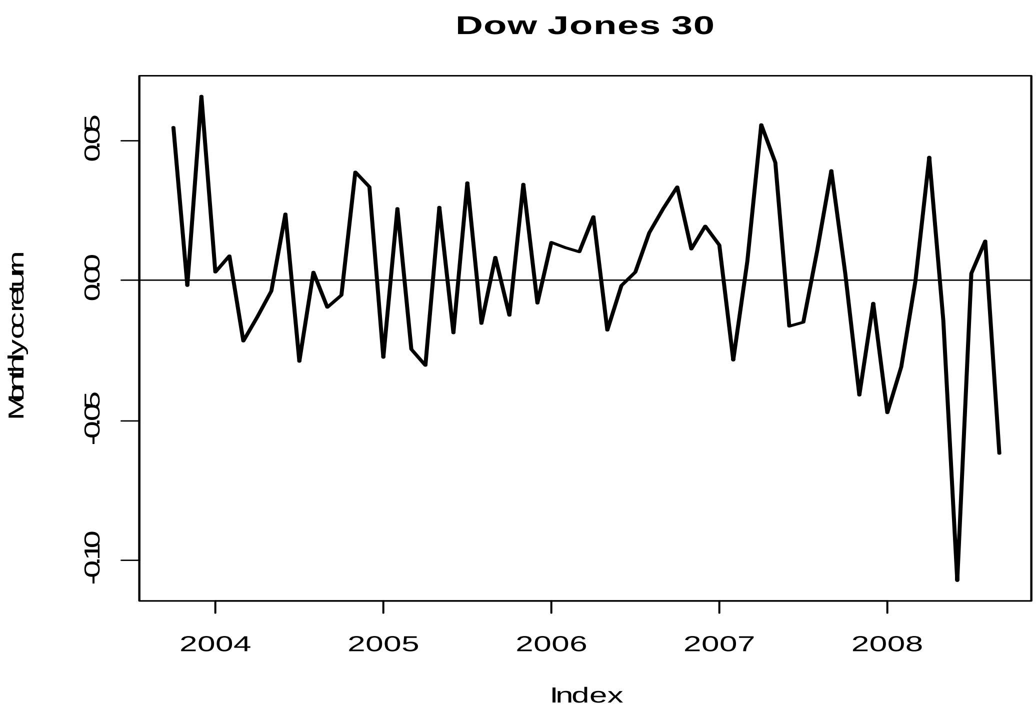
`The SACF decays toward zero and does not cut off at lag 1. Therefore, it looks more like an AR(1) process than an MA(1) process.`

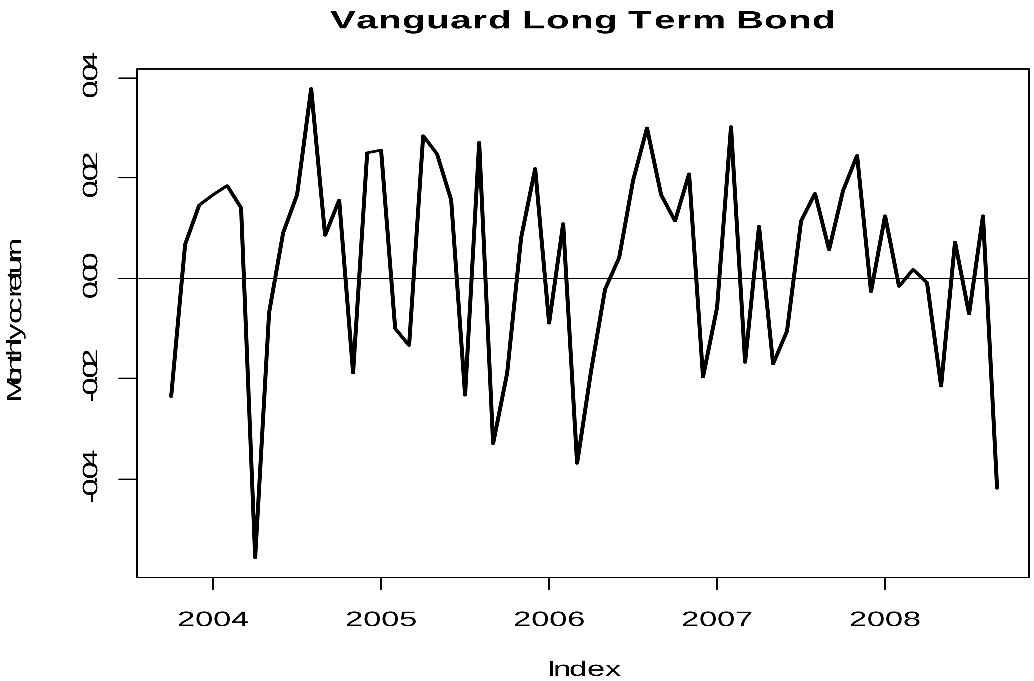
IV. Constant Expected Return Model (45 points, 5 points each)

Consider the constant expected return model

$$r_{it} = \mu_i + \varepsilon_{it},\ \varepsilon_{it} \sim iid\ N(0, \sigma_i^2)$$
$$\mathrm{cov}(r_{it}, r_{jt}) = \sigma_{ij},\ corr(r_{it}, r_{jt}) = \rho_{ij}$$

for the *monthly* continuously compounded returns on the Dow Jones Industrial Average (dji) and the Vanguard long-term bond index (vbltx) over the period September 2003 through September 2008. For this period there are *T*=60 monthly observations. The data are shown in the graphs below.

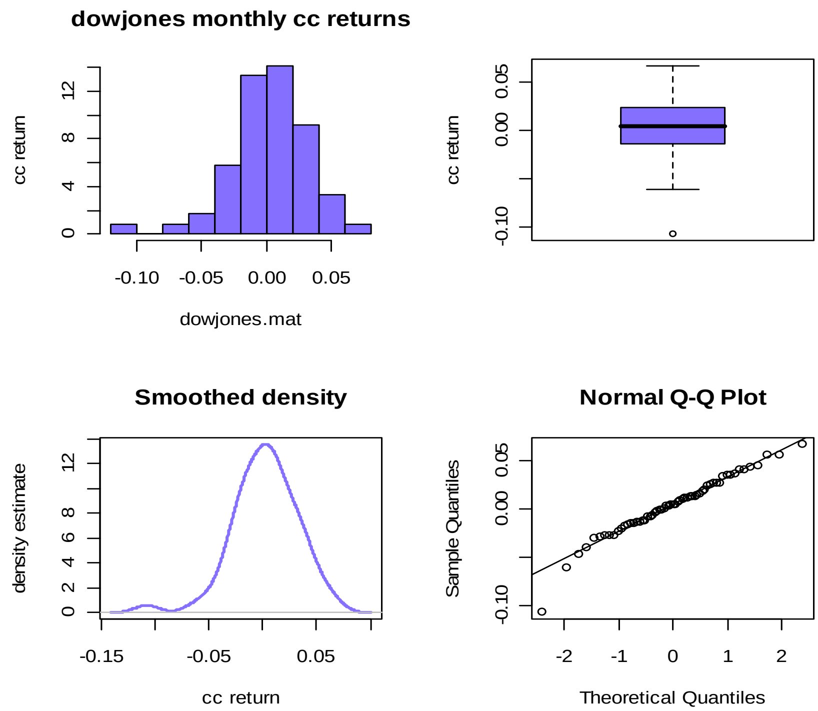


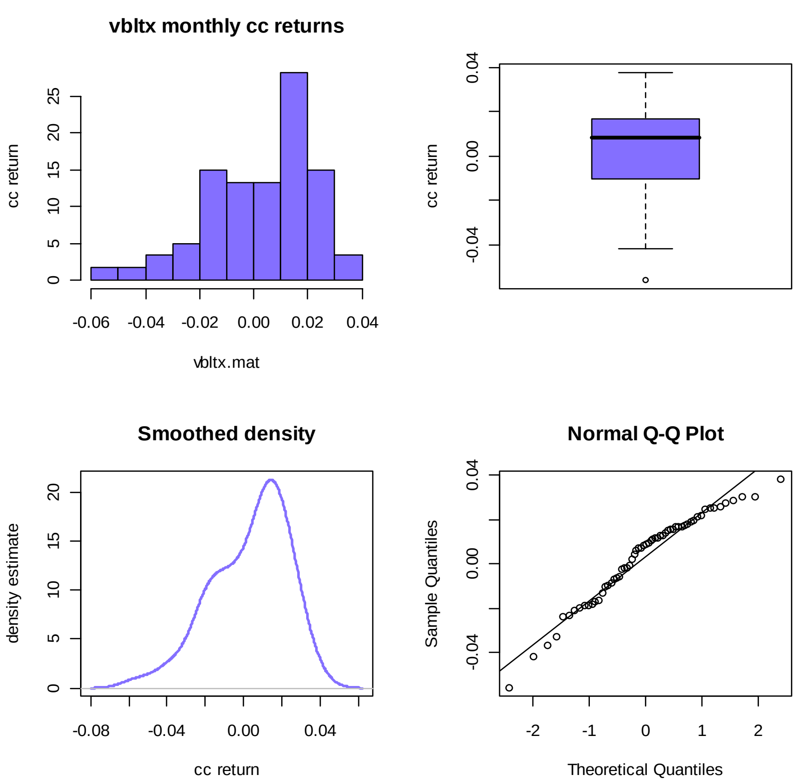


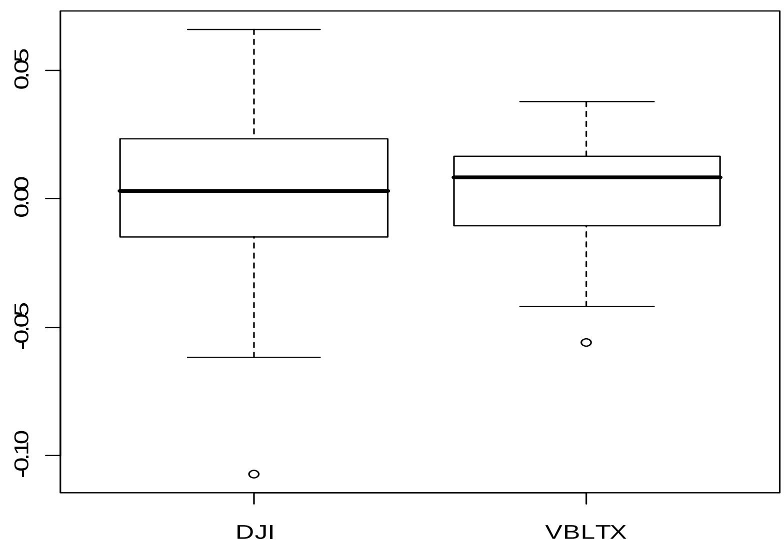
a) Do the monthly continuously compounded return data look like they come from the CER model? Why or why not?

The CER model postulates that cc returns are (covariance stationary) iid normal random variables with constant means, variances and covariances (correlations). The above two return series look a bit like computer simulations from the CER model. The returns appear to fluctuate randomly about a constant mean. The mean and the variance for the Vanguard bond index appears to be constant over time. However, for the Dow, the mean appear appears to be slightly lower and the volatility appears to be slightly higher after 2007 suggesting that the mean and variance of the returns are not constant through time.

b) The figures below gives some graphical diagnostics of the return distributions for dji and vbltx.


dowjones monthly cc returns
cc return
dowjones.mat
Smoothed density
density estimate
cc return
Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles


vbltx monthly cc returns
cc return
vbltx.mat
Smoothed density
density estimate
cc return
Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles



Sample descriptive statistics are (note: the reported kurtosis is excess kurtosis)

```
> apply(ret.z, 2, mean)
   dowjones       vbltx
0.002614947 0.003035592

> apply(ret.z, 2, sd)
  dowjones      vbltx
0.03014207 0.01990333

> apply(ret.z, 2, skewness)
  dowjones      vbltx
-0.6686971 -0.7185054

> apply(ret.z, 2, kurtosis)
  dowjones      vbltx
1.76144376 0.07748463
```

Based on this information, do you think the monthly cc returns on dji and vbltx are normally distributed? Briefly justify your answer.

Recall, the normal distribution is symmetric (zero skewness) and has a kurtosis equal to three.

Dow Jones: The histogram and qq-plot show a negative skweness. The boxplot indicates one large negative outlier. The sample skewness is moderate and negative and the excess kurtosis is large. However, these values appear to be driven by the one large negative return. The negative skewness and kurtosis greater than three cast doubt on the appropriateness of the normal distribution.

Vanguard long term bond: The histogram and boxplot shows that the returns are highly negatively skewed, and the qq-plot has a curved shape that reflects the negative skewness. The sample skewness is negative while the excess kurtosis is close to zero. The large negative skewness casts doubt on the appropriateness of the normal distribution.

c) The following R output gives the estimates of $\mu_i, \sigma_i, \sigma_{ij}$ and ρ_{ij} for dji and vbltx from the 5 years of monthly data.

```
> muhat.vals
   dowjones       vbltx
0.002614947 0.003035592

> sigmahat.vals
  dowjones      vbltx
0.03014207 0.01990333

> covhat.vals
    dji,vbltx
-3.683218e-05

> rhohat.vals
  dji,vbltx
-0.06139438
```

Which asset appears to be riskier? Do you think there is any benefit of holding a portfolio consisting of dji and vbltx? Justify your answer.

Risk is generally measured by the return standard deviation. Here, the sd for dji is 0.030 and the sd for vbltx is 0.020. Therefore dji is the riskier asset. In a two asset portfolio, the portfolio variance is $x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\sigma_{12}$. If the two assets are negatively correlated then this reduces the portfolio variance (diversification effect). Here, dji and vbltx are slightly negatively correlated so there is a clear risk reduction benefit of holding these two assets in a portfolio.

d) Consider a 12-month (one year) investment. Let $r_t(12)$ denote the 12-month (annual) continuously compounded return. Using the monthly CER estimates for dji and vbltx, give the estimates for the annual mean and standard deviation.

```
> muhat.vals*12
  dowjones      vbltx
0.03137937 0.03642711

> sigmahat.vals*sqrt(12)
  dowjones      vbltx
0.10441518 0.06894717
```

e) Using the above output, compute for both assets estimated standard errors for μ, σ and ρ. Briefly comment on the precision of the estimates.

```
> nobs = length(ret.z[,1])
> se.muhat = sigmahat.vals/sqrt(nobs)
> se.sigma = sigmahat.vals/sqrt(2*nobs)
> se.rho = (1 - rhohat.vals^2)/sqrt(nobs)

> se.muhat
   dowjones       vbltx
0.003891324 0.002569509
> se.sigma
   dowjones       vbltx
0.002751582 0.001816917

> se.rho
dji,vbltx
0.1286128
```

The standard errors for the mean are about the same magnitude as the estimates for the mean indicating that the means are not estimated well. The standard errors for the standard deviations are much smaller than the estimated values and indicate that the standard deviations are estimated more precisely than the means. The standard error for the estimated correlation is quite large relative to the estimate and shows that the correlation is not estimated precisely.

f) For dji only compute 95% confidence intervals for μ and σ. Briefly comment on the precision of the estimates. In particular, note if both positive and negative values are in the respective confidence intervals.

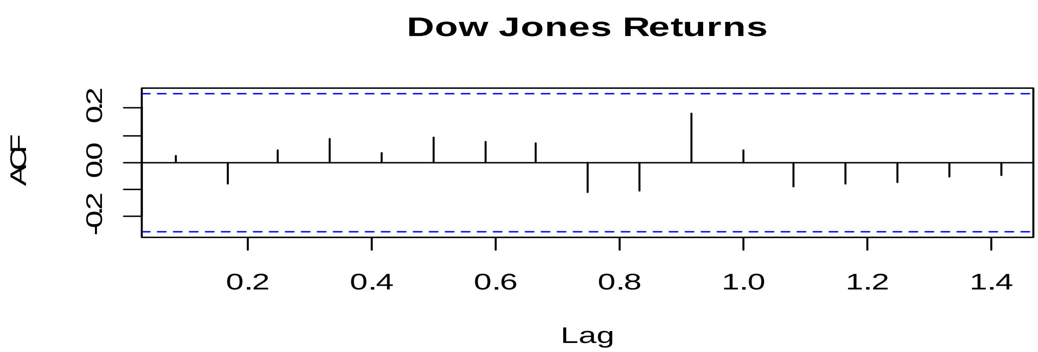
```
> # 95% ci for mu
```

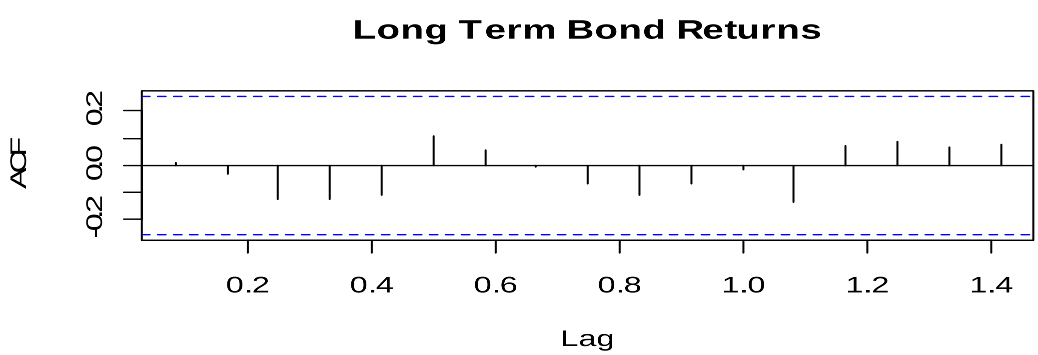
```
> upper = muhat.vals + 2*se.muhat
> lower = muhat.vals - 2*se.muhat
> cbind(lower[1],upper[1])
                 [,1]       [,2]
dowjones -0.005167701 0.01039760

> # 95% ci for sigma
> upper = sigmahat.vals + 2*se.sigma
> lower = sigmahat.vals - 2*se.sigma
> cbind(lower[1],upper[1])
               [,1]       [,2]
dowjones 0.02463890 0.03564523
```

The wide confidence interval for the mean containing positive and negative values indicates imprecise estimation. In contrast, the confidence interval for the sd is fairly narrow indicating a precise estimation.

g) Below are the sample autocorrelation functions for dji and vbltx. Using the information in these graphs, would you say that the CER model assumption that returns are uncorrelated over time is appropriate? Briefly justify your answer.





The estimates autocorrelations are small for both dji and vbltx. Note of the estimates are bigger than 0.2. There appears to be no compelling evidence for exploitable autocorrelation in these returns.

h) Suppose you currently hold $2M (million) in the Dow Jones Index. That is, your initial wealth at the beginning of the month is $W_0 = \$2M$. Using the estimates from the CER model compute the 1% and 5% value-at-risk (VaR) associated with a one-month investment in the Dow. Hint: the 1% and 5% quantiles of the standard normal distribution are -2.326 and -1.645.

```
> W0 = 2
> q.05 = muhat.vals + sigmahat.vals*qnorm(0.05)
> q.01 = muhat.vals + sigmahat.vals*qnorm(0.01)
> VaR.05 = (exp(q.05[1]) - 1)*W0
> VaR.01 = (exp(q.01[1]) - 1)*W0

> VaR.05
   dowjones
-0.09175716
> VaR.01
  dowjones
-0.1305558
```

i) Describe briefly how you could compute an estimated standard error for the estimate of 5% VaR found in part (h) above.

You could use bootstrapping to obtain an estimated SE for the 5% VaR. This involves sampling with replacement from the original data to create B different samples. On each sample the 5% VaR is computed, and the estimated bootstrap SE is the sample standard deviation of these B 5% VaR values.