University of Washington
Department of Economics

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Economics 424/Applied Mathematics 462

Final Exam Solutions

I. Matrix Algebra and Portfolio Math (36 points, 4 points each)

Let R_i denote the continuously compounded return on asset i (i = 1, …, N) with $E[R_i] = \mu_i$, var(R_i) = σ_i^2 and cov(R_i, R_j) = σ_{ij}. Define the ($N \times 1$) vectors $\boldsymbol{R} = \left(R_1,\ldots,R_N\right)'$, $\boldsymbol{\mu} = (\mu_1,\ldots,\mu_N)'$, $\boldsymbol{m} = \left(m_1,\ldots,m_N\right)'$, $\boldsymbol{x} = \left(x_1,\ldots,x_N\right)'$, $\boldsymbol{y} = \left(y_1,\ldots,y_N\right)'$, $\boldsymbol{t} = \left(t_1,\ldots,t_N\right)'$, $\mathbf{1} = \left(1,\ldots,1\right)'$ and the ($N \times N$) covariance matrix

$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{12} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1N} & \sigma_{2N} & \cdots & \sigma_N^2 \end{pmatrix}.$$

The vectors $\boldsymbol{m}$, $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{t}$ contain portfolio weights that sum to one. Using simple matrix algebra, answer the following questions.

a. For the portfolios defined by the vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ give the expression for the portfolio returns, ($R_{p,x}$ and $R_{p,y}$), the portfolio expected returns ($\mu_{p,x}$ and $\mu_{p,y}$), the portfolio variances ($\sigma_{p,x}^2$ and $\sigma_{p,y}^2$), and the covariance between $R_{p,x}$ and $R_{p,y}$ (σ_{xy}).

$$\begin{aligned} R_{p,x} &= \boldsymbol{R}'\boldsymbol{x},\ \mu_{p,x} = \boldsymbol{x}'\boldsymbol{\mu},\ \sigma_{p,x}^2 = \boldsymbol{x}'\boldsymbol{\Sigma}\boldsymbol{x} \\ R_{p,y} &= \boldsymbol{R}'\boldsymbol{y},\ \mu_{p,y} = \mathbf{y}'\boldsymbol{\mu},\ \sigma_{p,y}^2 = \mathbf{y}'\boldsymbol{\Sigma}\boldsymbol{y} \\ \sigma_{xy} &= \mathbf{x}'\Sigma\mathbf{y} \end{aligned}$$

b. For portfolio $\boldsymbol{x}$, derive the n x 1 vector of marginal contributions to portfolio volatility defined by

$$\mathbf{MCR}^{\sigma} = \frac{\partial \sigma_p(\mathbf{x})}{\partial \mathbf{x}}.$$

By the chain rule we have

$$
\begin{aligned}
\frac{\partial \sigma_p(\mathbf{x})}{\partial \mathbf{x}} &= \frac{\partial}{\partial \mathbf{x}}\left(\mathbf{x}'\boldsymbol{\Sigma}\mathbf{x}\right)^{1/2} = \frac{1}{2}\left(\mathbf{x}'\boldsymbol{\Sigma}\mathbf{x}\right)^{-1/2} 2\boldsymbol{\Sigma}\mathbf{x} \\
&= \frac{\boldsymbol{\Sigma}\mathbf{x}}{\left(\mathbf{x}'\boldsymbol{\Sigma}\mathbf{x}\right)^{1/2}} = \frac{\boldsymbol{\Sigma}\mathbf{x}}{\sigma_p(\mathbf{x})}
\end{aligned}
$$

c. Write down the optimization problem and give the Lagrangian used to determine the global minimum variance portfolio assuming short sales are allowed. Let ***m*** denote the vector of portfolio weights in the global minimum variance portfolio.

$$
\begin{aligned}
&\min_{m}\ \boldsymbol{m}'\boldsymbol{\Sigma}\boldsymbol{m} \text{ s.t. } \boldsymbol{m}'\boldsymbol{1} = 1 \\
&L(\mathbf{m},\lambda) = \boldsymbol{m}'\boldsymbol{\Sigma}\boldsymbol{m} + \lambda(\boldsymbol{m}'\boldsymbol{1} - 1)
\end{aligned}
$$

d. Continuing with part b., derive the first order conditions for determining the minimum variance portfolio ***m***. Write these first order conditions as a system of linear equations in the form **Az = b** and show how the portfolio **m** can be determined from this system.

The first order conditions are

$$
\begin{aligned}
\frac{\partial L}{\partial \mathbf{m}} &= 2\boldsymbol{\Sigma}\mathbf{m} + \lambda \cdot \mathbf{1} = \mathbf{0} \\
\frac{\partial L}{\partial \lambda} &= \mathbf{m}'\mathbf{1} - 1 = 0
\end{aligned}
$$

These first order conditions can be written in matrix form as

$$
\begin{bmatrix} 2\boldsymbol{\Sigma} & \mathbf{1} \\ \mathbf{1}' & 0 \end{bmatrix} \begin{bmatrix} \mathbf{m} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} \text{ or } \mathbf{Az} = \mathbf{b}
$$

This linear system has solution

$$
\mathbf{z} = \mathbf{A}^{-1}\mathbf{b}
$$

*The vector **m** is the first n elements of the n+1 x 1 vector **z**.*

e. Write down the optimization problem and give the Lagrangian used to determine an efficient portfolio with target return equal to μ_0 assuming short sales are allowed. Let ***x*** denote the vector of portfolio weights in the efficient portfolio.

$$
\begin{aligned}
&\min_{x}\ \boldsymbol{x}'\boldsymbol{\Sigma}\boldsymbol{x} \text{ s.t. } \boldsymbol{x}'\boldsymbol{1} = 1 \text{ and } \boldsymbol{x}'\boldsymbol{\mu} = \mu_0 \\
&L(\mathbf{x},\lambda_1,\lambda_2) = \boldsymbol{x}'\boldsymbol{\Sigma}\boldsymbol{x} + \lambda_1(\boldsymbol{x}'\boldsymbol{1} - 1) + \lambda_2(\boldsymbol{x}'\boldsymbol{\mu} - \mu_0)
\end{aligned}
$$

f. Briefly describe how you would compute the efficient frontier containing only risky assets (Markowitz bullet) when short sales are allowed.

Because short sales are allowed, the Markowitz bullet can be constructed using a convex combination of any two efficient (frontier) portfolio: $\mathbf{z} = \alpha \cdot \mathbf{m} + (1-\alpha) \cdot \mathbf{x}$ *, where* $\boldsymbol{m}$ *and* $\boldsymbol{x}$ *are any two frontier portfolios. To get a nice picture, choose portfolio* $\mathbf{m}$ *to be the global minimum variance portfolio and choose the other efficient portfolio* $\mathbf{x}$ *to be the efficient portfolio with target return equal to the highest expected return of the available assets. Then, for example, vary* α *from 1 to -1 in increments of 0.1 and compute z. For each z, compute* $\mu_{p,z} = \mathbf{z}'\boldsymbol{\mu}$ *and* $\sigma_{p,z} = \left(\mathbf{z}'\boldsymbol{\Sigma}\mathbf{z}\right)^{1/2}$ *and then plot these pairs of points.*

g. Write down the optimization problem used to determine the tangency portfolio, assuming short sales are *not* allowed and the risk free rate is given by r_f. Let $\boldsymbol{t}$ denote the vector of portfolio weights in the tangency portfolio.

$$\max_{t} \frac{\boldsymbol{t}'\boldsymbol{\mu} - r_f}{\left(\boldsymbol{t}'\boldsymbol{\Sigma}\boldsymbol{t}\right)^{1/2}} \text{ s.t. } \boldsymbol{t}'\boldsymbol{1} = 1 \text{ and } t_i \geq 0, i = 1, \ldots, N$$

h. Continuing with part g., is there an analytical solution (i.e., matrix algebra mathematical formula) for the tangency portfolio when short sales are not allowed?

There is no analytical solution for the tangency portfolio when short sales are not allowed. The inequality constraints do no allow the method of Lagrange multipliers to be used to determine an analytic solution using matrix algebra.

i. Write down the equations for the expected return (μ_p^e) and standard deviation (σ_p^e) of efficient portfolios consisting of the tangency portfolio and T-bills, where the T-bill rate (risk-free rate) is given by r_f and $\boldsymbol{t}$ denotes the vector of portfolio weights in the tangency portfolio.

$$\mu_p^e = r_f + x_{\tan}(\mu_{\tan} - r_f),\ \mu_{\tan} = \mathbf{t}'\mu$$
$$\sigma_p^e = x_{\tan}\sigma_{\tan},\ \sigma_{\tan}^2 = \mathbf{t}'\Sigma\mathbf{t}$$

II. Efficient Portfolios (32 points points, 4 points each)

The graph below shows the efficient frontier computed from three Vanguard mutual funds: Pacific Stock Index (vpacx), US Long Term Bond Index (vbltx), and Emerging Markets Fund (veiex).

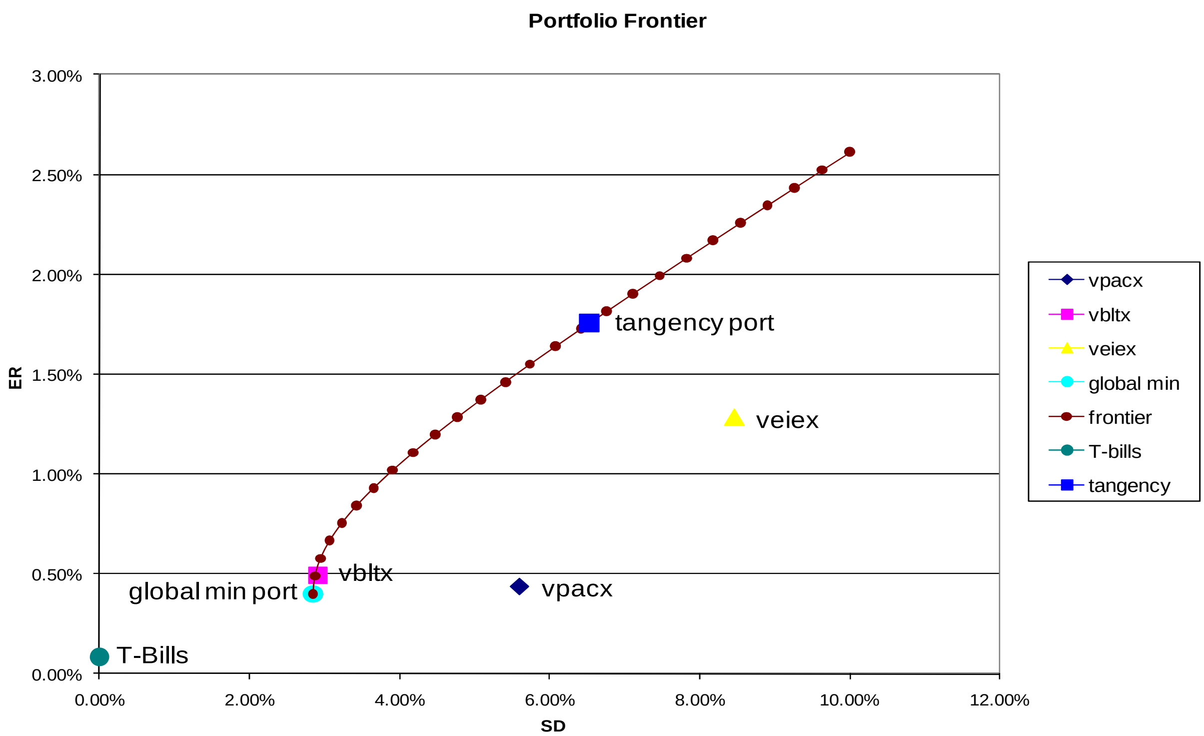


Figure 1 Markowitz Bullet

Expected return and standard deviation estimates for specific assets are summarized in the table below. These estimates are based on monthly *continuously compounded* return data over the five year period September 2004 – September 2009.

Table 1 Portfolio Statistics

Asset	Mean E[R])	Standard deviation (SD(R))	Weight in Global Min Portfolio	Weight in Efficient Portfolio Mean = 1.28%	Weight in Tangency portfolio
VPACX	0.43%	5.59%	23%	-120%	-197%
VBLTX	0.49%	2.90%	87%	129%	151%
VEIEX	1.28%	8.45%	-10%	91%	145%
T-Bills	0.08%	0%			
Global Min Portfolio	0.40%	2.84%			

Efficient Portfolio with Mean=1.28%	1.28%	4.77%			
Tangency Portfolio	1.76%	6.53%			

Using the above information, please answer the following questions.

a. Compute annualized means and standard deviations from the monthly statistics in Table 1 for the three portfolios vpacx, vbltx, and veiex (Remember, the returns are continuously compounded)

The annualized cc mean return is $\mu_A = 12 \cdot \mu_m$. *For the three portfolios we have*

$$\begin{aligned}
\mu_{A,vpacx} &= 12 \cdot (.0043) = .0516 \\
\mu_{A,vbltx} &= 12 \cdot (.0049) = .0588 \\
\mu_{A,veiex} &= 12 \cdot (.0128) = .1536
\end{aligned}$$

The annualized cc standard deviation is $\sigma_A = \sqrt{12}\sigma_m$

$$\begin{aligned}
\sigma_{A,vpacx} &= \sqrt{12} \cdot (.0559) = .1936 \\
\sigma_{A,vbltx} &= \sqrt{12} \cdot (.0290) = .1005 \\
\sigma_{A,veiex} &= \sqrt{12} \cdot (.0845) = .2927
\end{aligned}$$

The annualized T-Bill rate is $r_{A,f} = 12 \cdot 0.0008 = 0.01$

b. Using the annualized information from part a., compute the annualized Sharpe ratios/slopes for each of the three portfolios. Which portfolio is ranked best using the Sharpe ratio?

The annualized Sharpe ratio is

$$SR_A = \frac{\mu_A - r_{f,A}}{\sigma_A},\ r_{f,A} = 12 \cdot r_{f,m}$$

Using the results from part a., we get

$$\begin{aligned}
SR_{A,vpacx} &= \frac{0.0516 - 0.01}{0.1936} = 0.2149 \\
SR_{A,vbltx} &= \frac{0.0588 - 0.01}{0.1005} = 0.4856 \\
SR_{A,veiex} &= \frac{0.1536 - 0.01}{0.2927} = 0.4906
\end{aligned}$$

The asset with the highest annual Sharpe ratio is veiex.

c. Find the efficient portfolio of risky assets only (e.g. a portfolio on the Markowitz bullet) that has an expected monthly return equal to 1%. In this portfolio, how much is invested in vpacx, vbltx, and veiex?

Here, we make use of the fact that the global minimum variance portfolio and the tangency portfolio are on the Markowitz bullet. Therefore, the expected return for any portfolio on the Markowitz bullet can be expressed as

$$\mu_{p,z} = \alpha\mu_{p,m} + (1-\alpha)\mu_{p,t}$$

Setting $\mu_{p,z} = 0.01$ *and using* $\mu_{p,m} = 0.0040,\ \mu_{p,t} = 0.0176$ *we can solve for* α:

$$\alpha = \frac{\mu_{p,z} - \mu_{p,t}}{\mu_{p,m} - \mu_{p,t}} = \frac{0.01 - 0.0176}{0.0040 - 0.0176} = 0.56$$
$$1 - \alpha = 0.44$$

The weights on vfinx, veurx and vbltx in this portfolio are

$$\begin{aligned} \boldsymbol{z} &= \alpha\boldsymbol{m} + (1-\alpha)\mathbf{t} \\ &= (0.56)\begin{pmatrix} .23 \\ .87 \\ -.10 \end{pmatrix} + (0.44)\begin{pmatrix} -1.97 \\ 1.51 \\ 1.45 \end{pmatrix} = \begin{pmatrix} -.74 \\ 1.16 \\ .59 \end{pmatrix} \end{aligned}$$

We can get the same answer if we use the global minimum variance portfolio and the efficient portfolio with expected return equal to 1.28%. The calculations are essentially the same:

$$\mu_{p,z} = \alpha\mu_{p,m} + (1-\alpha)\mu_{p,e}$$

Setting $\mu_{p,z} = 0.01$ *and using* $\mu_{p,m} = 0.0040,\ \mu_{p,e} = 0.0128$ *we can solve for* α:

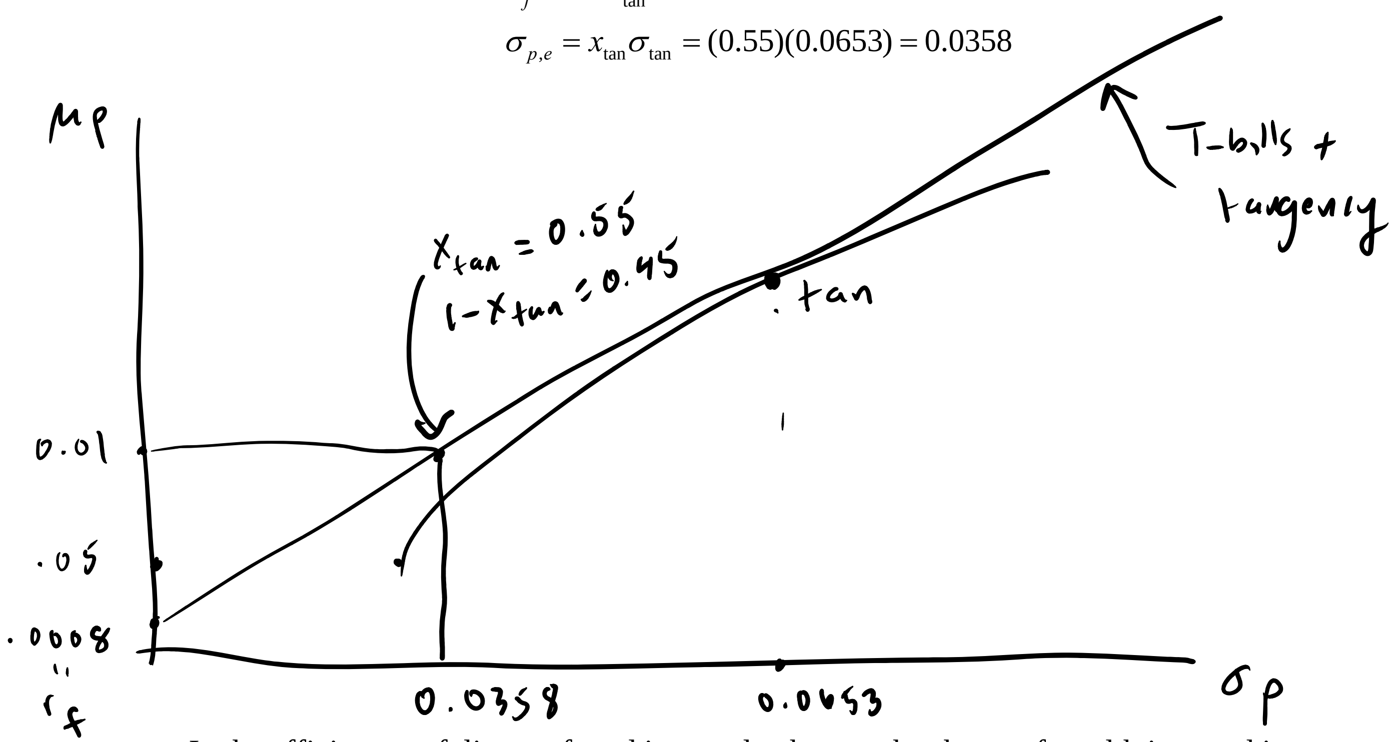
$$\alpha = \frac{\mu_{p,z} - \mu_{p,e}}{\mu_{p,m} - \mu_{p,e}} = \frac{0.01 - 0.0128}{0.0004 - 0.0128} = 0.32$$
$$1 - \alpha = 0.68$$

The weights on vfinx, veurx and vbltx in this portfolio are

$$
\begin{aligned}
\boldsymbol{z} &= \alpha \boldsymbol{m} + (1-\alpha)\boldsymbol{x} \\
&= (0.32)\begin{pmatrix} .23 \\ .87 \\ -.10 \end{pmatrix} + (0.68)\begin{pmatrix} -1.20 \\ 1.29 \\ .91 \end{pmatrix} = \begin{pmatrix} -.74 \\ 1.16 \\ 0.59 \end{pmatrix}
\end{aligned}
$$

d. How much should be invested in T-bills and the tangency portfolio to create an efficient portfolio with expected return equal to 1%? What is the standard deviation of this efficient portfolio? Transfer the graph of the Markowitz bullet to your blue book and indicate the location of this efficient portfolio on the graph.

$$
\begin{aligned}
x_{\tan} &= \frac{.01 - r_f}{\mu_{\tan} - r_f} = \frac{.01 - 0.0008}{.0176 - 0.0008} = 0.55 \\
x_f &= 1 - x_{\tan} = 1 - 0.55 = 0.45 \\
\sigma_{p,e} &= x_{\tan}\sigma_{\tan} = (0.55)(0.0653) = 0.0358
\end{aligned}
$$



e. In the efficient portfolio you found in part d, what are the shares of wealth invested in T-Bills, vpacx, vbltx, and veiex?

vpacx	vbltx	veiex	T-Bills
0.55*(-1.97)=-1.08	0.55*(1.51)=0.83	0.55*(1.45)=0.80	0.45

f. Assuming an initial $100,000 investment for one month, compute the 5% value-at-risk on the global minimum variance portfolio.

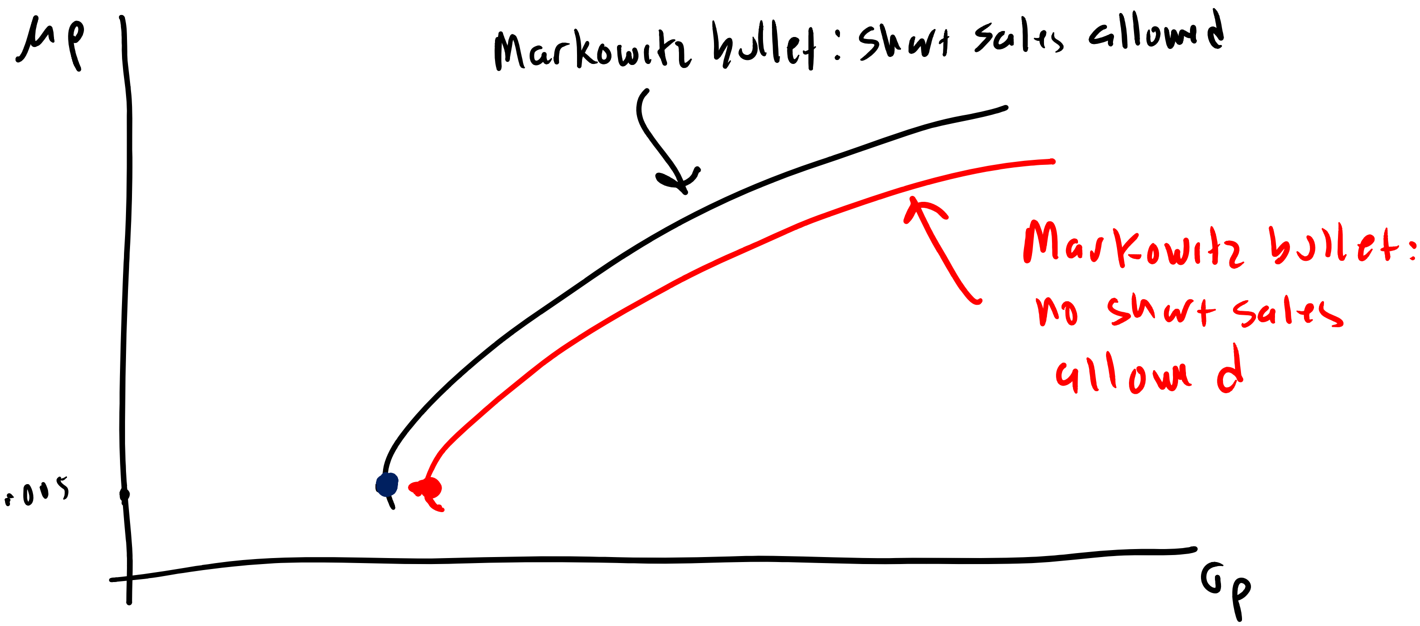
First we find the 5% quantiles for the global minimum variance portfolio

$$port: \; q_{.05} = 0.0004 + (0.0284)(-1.645) = -0.0427$$

Then we compute the 5% VaR

$$port: \; VaR_{.05} = \$100,000 \times \left(e^{-0.0427} - 1\right) = -\$4,182.95$$

g. The efficient frontier of risky assets shown in Figure 1 allows for short sales (see the weights in the portfolios listed in Table 1). Transfer this graph to your blue book. On this graph, indicate roughly the location of the efficient frontier of risky assets that *does not* allow short sales.



h. Suppose you want to find the efficient portfolio of risky assets only that has an expected monthly return equal to 2% but that you are prevented from short selling. Is it possible to find such an efficient portfolio? Briefly explain why or why not.

It is not possible to find an efficient portfolio of risky assets only that has an expected return of 2%. To see this, from Table 1 above notice that the efficient portfolio with expected return equal to 1.28% has a short position in vpacx and the tangency portfolio (with mean 1.78%) has a short position in vpacx as well. Hence, for an efficient portfolio with mean 2% we would need a short position in vpacx.

III. Empirical Analysis of the single index model and the CAPM (36 points, 4 points each)

The single index model for asset returns has the form

$$
\begin{aligned}
&R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it} \\
&i = 1, \ldots, n \text{ assets} \\
&t = 1, \ldots, T \text{ time periods}
\end{aligned}
$$

where R_{it} denotes the cc return on asset *i* at time *t*, and R_{Mt} denotes the return on a market index portfolio at time *t*.

a. In the SI model, what is the interpretation of β_i?

β_i measures the contribution of asset i to the risk of the market index measured by the standard deviation of the market index. Explicitly,

$$
\beta_i = \frac{\text{cov}(R_{it}, R_{Mt})}{\text{var}(R_{Mt})}
$$

The following represents R linear regression output from estimating the single index model for the Vanguard Pacific Stock Index (vpacx), the Vanguard long-term bond index (vbltx) and the Vanguard Emerging Markets Fund (veiex) using monthly continuously compounded return data over the 5 year period September 2004 – September 2009. In the regressions, the market index is the Vanguard S&P 500 index (vfinx).

```
> summary(vpacx.fit)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.00356    0.00385    0.93     0.36
vfinx        1.00139    0.08194   12.22   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0298 on 58 degrees of freedom
Multiple R-squared: 0.72,      Adjusted R-squared: 0.715
F-statistic:  149 on 1 and 58 DF,  p-value: <2e-16

> summary(vbltx.fit)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.00479    0.00364    1.32    0.193
vfinx        0.16639    0.07744    2.15    0.036 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 0.0282 on 58 degrees of freedom
Multiple R-squared: 0.0737,     Adjusted R-squared: 0.0578
F-statistic: 4.62 on 1 and 58 DF,  p-value: 0.0358



> summary(veiex.fit)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.0117     0.0057    2.04    0.046 *
vfinx         1.5262     0.1214   12.57   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0442 on 58 degrees of freedom
Multiple R-squared: 0.732,      Adjusted R-squared: 0.727
F-statistic:  158 on 1 and 58 DF,  p-value: <2e-16
```

b. Make a table (see example below) showing the estimated values of β, its estimated standard error, the estimate of σ_{ε}, and the R^2 values from the three regression equations. How accurate are the estimates of β?

Asset	β	SE(β)	σ_{ε}	R^2
vpacx	1.001	0.08194	0.0298	0.72
vbltx	0.16639	0.07744	0.0282	0.0737
veiex	1.5262	0.1214	0.0442	0.732

SE(β) is largest for veiex and smallest for vbltx. However, the values of SE(β) are fairly small for vpacx and veiex relative to the value of β, so I would say that β is estimated reasonably precisely for these assets. However, the value of SE(β) for vbltx is about half the value of β and so β is not estimated as precisely.

c. What can you say about the risk characteristics of the three assets relative to the S&P 500 index (vfinx)?

Using β as a measure of risk, we see that vbltx has the lowest risk and veiex has the highest risk. The bond index vbltx is the most beneficial asset to hold in terms of diversification since it has the lowest β and the lowest R^2.

d. For each asset, give the percentage of total risk due to the market (non-diversifiable risk) and the percentage not due to the market (diversifiable risk). Which asset is most beneficial to hold in terms of diversification?

The market risk is measured by R^2 *and non-market risk is measured by* $1 - R^2$. *The following table summarizes the results for the three assets*

Asset	R^2 (market risk)	$1-R^2$ (non-market risk)
vpacx	0.72	0.28
vbltx	0.0737	0.9263
veiex	0.732	0.268

e. For each asset, test the null hypothesis that $\beta = 1$ against the alternative that $\beta \neq 1$ using a 5% significance level. What do you conclude?

$$vpacx: t_{\beta=1} = \frac{\hat{\beta}-1}{SE(\hat{\beta})} = \frac{1.001309-1}{.08194} = 0.0169$$

$$vbltx: t_{\beta=1} = \frac{\hat{\beta}-1}{SE(\hat{\beta})} = \frac{0.16639-1}{.07744} = -10.76$$

$$veiex: t_{\beta=1} = \frac{\hat{\beta}-1}{SE(\hat{\beta})} = \frac{1.5262-1}{.1214} = 4.33$$

We reject the null hypothesis that $\beta = 1$ against the alternative that $\beta \neq 1$ using a 5% significance level whenever $|t_{\beta=1}| > 2$. Therefore, we reject the null for vbltx and veiex but not for vpacx.

If an asset has $\beta = 1$ then it has the same risk as the market index: adding the asset to the market index does not change the volatility of the index.

f. Using the information listed in Table 1 (from Portfolio theory section), what is the β of the global minimum variance portfolio?

$$\begin{aligned}\beta_p &= m_{vpacx}\beta_{vpacx} + m_{vbltx}\beta_{vbltx} + m_{veiex}\beta_{veiex} \\ &= (.23)(1.001) + (.87)(0.1664) + (-.10)(1.5262) = 0.22\end{aligned}$$

g. Using the information in the table in question b. above and the standard deviation of vfinx of 0.04737, show how you could estimate the 3 x 3 covariance matrix of the return vector consisting of vpacx, vbltx and veiex using the single index model where the market return is vfinx. You only have to write out the matrix algebra formula with the appropriate information from the table in the vectors and matrices. You do not have to numerically compute the covariances.

Using matrix algebra, the single index covariance has the form

$$\Sigma_{SI} = \sigma_M^2 \boldsymbol{\beta}\boldsymbol{\beta}' + \mathbf{D}$$

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_{vpacx} \\ \beta_{vbltx} \\ \beta_{veiex} \end{pmatrix}, \mathbf{D} = \begin{pmatrix} \sigma^2_{\varepsilon,vpacx} & 0 & 0 \\ 0 & \sigma^2_{\varepsilon,vbltx} & 0 \\ 0 & 0 & \sigma^2_{\varepsilon,veiex} \end{pmatrix}$$

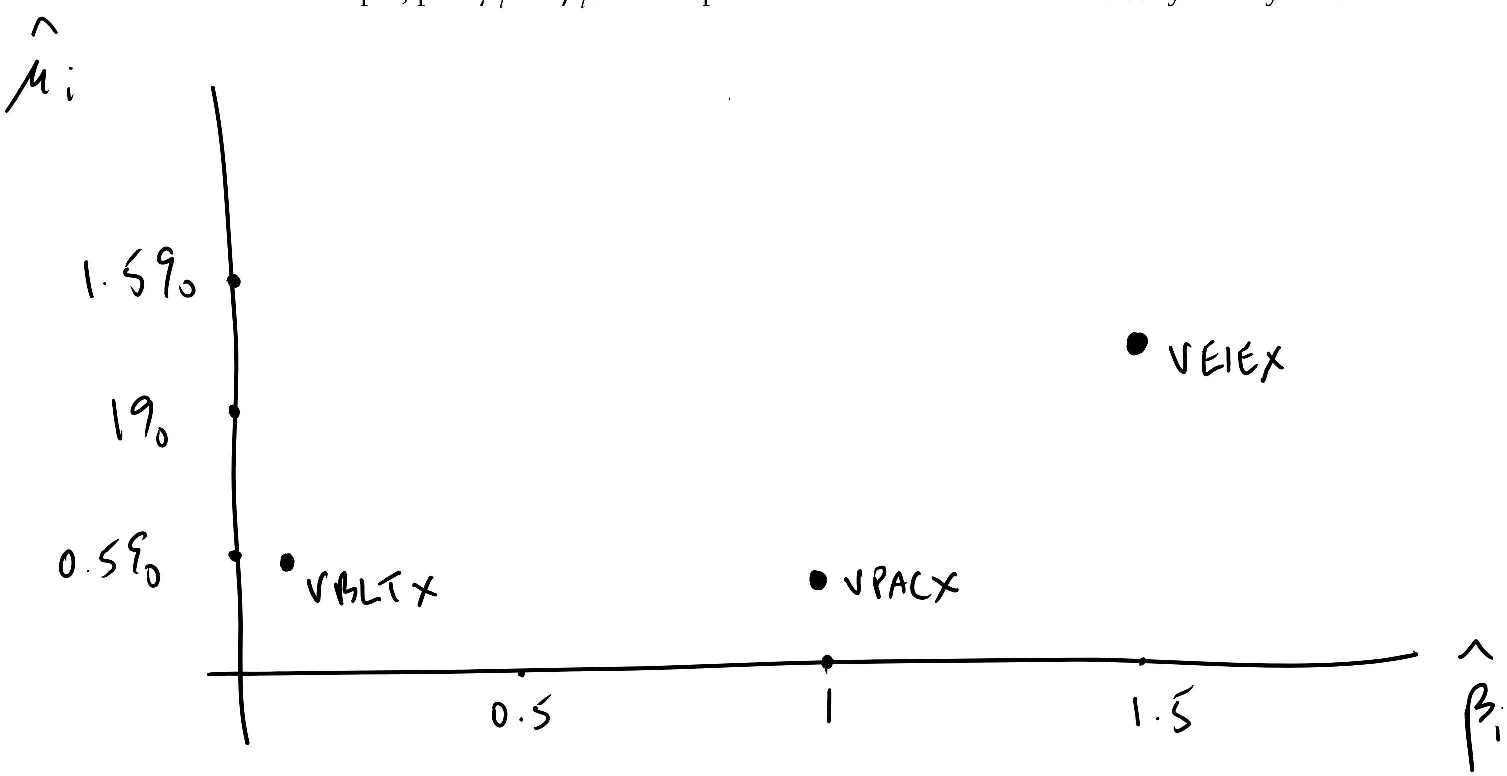
Using the information from the table in question b we have

$$\sigma_M^2 = (0.04737)^2,\ \boldsymbol{\beta} = \begin{pmatrix} 1.001 \\ 0.16639 \\ 1.5262 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} (0.0298)^2 & 0 & 0 \\ 0 & (0.0282)^2 & 0 \\ 0 & 0 & (0.0442)^2 \end{pmatrix}$$

h. In the Capital Asset Pricing Model (CAPM), what is the relationship between the expected return μ_i on any asset and its risk as measured by β_i?

$$E[R_i] = r_f + \beta_i (E[R_M] - r_f)$$

i. Using the estimated expected returns $\hat{\mu}_i$ from Table 1 above and the estimated $\hat{\beta}_i$ values from the R output, plot $\hat{\mu}_i$ vs. $\hat{\beta}_i$. Is this plot consistent with the CAPM? Why or why not?



The CAPM predicts an exact upward sloping linear relationship between average return and beta (with slope equal to the market risk premium). Here we don't see an exact linear relationship *but the asset with the highest beta (veiex) does have the highest average return. However, the asset with the lowest beta (vbltx) has an average return slightly higher than vpacx whose beta is 1. Due to estimation errors in average returns and betas it is not possible to rule out the CAPM relationship from the above plot.*