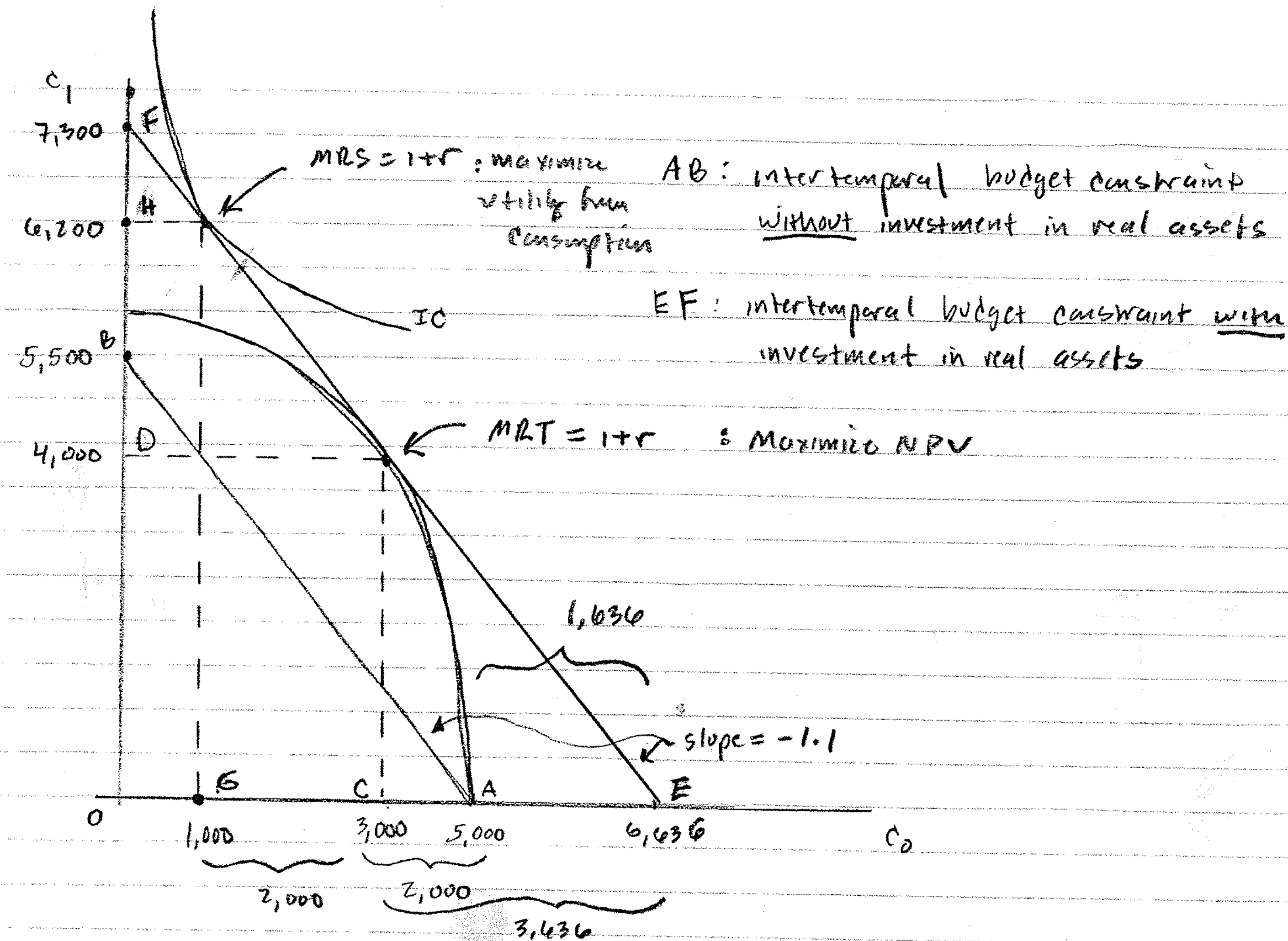
Example: Fisher Separation



r = 10% = annual interest rate on 1 yr risk free bonds

OA : 5,000 = initial wealth w_0 = max cons. this yr. without inv in real assets

OB : 5,500 = maximum consumption next year without inv. in real assets $w_0(1+r)$

CA : 2,000 = optimal investment (max NPV) $I^* = w_0 - x_0^*$

OD : 4,000 = return from optimal investment x_1^*

CE : 3,636 = PV of x_1^* = $\frac{x_1^*}{1+r}$

OE : 6,636 = max consumption this yr, with inv. in real assets : $x_0^* + \frac{x_1^*}{1+r}$

OF : 7,300 = max " next yr " $x_1^* + (1+r)x_0^*$

AE : 1,636 = NPV of optimal investment : $\frac{x_1^*}{1+r} - I^* = x_0^* + \frac{x_1^*}{1+r} - w_0$

OG : 1,000 = optimal current consumption c_0^*

GC : 2,000 = current lending $x_0^* - c_0^*$

OH : 6,200 = optimal cons. next period = c_1^*

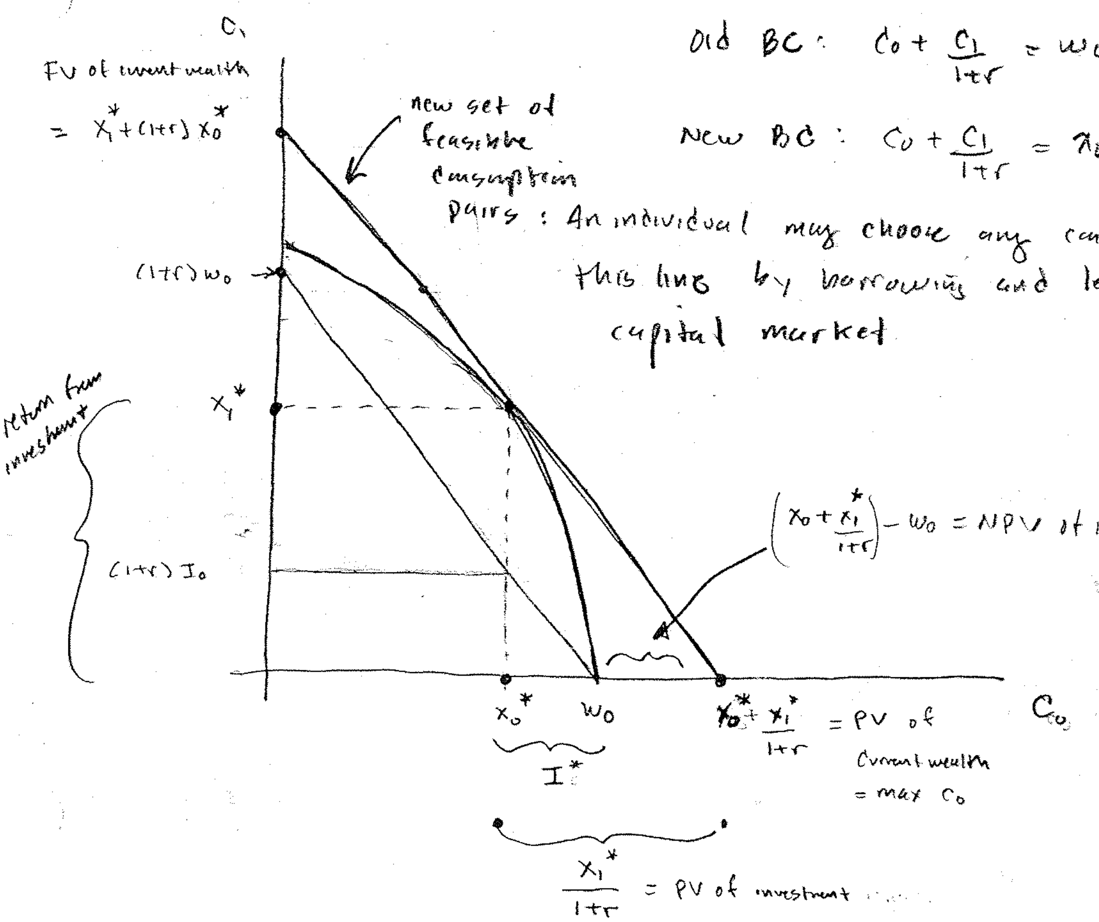
DH : 2,200 = FV of current lending = $(1+r)(x_0^* - c_0)^*$

Fisher Separation: (1) optimal investment maximizes NPV independently of the optimal consumption decision

Summary: Fisher model with Production

Old BC: $c_0 + \frac{c_1}{1+r} = w_0$

New BC: $c_0 + \frac{c_1}{1+r} = x_0^* + \frac{x_1^*}{1+r}$



(1) $x_0^* + \frac{x_1^*}{1+r}$ = maximum current wealth (consumption)

$x_1^* + (1+r)\, x_0^*$ = maximum end of period wealth (consumption)

(2) PV of investment$_0$ = $\frac{x_1^*}{1+r}$

(3) Cost of investment $= I_0 = w_0 - x_0^* \Rightarrow w_0 = I_0 + x_0$

(4) NPV of investment = PV − Cost $= \frac{x_1^*}{1+r} - w_0 + x_0^* = \left(x_0^* + \frac{x_1^*}{1+r}\right) - w_0$

(5) Current wealth is maximized if NPV is maximized!