14. Distinguish between the costs of transferring goods from one individual to another and exchange costs. How would you class transportation of goods between producer and consumer? What about the costs of negotiating a contract? Enforcing a contract?

For Further Thought and Discussion

1. Omar Khayyam wrote:

   I often wonder what the Vintners buy
   One half so precious as the Goods they sell.

   Khayyam seems to be suggesting that vintners ought not engage in exchange, because wine is more precious than anything else. Where is the fallacy in his reasoning?

†2. Compare the likely effects of taxes levied upon consumption, upon production, and upon exchange of a commodity.

†3. Suppose Robinson Crusoe is superior to Friday in producing both fish and bananas. For example, it may take Crusoe one hour to catch a fish and two hours to pick a bunch of bananas, whereas it takes Friday four hours to do either. Show that they still can engage in mutually beneficial exchange. (In international trade this is called *the principle of comparative advantage.*)

†4. Imagine an initial Edgeworth-box competitive equilibrium, starting from an endowment position where individual i has all of commodity G (grain) and individual j has all of commodity Y (the *numeraire*). Suppose i's endowment of grain doubles, everything else remaining unchanged. Can we be sure that i is better off at the new competitive equilibrium? That j is better off? That at least one of them is better off? (*Hint:* Are wheat farmers necessarily better off if the crop is large? What about consumers of wheat?)

5. Give examples of markets that have existed historically, but do not now exist. Can you explain their disappearance?

6. According to the economist Kenneth E. Boulding, a tariff can be regarded as a "negative railroad." Whereas a railroad connects trading communities, a tariff separates them. Is the analogy valid?

†7. The text stated that, in a world of two commodities X and Y, trade would normally lead to greater quantities produced of both goods. Is this necessarily the case? Might there possibly be, say, greater production of X but reduced production of Y? Explain. (*Hint:* Consider the case where only trader i [Ida] has productive opportunities, whereas trader j [John] has a fixed endowment.)

†8. Flowers provide bees with nectar, while bees facilitate the pollination of flowers. Is this exchange?

9. Fresh fruits are cheaper at farm roadstands than in city markets. Does the price difference reflect transfer costs or trading costs, or are there elements of each?

10. Give examples of exchange costs that are reduced by the existence of money as a medium of exchange. Of money as a store of value.

†11. If rationing is introduced, money is no longer fully effective as a medium of exchange. What types of additional exchange costs emerge in a world where ration coupons and cash are *both* required in order to effectuate a transaction?

†12. According to elementary textbooks, a commodity selected to serve as money should be portable, divisible, storable, generally recognizable, and homogeneous. In terms of the discussion in this chapter, why are these desirable qualities? Can you think of other desirable qualities? (*Hint:* Think of cigarettes serving as money in a prisoner-of-war camp.)

Chapter 14

THE ECONOMICS OF TIME

Examples

So far we have studied two types of decisions: (1) how to spend income (what consumption goods to purchase) and (2) how to earn income (what amounts of resource services to offer on the market). This chapter examines another economic choice: (3) how to strike a balance between consuming in the present and consuming in the future.

To balance between present and future, people "save" and "invest." *Saving* is refraining from current consumption. *Investment* means actually creating new resources that will generate more income in the future. These resources can take the form of physical capital (tangible assets such as factories and machines) or human capital (e.g., a course of training that improves personal skills).

For an isolated Robinson Crusoe, the amounts saved and invested must correspond. Crusoe *saves* by not eating up all of his current corn crop; he *invests* by putting the saved corn into the ground as seed for next year. In an exchange economy, however, those who save and those who invest need not be the same people. Savers put money in the bank or purchase corporate securities; the financial system then transfers this purchasing power to investors for building a house, enlarging a factory, or acquiring a college education.[1] However, as will be seen, the aggregate *totals* of saving and investment must correspond.

Why save or invest rather than consume today? Only so you, or someone you designate, or your heirs, can consume more in the future. A person builds a house in order to obtain future shelter; an orchardist plants a tree to harvest fruit in the future. Similarly, it is in order to achieve higher future incomes that businesses construct physical capital and students take courses of training to build up their human capital.

Section 14.1 of this chapter covers the elements of the economics of time. As an extension of standard microeconomics, we will see how the interest rate serves as a kind of price in relation to time. The next two sections cover the logic of choices between present and future. Section 14.4 discusses an important practical topic: the criteria used by business firms and by government agencies for evaluating investment projects. In section 14.5 we will see how *money*, an artificial commodity serving as numeraire, affects investment and interest. Section 14.6 looks into why interest rates vary as among different financial instruments, for example, the rates on long-term versus short-term bonds. Finally, section 14.7 reexamines the fundamental considerations that cause saving and investment to be large or small and interest rates to be high or low.

[1] In ordinary discussions, this distinction between saving and investment is not always carefully maintained. A financial writer may recommend that readers "invest" in stocks or bonds or savings deposits—although, strictly speaking, buying such financial instruments represents saving, not investment.

14.1 ELEMENTS OF THE ECONOMICS OF TIME

For simplicity, suppose corn C is the only consumption good. The objects of choice are *dated* quantities of corn: C_0 (this year's corn), C_1 (corn one year from now), C_2 (corn two years from now), and so on. The corresponding prices will be denoted P_0, P_1, P_2, and so on. (*Note:* These are the prices *today* for corn to be delivered at the specified dates.) Let current corn C_0 be the numeraire or basis of pricing, so that $P_0 \equiv 1$.

The economics of time falls easily into place once it is realized that choosing between this year's corn and next year's corn (between C_0 and C_1) is completely parallel to the logic previously employed in choosing between this year's wheat and this year's manufactures. Just as supply and demand determines the price ratio between wheat and manufactures, supply and demand will determine the price ratio P_1/P_0 between this year's corn and next year's corn.

The *rate of interest* is related to this price ratio. Specifically, the current annual rate of interest r_1 is the *extra* amount of one-year future corn that has to be offered in the market per unit of current corn:

$$-\frac{\Delta c_1}{\Delta c_0} \equiv 1 + r_1 \tag{14.1}$$

The minus sign indicates that a person can get more current corn (Δc_0) only by giving up some future corn (Δc_1), and vice versa. Furthermore, because in market exchange the values offered and received must be the same, we know that $P_0 \Delta c_0 = P_1 \Delta c_1$. Substituting, and given that C_0 is the numeraire so that $P_0 \equiv 1$, we can write:

$$\frac{1}{1 + r_1} \equiv \frac{P_1}{P_0} \equiv P_1 \tag{14.1'}$$

Thus P_1, the value of one-year future corn, is "discounted" by the factor $1 + r_1$ relative to current corn. Equivalently, r_1 is the "premium" on current corn relative to future corn.

EXERCISE 14.1

(a) If the interest rate is $r_1 = 10$ percent, what is P_1, the price today of a one-year future claim to corn? (b) What if $r_1 = 100\%$? (c) If future claims become almost valueless (P_1 approaches zero), what would happen to r_1?

Answer (a) Using equation (14.1′), if $r_1 = 10\%$, then $P_1 \equiv 1/(1 + r_1) = \frac{1}{1.1} = 0.909$, approximately. (b) If $r_1 = 100\%$, then $P_1 = \frac{1}{2} = 0.5$. (c) As P_1 approaches zero, r_1 goes to infinity.

As a check upon understanding, consider the question: Are negative interest rates ($r_1 < 0$) possible? From equation (14.1′), negative interest rates mean that future claims are worth more than current claims in today's market. Although unusual, this is not impossible. It might come about if people anticipated great scarcities in the future. However, there is a limit: interest rates less than −100 percent are not possible. If $r < -1$, in equation (14.1′) either P_0 or P_1 would have to be negative—contradicting our assumption that current corn and future corn are both *goods* rather than bads.

So far we have considered only two periods: "now" (date 0) versus "one year from now" (date 1). More generally, individuals will be making choices about corn at various dates $C_0, C_1, C_2, \ldots$ in the light of prices $P_0, P_1, P_2, \ldots$—up to some future horizon T. So we need to generalize equations (14.1) and (14.1′). There are two different useful ways of doing this, as illustrated in Table 14.1. First, consider the successive year-to-year price ratios $P_1/P_0, P_2/P_1, \ldots, P_T/P_{T-1}$. These can be used to define the one-year *short-term* interest rates $r_1, r_2, \ldots, r_T$ shown in column 1 of the table. Here $1 + r_1$ is the discount factor for transactions between date 0 and date 1, $1 + r_2$ is the discount factor between date 1 and date 2, and so on.

Alternatively, consider the ratios $P_1/P_0, P_2/P_0, \ldots, P_T/P_0$, where P_0 appears in all the denominators. In column 2 of Table 14.1 these ratios are used to define the *long-term* interest rates $R_1, R_2, \ldots, R_T$ associated with transactions between date 0 and any future date up to T. Notice that R_2 is a kind of average of r_1 and r_2. Put another way, the worth today of a unit of corn two years off could be found by using either the long-term discounting formula for P_2/P_0 in column 2 of the table or else by successively using the short-term one-year discounting formulas for P_2/P_1 and P_1/P_0.

Table 14.1 Interest-Rate Equivalents

SHORT-TERM INTEREST RATES	*LONG-TERM INTEREST RATES*
$\frac{P_1}{P_0} = \frac{1}{1 + r_1}$	$\frac{P_1}{P_0} = \frac{1}{1 + R_1}$
$\frac{P_2}{P_1} = \frac{1}{1 + r_2}$	$\frac{P_2}{P_0} = \frac{1}{(1 + R_2)^2}$
$\ldots$	$\ldots$
$\frac{P_T}{P_{T-1}} = \frac{1}{1 + r_T}$	$\frac{P_T}{P_0} = \frac{1}{(1 + R_T)^T}$

EXERCISE 14.2

(a) Let $P_0 \equiv 1$, and suppose that $r_1 = 10\%$ and $r_2 = 20\%$. (a) What is the worth *at date 1* of a bushel of corn at date 2? (b) What is the worth *today* of a bushel of corn at date 2? (c) What is the implied long-term interest rate R_2?

Answer (a) We want to compute P_2/P_1. Using Table 14.1, when $r_2 = 20\%$ this ratio equals $\frac{1}{1.2} = 0.833$. (b) $P_2/P_0 = (P_2/P_1) \times (P_1/P_0)$, and in the previous exercise we found hat $P_1/P_0 = 0.909$ when $r_1 = 10\%$. Thus, a bushel two years off is worth $0.833 \times 0.909 = 0.758$ today. (c) Using column 2 of Table 14.1, set $0.758 = 1/(1 + R_2)^2$. The nu

The next two exercises illustrate the power of compound interest.

EXERCISE 14.3

(a) If $P_0 \equiv 1$ as usual, and supposing that all the short-term interest rates $r_1, r_2, \ldots$ are equal to some common value $r = 10\%$, what is the value of a bushel of corn to be received in 5 years? In 10 years? In 20 years? (b) Same questions, if $r = 20\%$?

Answer (Questions such as these are most easily answered by reference to the interest tables published in many financial and accounting texts.) (a) At 10%, a payment 5 years in the future is worth 0.683 today, a payment 10 years off is worth 0.386, and a payment 20 years off is worth 0.149. (b) At 20%, the corresponding numbers are 0.402, 0.162, and 0.026.

So, we see, as the length of the term increases, the force of compound interest more and more drastically erodes the value today of a future payment. Furthermore, the effect becomes disproportionately stronger at higher interest rates.

EXERCISE 14.4

Suppose you had to plan ahead whether to cut a tree for timber after 10 years or after 20 years. (a) If $r = 5\%$, how much greater would the timber value have to be at the later date to justify letting the tree grow that long? (b) If $r = 10\%$?

Answer (a) Using interest tables, at 5 percent a payment 10 years off is worth 0.614 today, whereas a payment 20 years off is worth only 0.377. To justify not cutting until the later date, the timber value at that date would have to be greater by at least the proportion $\frac{0.614}{0.377} = 1.63$, approximately. (b) At $r = 10\%$ the corresponding ratio is $\frac{0.463}{0.215} = 2.15$.

CONSUMPTION CHOICES OVER TIME: PURE EXCHANGE

The analysis that follows deals with choices between consumption now versus one year from now: between c_0 and c_1. Using the same kind of simplification as in chapter 13 (section 13.2), we can begin with a hypothetical "pure exchange" economy. In the intertemporal context, pure exchange means that no net investment is taking place. (Productive activities such as building a house or planting a tree are ruled out.) If aggregate investment is zero, then aggregate saving must also equal zero, which means that the saving of some people must be balanced by the "dissaving" of others. (If some are lending, others must be borrowing.) Intertemporal *productive* opportunities, and their implications for saving and investment, will be covered in the next section.

Borrowing-Lending Equilibrium with Zero Net Investment

Figure 14.1, which pictures choices between consumption this year and consumption next year (between C_0 and C_1), closely resembles the "optimum of the consumer" diagrams in chapter 4. Once again there are indifference curves U', U'', U''', and a

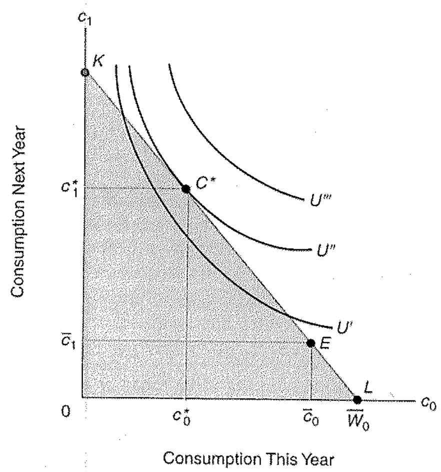


FIGURE 14.1
Consumption This Year versus Consumption Next Year

The choice between C_0 and C_1 involves the preference map (indifference curves U', U'', U'''), endowment E, and budget line KL. The intertemporal consumption optimum is C^*. The individual shown here chooses to lend the amount $\bar{c}_0 - c_0^*$ of current claims, receiving in repayment the amount $c_1^* - \bar{c}_1$ of future claims. $\overline{W}_0$ is the endowed wealth measured in units of current claims C_0.

endowment position on the vertical axis, here the endowment position $E \equiv (\bar{c}_0, \bar{c}_1)$ is in the interior. Thus, the individual—let us call him Karl—has an initial entitlement to positive amounts $\bar{c}_0$ of this year's corn and $\bar{c}_1$ of next year's corn.

Suppose Karl wants to consume more than his endowed $\bar{c}_0$ units of corn in the current period. Since productive investment is ruled out under the pure-exchange assumption, Karl can do so only by borrowing from someone else. If instead he prefers to consume more than $\bar{c}_1$ next year, he can achieve this only by lending to others. In the diagram, *lending* means moving northwest along the budget line KL from E—giving up current corn in exchange for future corn.[2] Moving southeast along KL, increasing c_0 at the expense of c_1, is *borrowing*. As pictured in Figure 14.1, the utility-maximizing consumption basket is the tangency point $C^* \equiv (c_0^*, c_1^*)$. Karl's optimum involves lending $\bar{c}_0 - c_0^*$ units of current corn, the anticipated future repayment being $c_1^* - \bar{c}_1$ units of next year's corn.

What if, with the same preferences and budget line, the endowment position E had instead been located northwest of C^* along the budget line? Karl would then borrow $c_0^* - \bar{c}_0$ and repay $\bar{c}_1 - c_1^*$. Such a person, with an endowment mainly in the form of future income, is like the "heir with great expectations" in Charles Dickens's novel: short of funds today, but in a position to borrow thanks to his future prospects.[3]

In the "optimum of the consumer" diagrams of chapter 4, the absolute slope of the budget line was $-\Delta y/\Delta x = P_x/P_y$, the price ratio between goods X and Y. Similarly here in Figure 14.1, the absolute slope of the budget line—the market rate at which individuals can trade current corn against future corn (can lend or borrow)—is $-\Delta c_1/\Delta c_0 = P_0/P_1$. From equation (14.1′) we know that this ratio also equals $1 + r_1$. Thus, a higher interest rate means a steeper slope of the budget line.

In chapter 4 it was the individual's *income* I that constrained choices, as shown by the budget equation $P_x x + P_y y = I$. That equation was valid only because of an implicit assumption that all decisions concerned only a single time period. For choices over time, it is not a single period's income that limits consumption choices but rather overall *wealth* W_0. More specifically, under the pure-exchange assumption, the constraint on consumption now and in the future is *endowed wealth* $\overline{W}_0$.

$$\overline{W}_0 \equiv P_0\bar{c}_0 + P_1\bar{c}_1 \tag{14.2}$$

(The subscript 0 is attached because wealth signifies a *present* market value, the worth today of a person's current and future income claims.)

DEFINITION: **Endowed wealth $\overline{W}_0$ is the present value of an individual's endowment of present and future claims.**

Since $P_0 \equiv 1$ and $P_1 \equiv 1/(1 + r_1)$, the definition in equation (14.2) can also be written:

$$\overline{W}_0 \equiv \bar{c}_0 + \frac{\bar{c}_1}{1 + r_1} \tag{14.2′}$$

Endowed wealth, as just defined, is the horizontal intercept of the budget line KL in Figure 14.1. The equation of the budget line can also be expressed in two ways:

$$P_0c_0 + P_1c_1 = \overline{W}_0 \tag{14.3}$$

$$c_0 + \frac{c_1}{1 + r_1} = \overline{W}_0 \tag{14.3′}$$

Equation (14.3) looks very much like (14.2), and (14.3′) looks very much like (14.2′). Do not confuse them, however. Equations (14.2) and (14.2′) *define* endowed wealth $\overline{W}_0$ (notice the identity sign $\equiv$) in terms of the endowments $\bar{c}_0$ and $\bar{c}_1$ as known constants. But in equations (14.3) and (14.3′) c_0 and c_1 are variables, chosen subject to the fixed wealth $\overline{W}_0$ constraint.

To find the market equilibrium, think in terms of supply and demand for current corn C_0 as functions of the rate of interest r_1. (The analysis here parallels section 13.2 of the preceding chapter). Karl's *supply of lending* is his *transaction supply of current corn.* If he is endowed with $\bar{c}_0$ units and chooses to consume only c_0 units, he will be lending the difference $\bar{c}_0 - c_0$. Similarly, his *demand for borrowing* is his *transaction demand for current corn,* the difference $c_0 - \bar{c}_0$. Whether he wants to be a borrower or a lender will depend upon the interest rate, as shown by the b curve (demand for borrowing) and the ℓ curve (supply of lending) in panel (a) of Figure 14.2.

Panel (b) shows the market aggregate B and L curves, the horizontal summations of the individual b and ℓ curves. The intersection of B and L determines the equilibrium amounts of borrowing and lending ($B^* = L^*$) and the equilibrium rate of interest r_1^*.

[2]As the term is used here, lending includes depositing funds in a savings account or purchasing financial instruments such as stocks and bonds.

[3]In nineteenth-century Britain there was a popular financial instrument known as the "post-obit." Prospective heirs could borrow, repayment being postponed until after succession to an entailed estate. (The *entail* certified to lenders that the borrower could not be disinherited, making him a relatively good credit risk despite possible improvident habits.)

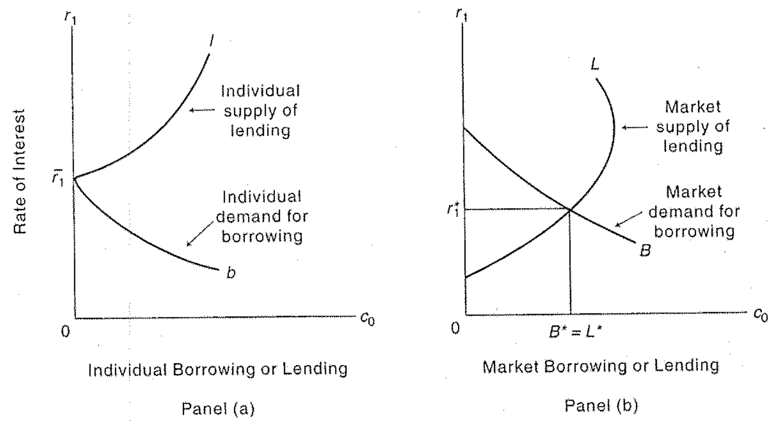


FIGURE 14.2

Supply of Lending, Demand for Borrowing

Panel (a) depicts an *individual's* supply of lending curve ℓ (the transaction supply of C_0) and demand for borrowing curve b (the transaction demand for C_0), as functions of the interest rate r_1. In panel (b) the intersection of the *market* supply of lending curve L and the *market* demand for borrowing curve B determines the equilibrium interest rate r_1^* and the amount of borrowing and lending $B^* = L^*$.

An Application: Double Taxation of Saving?

It has been asserted that income taxes tend to discourage saving. Income is taxed once when initially earned, but then any amount saved is taxed a second time when it generates earnings in future years. Is this contention valid? To resolve the issue we need to consider the effects upon consumption and saving choices.

Figure 14.3 shows an individual's (Karl's) indifference curves. The endowment position E is assumed to lie along the horizontal axis: $E \equiv (\bar{c}_0, 0)$. Therefore, Karl has to save if he is to make any provision at all for future consumption. In the absence of taxes his budget line would be EK with absolute slope $1 + r_1$. The line EK intercepts the vertical axis at $\bar{c}_0(1 + r_1)$, the maximum attainable amount of future corn. C^* is his consumption optimum.

To verify whether income taxation is biased against saving, we need a baseline for comparison. An appropriate baseline is a tax levied upon *consumption* rather than on income. (For the moment, assume that imposition of income or consumption taxes leaves the market interest rate r_1 unchanged.)

Consider a consumption tax at the rate of 50 percent. This is taken to mean that current endowed amounts *intended* for consumption (any amounts not saved) are reduced by one-half. (We might also think of this as a 100 percent tax on *actual* con

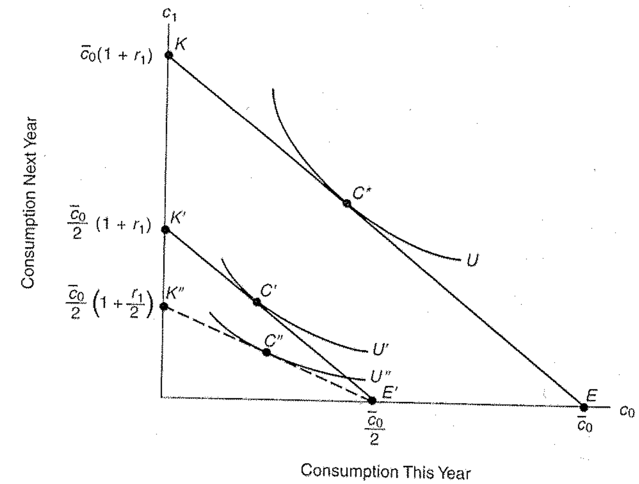


FIGURE 14.3

Consumption Tax versus Income Tax

A *consumption tax*, that bears equally heavily upon C_0 and C_1, shifts the budget line proportionately downward from EK to $E'K'$. The original consumption optimum C^* shifts to C'—present consumption, future consumption, and saving all fall about proportionately. An *income tax* bears upon all of current income (whether or not saved) as well as upon future consumption and income, so the budget line becomes $E'K''$. Comparing the income tax optimum C'' with the consumption tax optimum C', the amount of saving (horizontal distance between E' and C' or C'') may be about the same, but net provision for the future is less.

For such a consumption tax the relevant budget line is $E'K'$, where both intercepts of EK have been shifted halfway to the origin. If Karl were to maximize his current consumption and not save at all, his attainable c_0 would equal half of the endowed $\bar{c}_0$. If he chose instead to forgo current consumption entirely, saving as much as possible, the maximum attainable c_1 would be half of $\bar{c}_0(1 + r_1)$. Any intermediate choice would lie upon the line $E'K'$ connecting these two intercepts. As can be seen, a consumption tax has no systematic bias between current and future consumption. The new optimum C' would be shifted more or less proportionately inward to the origin from the original C^*.

If instead a 50 percent *income* tax is imposed, the relevant budget line would become $E'K''$. Why? The key point is that 50 percent of current income $\bar{c}_0$ has to be paid at date 0 *regardless* of the consumption/saving decision. (Whereas under a consumption tax, if Karl saves the maximum possible amount $\bar{c}_0/2$ no tax payment would be due would be $\ell r_1/2$, where ℓ is the amount lent (saved). In the extreme case where he saves as much as possible ($\ell = \bar{c}_0/2$), the net after-tax amount remaining is the intercept of the budget line $E'K''$ on the vertical axis: $\bar{c}_0/2 \times (1 + r_1/2)$. Finally, along this budget line the consumption optimum is shown as C''.

Returning to the original issue, as compared with a consumption tax is an income tax *biased against saving*? Is it *biased against future consumption*? It turns out these are two different questions. Geometrically, comparing C' (the optimum under the consumption tax) and C'' (the optimum under the income tax), it appears that the amount saved (as defined by the horizontal distance from point E') is about the same. However, C'' lies considerably below C', which means that provision for future consumption is considerably less.

We can interpret this result in terms of the *income effect* and *substitution effect* of the tax regarded as a price change. A 50 percent income tax is a heavier burden than a 50 percent consumption tax; the opportunity set for $E'K''$ is shrunken relative to $E'K'$. The income effect therefore suggests that Karl should want to consume less of both C_0 and C_1.[4] However, the income tax also makes C_1 more expensive relative to C_0 (flatter slope of the budget line), encouraging current consumption relative to future consumption. As the income and substitution effects *counterbalance* one another with regard to current consumption, we are not surprised that the amount of current saving might remain about the same. However, the income and substitution effects *reinforce* one another when it comes to future consumption.

▶ CONCLUSION

Income taxes might or might not discourage saving, compared with taxes on consumption, but they definitely reduce provision for the future.

A more complete analysis would next trace out the effects of the different tax systems upon the before-tax market interest rate, assumed to be constant in the foregoing discussion. (**Challenge to the Student:** Would a consumption tax tend to raise the rate of interest r_1? Would an income tax? *Hint:* Since a *consumption* tax reduces current and future consumption more or less proportionately, there would be no systematic effect upon r_1.)

Example 14.1

SCHOLARSHIPS AND SAVING

One of the most important motives for family saving is to finance children's college studies. Although higher education has become fiercely expensive, universities and colleges generally offer scholarships or other financial assistance to ease the burden. Such financial aids are almost always "means-tested"; that is, the richer the family, the less the assistance provided. It is not widely realized that the effect upon parents is like a *tax upon future income.*

[4]This assumes that current consumption and future consumption are both normal superior goods.

Using data from the 1986 Survey of Consumer Finances, Martin Feldstein[a] analyzed the "uniform methodology" employed until the late 1980s by the College Entrance Examination Board. This methodology was a formula for reducing college financial aid on the basis of family assets and income. Feldstein calculated that the 1986 aid reduction was equivalent to an implicit 22 percent tax on the first $7,300 of what was called "adjusted available income," rising to 47 percent above $14,500. (This "tax" is quite apart from ordinary federal and state income taxes.) The calculated aid reduction would then be multiplied by the number of children in college each year.

Thus, for families with college-bound children, provision for the future is heavily discouraged. (The more family saving, the greater the accumulated assets, and thus the less the financial aid.) Feldstein estimated that for a typical household—with a head aged 45 years, two precollege children, and an annual income of $45,000—the effect would be to reduce family assets at the date of first entry to college by $23,124. The reduction is about 50 percent of the amount of financial assets that would otherwise have been accumulated. For the nation as a whole, the reduction amounts to about $66 billion!

Comment

The effect upon saving is so large because, whereas ordinary income taxes bear upon both current and future income, "means-tested" financial aid penalizes *only* future assets and income. A family that consumes all its current income and makes absolutely no provision for their children's college education would entirely escape the burden of this implicit tax.

[a]Martin Feldstein, "College Scholarship Rules and Private Saving," *American Economic Review* 85 (June 1995).

PRODUCTION AND CONSUMPTION OVER TIME: SAVING AND INVESTMENT

Because there are no investment opportunities in a pure exchange economy, in equilibrium the savings of lenders have to equal the dissavings of borrowers. What happens when investment (productive) opportunities are available?

Figure 14.4 pictures the situation of an isolated Robinson Crusoe, who has *only* productive opportunities as indicated by the Production-Possibility Curve *QQ*. (Note the similarity to Figure 13.8 of the previous chapter.) For Crusoe, corn *consumed* (c_0 and c_1) and corn *produced* (q_0 and q_1) must be the same: he can consume only what he produces. His endowment *E* can therefore be written either as $(\bar{c}_0,\bar{c}_1)$ or $(\bar{q}_0,\bar{q}_1)$. Robinson's optimum occurs at point $R^* = (c_0^*,c_1^*) = (q_0^*,q_1^*)$, where the Production-Possibility Curve is tangent to his highest attainable indifference curve.

Crusoe's saving (corn not currently consumed) is the horizontal distance $\bar{c}_0 - c_1^*$. Because he cannot lend (there is no one to lend to), Crusoe saves only in order to have seed corn for planting, that is, for productive investment. His scale of plantings (investment) exactly equals his sacrificed consumption of current corn (saving): $\bar{c}_0 - c_1^* = \bar{q}_0 - q_1^*$. (The same holds true for an isolated country: in the absence of foreign trade, a country's investment must come from its own saving.) The return on investment takes the form of an increment of future corn: $q_1^* - \bar{q}_1 = c_1^* - \bar{c}_1$.

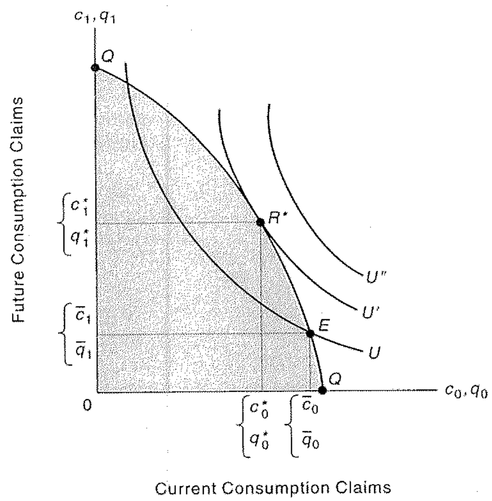


FIGURE 14.4
Robinson Crusoe Optimum over Time

Robinson Crusoe has no intertemporal exchange (borrowing-lending) opportunities, but can engage in productive transformations between consumption this year and consumption next year. *QQ* is the Production-Possibility Curve through his endowment *E*. The Crusoe optimum is at R^*, where *QQ* is tangent to the highest attainable indifference curve. This is an autarky solution: the amounts produced (q_0^*, q_1^*) equal the amounts consumed (c_0^*, c_1^*).

Figure 14.5 now pictures an individual—say, Ida—with access to *both* productive opportunities and market opportunities. (Compare Figure 13.9 in the preceding chapter.) As before, the productive opportunities are represented by the Production-Possibility Curve *QQ*. The market opportunities are shown by budget lines of slope $-P_0/P_1 \equiv -(1 + r)$ through attainable points on *QQ*. Each budget line is associated with a specific level of wealth as defined in:

$$W_0 \equiv q_0 + \frac{q_1}{1 + r_1} \tag{14.4}$$

In Figure 14.5, for each budget line the level of wealth W_0 is the intercept along the horizontal axis. Two of these budget lines are of special interest. *MM* through the endowment position *E* is like the budget line *KEL* of Figure 14.1: it shows what could be achieved by lending or borrowing alone. The horizontal intercept of *MM* is the endowed wealth $\overline{W}_0$. *NN* in Figure 14.5, the highest attainable budget line, is tangent to the Production-Possibility Curve *QQ* at point Q^*, that is, at Ida's production *optimum*. The horizontal intercept of *NN* is the *maximum attainable* level of wealth W_0^*.

$$W_0^* \equiv q_0^* + \frac{q_1^*}{1 + r_1} \tag{14.5}$$

Having maximized wealth by choosing the production optimum, Ida can then engage in market exchange (borrowing or lending) along *NN*, attaining her *consump-*

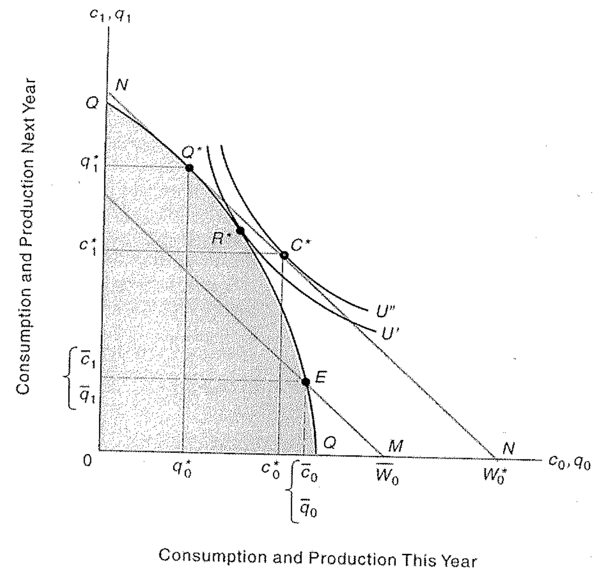


FIGURE 14.5
Intertemporal Production Optimum and Consumption Optimum, with Exchange

The individual here has intertemporal productive opportunities (the Production-Possibility Curve *QQ*) as well as exchange opportunities indicated by budget lines such as *MM* and *NN* of slope $-P_0/P_1 = -(1 + r)$. The production optimum Q^* involves investment in the amount $\bar{q}_0 - q_0^*$. The consumption optimum C^* indicates that the individual saves only $\bar{c}_0 - c_0^*$; the remainder of the investment is financed by borrowing in the market.

tion optimum at point C^*. (Note that C^* is on a higher indifference curve than R^*.) The budget-line equation corresponding to *NN* is:[5]

$$c_0 + \frac{c_1}{1 + r_1} = W_0^* \tag{14.6}$$

In Figure 14.5 Ida is *investing* (planting seed corn) in the amount indicated by the horizontal distance $\bar{q}_0 - q_0^*$. However, her *saving* is only the horizontal distance $\bar{c}_0 - c_0^*$. She "finances" the remainder of her investment by borrowing some of the savings (consumption sacrifices) of other individuals—to be repaid out of the investment yield $q_1^* - \bar{q}_1$.

From the picture in Figure 14.5 it would be possible to generate curves showing Ida's saving *s*, investment *i*, lending ℓ, and borrowing *b*—all as functions of the rate of interest r_1. We omit this step and move directly to the aggregate level of analysis. For the market as a whole the curves for *S*, *I*, *L*, and *B* in Figure 14.6 are the horizontal summations of the individual *s*, *i*, ℓ, and *b* curves. The market equilibrium is shown in two ways: (1) as a balance between *S* and *I* (between the supply of saving and the demand for investment), and (2) as a balance between *L* and *B* (between the supply of lending and the demand for borrowing). (Note the similarity to Figure 13.11 of the preceding chapter.) The difference between the equilibrium

[5] The maximum attainable level of wealth W_0^* is *defined* in the identity (14.5), but appears in the conditional equation (14.6) as a *constraint* upon the possible consumption choices (c_0, c_1).

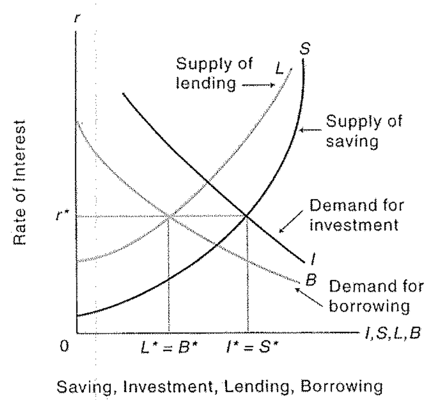


FIGURE 14.6
Intertemporal Equilibrium with Productive Investment

When productive investment takes place, the equilibrium interest rate r^* simultaneously balances (1) the aggregate supply of saving S with the aggregate demand for investment I, and (2) the aggregate supply of lending L with the aggregate demand for borrowing B. The difference between the two magnitudes, at any interest rate r, is accounted for by the amount of investment self-financed out of investors' own savings.

magnitude of saving and investment ($S^* = I^*$) as compared with the amount of borrowing and lending ($L^* = B^*$) is accounted for by the investments that people "self-finance" through their own saving. (**Challenge to the Student:** Whereas Figure 14.6 shows $S^* = I^*$ as larger than $L^* = B^*$, this is not *necessarily* the case. Explain why. *Hint:* As shown for the pure-exchange economy discussed in the previous section, lending and borrowing can take place even when there is no aggregate investment at all.)

CONCLUSION

In a regime of pure exchange, a person can achieve a preferred intertemporal pattern of consumption only by borrowing or lending. At the equilibrium interest rate, the overall market supply of lending equals the overall market demand for borrowing ($L^* = B^*$). But in a regime of production and exchange, each individual chooses a level of investment as well as an amount of lending or borrowing. The scale of investment is chosen to maximize wealth, and it is this maximized wealth that serves as the constraint in achieving a preferred time pattern of consumption. The equilibrium interest rate balances the aggregate supply of saving with the aggregate demand for investment ($S^* = I^*$) while still equating the aggregate supply of lending with the aggregate demand for borrowing ($L^* = B^*$).

Example 14.2 GROWTH VERSUS INVESTMENT: INTERNATIONAL COMPARISONS

Some nations save and invest more than others. We would expect nations to grow more rapidly the more they invest. The table below indicates that this tends to be the case, where growth is measured in terms of annual changes in Gross Domestic Product (GDP).

Growth, Investment, and Saving (1973–1984)

	GROWTH RATE OF GDP	INVESTMENT RATE	SAVING RATE
Five highest growth rates			
Egypt	8.5%	25%	12%
Yemen Arab Republic	8.1	21	−22
Cameroon	7.1	26	33
Syrian Arab Republic	7.0	24	12
Indonesia	6.8	21	20
Five lowest growth rates			
Zambia	0.4	14	15
El Salvador	−0.3	12	4
Ghana	−0.9	6	5
Zaire	−1.0	NA	NA
Uganda	−1.3	8	6

Source: Adapted from The World Bank, *The World Development Report* (1986).

Comment

A high saving rate by residents of a country would lead to increases in wealth, but not necessarily to increases in GDP—if, for example, the savings are invested abroad. Conversely, a country can have high investment despite low saving if funds flow in from abroad. Yemen clearly falls into the latter category; it had a very high investment rate despite a large negative saving rate. (Saudi Arabia provided large financial assistance to Yemen during this period.) Overall, however, there is some correlation between investment and saving, since it is generally easier and safer for people to invest at home rather than abroad. The countries with very low saving rates here were almost all subject to great political disturbances during this period. The countries with extraordinarily high investment rates were mainly beneficiaries of the oil boom.

The distinction between saving and investment is also central to Example 14.3.

Example 14.3 SOCIAL SECURITY, SAVING, AND NATIONAL INCOME

Before the Social Security program came into effect in the United States, individuals generally provided for retirement by saving out of income earned in their productive working years. Savers, by buying stocks and bonds or depositing money in banks, helped finance the productive investments that increased the real capital of the nation.

The expectation of Social Security benefits reduces the motive to provide for one's old age through private financial instruments. Also, Social Security taxes on earnings leave people with less income available for saving. So aggregate *private* saving would be expected to decline as a result of the Social Security program. On the other hand, Social Security taxes generate large revenues for the federal government. If the government had "funded" these contributions, the cash inflows would have built up a financial reserve for covering the anticipated future outlays. (Indeed, this is what private insurance companies do with premiums paid by individuals for retirement annuities.) The funds in the financial reserve could then have been channeled into productive investments. Thus, the collective saving of the Social Security Administration would have balanced the reduction in private savings.

However, a political decision was made early on not to fund Social Security. Instead, Social Security tax revenues were used to support the current expenses of government. On balance, therefore, aggregate savings (and hence aggregate investment) would be expected to decline for the nation as a whole.

Martin Feldstein[a] estimated that, as of 1992, the expectation of future Social Security benefits had cumulatively reduced private saving by $400 billion, with a further reduction of $84 billion through the effect of payroll taxes upon current disposable income. Overall, he calculated, the Social Security program decreased nationwide personal saving by an astonishing 66 percent. Even if corporate saving, presumably unaffected by Social Security, is counted, the reduction of national saving still amounts to 59 percent. Over the years, the reduced level of saving has substantially lowered the growth of U.S. national income.

[a]Martin Feldstein, "Social Security and Saving: New Time Series Evidence," working paper no. 5054, National Bureau of Economic Research, 1995.

14.4 INVESTMENT DECISIONS AND PROJECT ANALYSIS

It is one thing to decide *how much* to save or invest. Government and business decision makers, and private investors as well, also have to make *specific* choices among financial instruments and investment projects. Typically there will be many possibilities: some projects might be of larger and others of smaller scale, some offer quick and others deferred returns, some are more risky and others less. Finding the appropriate criterion for selecting and rejecting investment projects is an important practical issue in the world of affairs.

The Separation Theorem

A crucial implication of the previous analysis is:

THE SEPARATION THEOREM: **The production optimum position Q^* is entirely independent of personal preferences.**

In Figure 14.5, Ida's indifference curves did not matter for finding her production optimum; the location of Q^* depends only on the shape of the Production-Possibility Curve QQ and the slope of the market lines. (Once wealth has been maximized by choice of Q^*, however, time preferences do still matter when it comes to finding the intertemporal consumption optimum C^*.)

When the Separation Theorem is applicable, important practical results follow. Suppose an owner of a firm delegates all production decisions to a manager. The manager would not have to know anything about the time preferences of the owner. All the owner need care about is that the manager acts to maximize the firm's wealth, that is, achieves the production optimum Q^*. Even more important, the firm could have multiple owners with diverging time preferences. The Separation Theorem explains how it is that large numbers of owners can come together to form a corporation, allowing managers to make productive decisions on their behalf.

Strictly speaking, however, the Separation Theorem applies only if the markets for borrowing and lending are perfect and costless. Owing to *transaction costs* (see chapter 13) in the lending/borrowing market, individuals and firms can typically borrow only at a higher interest rate than the rate at which they can lend. The choice of Q^* on the Production-Possibility Curve would then depend upon time preferences after all. Take the extreme case of an isolated Robinson Crusoe, who can be thought of as facing infinite transaction costs (there are no market opportunities at all). Since all his investments must be self-financed, the shape of his indifference curves would surely affect his choice of Q^* (as shown in Figure 14.4).

The Separation Theorem is an ideal, and therefore is never *perfectly* applicable. If the borrowing and lending rates of interest differ substantially enough, individuals sharing particular types of time preferences will tend to group together as owners of firms, and managers will want to take these preferences into account. This explains why utility stocks, which typically pay high current dividends, are attractive to older people. In contrast, younger investors might prefer high-tech stocks paying little or no current dividends but offering better growth prospects.

The Present-Value Rule

What is the appropriate criterion or *investment decision rule* for choosing among projects? (In this discussion the Separation Theorem will be assumed to be valid unless indicated otherwise.)

Any project is characterized by a sequence of dated cash flows or payments $z_0, z_1, \ldots, z_T$ from the present up to some horizon T. Normally, projects begin with an *outlay phase* (one or more initial periods where the z_t are all negative or zero) followed by a *payoff phase* (periods with only positive or zero z_t). This time pattern characterizes an *investment project*, in the strict sense of the word. The opposite case, where the payoff phase precedes the outlay phase, would be a *disinvestment project*. More complicated patterns falling into neither of these simple categories are also possible, and indeed are important in the world of affairs. To begin with, however, we can start with simple investment projects.

The fundamental criterion for selecting among investment projects is known as the *Present-Value Rule*. For the simplest case of two periods—date 0 (now) and date 1 (one year from now)—Present Value V_0 is defined as:

$$V_0 \equiv z_0 + \frac{z_1}{1 + r_1} \tag{14.7}$$

There are different versions of the Present-Value Rule, depending upon whether the projects being considered are independent of one another.

PRESENT-VALUE RULE 1: **(Independent Projects) Adopt any project whose Present Value V_0 is positive; reject any project with negative Present Value V_0.**

The justification for this investment criterion is self-evident. The Present Value of a project represents the *additional wealth* it generates for its owners, as can be seen by comparing equation (14.7) with the definition of wealth in equation (14.4).

But what if projects are interdependent, meaning that adopting one may change the payoffs of another? Sowing corn, for example, may increase the benefit of digging an irrigation canal. An extreme of interdependence occurs when projects are *mutually exclusive*. A landowner may have to choose between putting a gas station or an office building on a city lot. For this case:

PRESENT-VALUE RULE 2: **(Mutually Exclusive Projects) Adopt the project with largest Present Value V_0, provided its V_0 is positive.**

Finally, the general rule, Present-Value Rule 3, subsumes rules 1 and 2:

PRESENT-VALUE RULE 3: **Tabulate *all* the possible combinations of projects available, including doing nothing, and then choose the *set* of projects that maximizes overall Present Value.**

This procedure will obviously maximize wealth for the owner or owners.

Rule 3 has the same form as rule 2, because the available projects can be grouped into mutually exclusive combinations. Suppose a firm has three possible projects *A*, *B*, and *C*. These can be sorted into the eight mutually exclusive combinations 0, *A*, *B*, *C*, *AB*, *AC*, *BC*, and *ABC* (where 0 represents adopting no project at all). Whichever combination has highest V_0 is the firm's wealth-maximizing investment choice.

EXERCISE 14.5

(a) A project has anticipated cash flows $z_0 = -100$, and $z_1 = 125$. Is this an investment or a disinvestment project? (b) What is its Present Value V_0 when the interest rate r_1 is 10 percent? At $r_1 = 20\%$?

Answer (a) Since z_0 is negative and z_1 is positive, this is an *investment* project. It involves current sacrifice for future benefit. (b) If $r_1 = 10\%$, using equation (14.7), V_0 $-100 + 125/(1 + 0.1) = 13.64$. If $r_1 = 20\%$, the Present Value is $V_0 = 4.167$.

Notice that Present Value falls as the discount rate *r* rises. This will be true for all two-date investment projects, as is evident from equation (14.7).

EXERCISE 14.6

The table here shows payment sequences for two projects *M* and *N*. The first two rows show the payments when either project is adopted separately, and the third row shows what happens when both are adopted. (Note that, owing to interdependence, the date 0 outlays and the date 1 payoffs shown for *MN* are *not* simple summations of the magnitudes for *M* and *N*.) (a) If the interest rate were 20 percent and you could adopt only *M* or *N*, which, if any, should you choose? (b) If you could adopt both together, would you want to do so?

PROJECT	z_0	z_1
M	−100	125
N	− 50	90
MN	−160	240

Answer (a) If only one of them can be adopted, *M* and *N* are mutually exclusive. At 20 percent, $V_0(M) = -100 + \frac{125}{1.2} = 4.17$ and $V_0(N) = -50 + \frac{90}{1.2} = 25$. Because *N* has the higher Present Value (and since this V_0 is positive), *N* should be adopted. (b) If the combination *MN* is also an available option, the calculation becomes $V_0(MN) = -160 + \frac{240}{1.2} = 40$. Thus, the combination is better than either project alone.

Generalizing equation (14.7) to any number of dates, the Present Value of a stream of payments from date 0 to a "horizon" date *T* can be expressed either in terms of the long-term or the short-term interest rates that were defined in Table 14.1:

$$V_0 \equiv z_0 + \frac{z_1}{1 + R_1} + \frac{z_2}{(1 + R_2)^2} + \ldots + \frac{z_T}{(1 + R_T)^T} \tag{14.8}$$

$$V_0 \equiv z_0 + \frac{z_1}{1 + r_1} + \frac{z_2}{(1 + r_2)(1 + r_1)} + \ldots + \frac{z_T}{(1 + r_T)\ldots(1 + r_2)(1 + r_1)} \tag{14.9}$$

For some purposes the first formulation is more convenient, for other purposes, the second. There is no logical difference between them, because the long-term rate R_T is an average of the short-term rates $r_1, r_2, \ldots, r_T$ between now and date *T*.

While the "term structure" of interest rates (the differences between r_1, r_2, etc.) can be important, in practical project analysis it is usually assumed that the current rate will maintain itself into the future. If all the rates r_t are all equal to some common value *r*, equations (14.8) and (14.9) both reduce to:

$$V_0 \equiv z_0 + \frac{z_1}{1 + r} + \frac{z_2}{(1 + r)^2} + \ldots + \frac{z_T}{(1 + r)^T} \tag{14.10}$$

EXERCISE 14.7

(a) Suppose it costs 10¢ to plant a tree. Let the timber value of the tree, if cut at any time *t* from the date of planting, be $g = \sqrt{t}$ net of harvesting cost. A partial tabulation would be:

Year of cut (*t*)	1	4	9	16
Value of timber (*g*)	1	2	3	4

Considering only the possibilities tabulated (i.e., not interpolating within the table, or extending it beyond 16 years), what is the best time to cut the tree if the interest rate is constant over time and equal to 5 percent? (b) As a more difficult problem, suppose that after a tree is harvested (and only then) a new one can be planted in its place. Of the possibilities above, which period represents the best cutting cycle?

Answer (a) Because the harvesting periods are mutually exclusive, we can use Present-Value Rule 2 to find the cutting period that maximizes V_0.

YEAR OF CUT	PRESENT VALUE
1	$-.10 + 1/1.05 = -.01 + 0.95 = .85$
4	$-.10 + 2/1.05^4 = -.10 + 1.65 = 1.55$
9	$-.10 + 3/1.05^9 = -.10 + 1.93 = 1.83$
16	$-.10 + 4/1.05^{16} = -.10 + 1.83 = 1.73$

Evidently, it's best to cut after nine years.

(b) For the one-year cycle, the Present-Value equation is:

$$V_0 = -.1 + \frac{1 - 0.1}{1 + r} + \frac{1 - 0.1}{(1 + r)^2} + \ldots$$

where the annual net return of 90¢ repeats itself forever. The Present Value works out to $V_0 = \$17.90$. For the four-year cycle:

$$V_0 = -0.1 + \frac{2 - 0.1}{(1 + r)^4} + \frac{2 - 0.1}{(1 + r)^8} + \ldots$$

Here the net return is \$1.90, repeating itself every four years forever. The Present Value is $V_0 = \$8.54$. By analogous calculations it can be shown that for the 9-year cycle $V_0 = \$5.16$ and for the 16-year cycle $V_0 = \$3.20$. Thus, allowing for the possibility of replanting drastically shortens the optimal cycling time from $t = 4$ to $t = 1$. (**Challenge to the Student:** Can you explain why?)

Example 14.4

EDUCATION AND EARNINGS

Assaf Razin and James D. Campbell[a] calculated present values of lifetime earnings for holders of bachelor's degrees in various fields, using National Science Foundation 1968 data on yearly incomes of scientific manpower. The authors assumed that earnings begin in years after admission to college, and terminate in year 44. So equation (14.10) was applied in the following special form:

$$V_0 = \frac{z_5}{(1 + r)^5} + \frac{z_6}{(1 + r)^6} + \ldots + \frac{z_{44}}{(1 + r)^{44}}$$

Calculating with an interest rate $r = 3\%$, the table indicates some of the results obtained. (The relatively favorable position of the economics degree is consistent with the salary standing of economists reported in Example 1.2.)

Present Values of Earnings for Holders of Bachelor's Degrees

	EARNINGS
Mathematics	\$342,068
Economics	339,482
Computer science	306,733
Political science	300,000
Physics	282,658
Psychology	262,127
Agriculture science	225,118
Biological science	215,691
Sociology	213,590

These numbers are not a firm indication of the value of education regarded as an investment project. For one thing, they do not allow for the *costs* of college, which include not only tuition fees but also earnings forgone during the four college years. As another important qualification, we cannot assume that the higher earnings of educated individuals are due to their education alone. People who go to college may earn more because they work more hours (see chapter 12), or because they are smart to begin with, or because they come from more affluent families.

To eliminate some of these biases, Jere R. Behrman, Robert A. Pollak, and Paul Taubman[b] compared the earnings of identical twins with different amounts of schooling. They found that the following formula best fitted the observations for men aged 47–57 in 1973:

$$\log(E_1/E_2) = 0.28 \log(S_1/S_2) + 0.014$$

Here E_1 and E_2 represent the annual earnings of the two twins in 1980 dollars, and S_1 and S_2 represent their years of schooling.

With this equation we can estimate that if an individual has a twelfth-grade education and earns \$18,400 per year, then the identical twin with four years of college will earn \$20,225 per year—an improvement of \$1,825 or only 9.9 percent. Assuming a 40-year working life and an interest rate of 3 percent, the present value of additional earnings attributable to attending four years of college works out to \$42,176. Note how much smaller this figure is than those in the Razin and Campbell study, and recall that neither study makes any allowances for the *costs* of college.

Does this suggest that college is a bad investment? Not necessarily. Presumably individuals derive some benefits from college apart from improvement in future earnings. Among the possibilities are intellectual enrichment, new friends, potential marriage partners, and fun and games.

[a] Assaf Razin and James D. Campbell, "Internal Allocation of University Resources," *Western Economic Journal* 10 (September 1972), p. 315.

[b] Jere R. Behrman, Robert A. Pollak, and Paul Taubman, "Parental Preferences and Provision for Progeny," *Journal of Political Economy* 90 (February 1982).

Women planning on an academic career suffer from several disadvantages. Perhaps the most important is the likelihood of one's career being interrupted by family obligations, especially childbearing and child rearing. Owing to this interruption, a woman's income payments stream in equation (14.10) is likely to have fewer positive entries than a man's, on average and other things being equal. Furthermore, because the Present Value of her academic career will be lower, a woman would rationally be somewhat less likely to make the necessary investment in training (in building human capital).

There is a subtler factor that aggravates this effect. Human capital tends to decay during career interruptions, since one's training may be forgotten or superseded by newer developments. The significance of this consideration is estimated in Example 14.5.

Example 14.5 *INTERRUPTED CAREERS AND THE DISCOUNTED VALUE OF KNOWLEDGE*

John M. McDowell[a] examined the "decay" of knowledge in different fields as an influence upon the specialization decisions of male and female academics. The decay rate of knowledge was estimated by the declining frequency in which research articles are cited by later authors. The annual average decay rate was found to be 18.30 percent and 14.50 percent for leading journals in physics and chemistry, respectively, but only 3.85 percent and 2.67 percent for corresponding journals in history and English. (E.g., a physics article that was cited 100 times in year t was on average cited only 100 − 18.3 = 81.7 times in year $t + 1$; for history, the corresponding numbers would be 100 versus 100 − 3.85 = 96.15.) These decay rates were then used to estimate, via a Present-Value calculation, the "discounted value of knowledge" in the various fields, as indicated in the table.

Present Value of Knowledge and Costs of Interrupted Careers

	DISCOUNTED VALUE (AT AGE 35)	*LOSS OF HUMAN CAPITAL DUE TO CAREER INTERRUPTION (3-YEAR)*
Physics	4.53	42.30%
Chemistry	5.08	35.27
History	8.03	10.91
English	8.63	7.70

Source: McDowell (1982), table 2, p. 757.

The high decay rates (discount rates) in physics and chemistry lead to correspondingly lower figures for the Present Values of knowledge in those fields. The decay rates also permitted computing the cost of career interruptions, in terms of the loss of accumulated knowledge during off-the-job interludes.

From the much higher cost of interruptions in physics and chemistry, it follows that women undertaking academic careers would tend to avoid those areas of specialization in favor of history and English, fields in which knowledge is more "durable." That this is indeed the case is of course well known.

Comment

It is also possible to infer from this analysis that, as the birthrate falls and as men increasingly share other family responsibilities, the proportion of women in fields such as physics and chemistry is likely to grow.

[a] John M. McDowell, "Obsolescence of Knowledge and Career Publication Profiles," *American Economic Review* 72 (September 1982).

Certain special assumptions lead to a useful simplification of equation (14.10). Suppose that the benefits of some asset or project begin at date 1 and continue on to infinity (the "economic horizon" is $T = \infty$), and that these positive future receipts z_1, z_2, . . . are all equal to some common z. The Present Value of these benefits, denoted B_0, is given by:[6]

$$B_0 \equiv \frac{z}{r} \quad \text{or} \quad r \equiv \frac{z}{B_0} \tag{14.11}$$

For example, if $r = 10\%$, the Present Value of \$10 per year forever is $B_0 = \frac{\$10.00}{0.10} = \$100$. Expressing this another way, if you put \$100 in the bank at 10 percent interest, it would yield $z = \$10$ per year forever.

As for the current-period element z_0, think of it as the asset's acquisition cost c_0 or price P. The asset's overall Present Value is evidently the amount by which the Present Value of the *benefits alone*, denoted B_0, exceeds the acquisition cost:

$$V_0 \equiv B_0 - P \equiv \frac{z}{r} - c_0 \tag{14.11$'$}$$

In equilibrium the acquisition cost or market price of any asset must equal the present value of the benefits it generates. Thus:[7]

$$P = \frac{z}{r} \quad \text{or} \quad r = \frac{z}{P} \tag{14.11$''$}$$

[6]*Mathematical footnote:* Under these special assumptions, equation (14.10) can be written:

$$B_0 = z\left[\frac{1}{1+r} + \frac{1}{(1+r)^2} + \ldots\right]$$

Let $1/(1 + r)$ be denoted k. Then:

$$B_0 = z(1 + k + k^2 + \ldots) - z$$

Using the formula for the sum of an infinite series, the expression within parentheses equals $1/(1 - k)$. From the definition of k, simple algebra shows that $1 - k = r/(1 + r)$. Making the indicated substitutions:

$$B_0 = z\left[\frac{1+r}{r}\right] - z = \frac{z}{r}$$

[7]Note that in these equations an equals sign (=) rather than an identity sign (≡) appears. The expression here is not an identity but a condition of equilibrium.

This justifies the discussion in chapter 12—see equation (12.4′) and (12.5)—that related the interest rate to the ratio between the hire-price h_A of a resource and the value or price P_A of the resource itself. Setting aside possible appreciation or depreciation ΔP_A, that discussion showed that $r = h_A/P_A$. The hire-price h_A corresponds to the benefit z in equation (14.11″), because h_A is the annual amount received if the asset is rented out, and obviously the asset price P_A corresponds to P here.

The Rate of Return (ROR) as Decision Criterion

The Present-Value (PV) Rule is not the only one in common use. Another decision rule for evaluating projects makes use of the measure known as the Rate of Return (ROR), sometimes also called the Internal Rate of Return (IRR). The PV and ROR rules often agree, but not always.

We have seen that the Present-Value Rule, in its most general form, calls for adoption of the overall set of investment (and disinvestment) projects that maximizes V_0. The underlying justification is that, because V_0 is an increment to owners' wealth, the best overall set of projects is the one that maximizes this increment. In the special case of independent projects (Present-Value Rule 1), each and every project of positive Present Value ($V_0 > 0$) should be adopted.

The Rate of Return (ROR) for any project is defined by setting up an equation in Present Value form, but with the discount rate treated as an unknown. The ROR is the value of the discount rate, denoted ρ, that makes this Present Value equal to zero. Thus, adapting equation (14.10):

$$0 \equiv z_0 + \frac{z_1}{1+\rho} + \frac{z_2}{(1+\rho)^2} + \ldots + \frac{z_T}{(1+\rho)^T} \tag{14.12}$$

The associated *rule*, in the simplest case of independent projects, is to adopt any project whose ROR exceeds the market rate of interest: that is, adopt if $\rho > r$. The underlying thought is that the ROR represents a kind of "growth rate" of value over time. If funds invested in a project grow in value more rapidly than compounding at the market interest rate, it makes sense to adopt the project.

For independent projects in the two-period (date 0 and date 1) case, the Present-Value Rule and the Internal-Rate Rule always agree: $V_0 > 0$ implies that $\rho > r$.[8] This is also true for the special case discussed earlier, where a project has a date 0 acquisition cost c_0 and yields annual benefits z forever.[9]

Furthermore, the equivalence between these PV and ROR rules holds for any sequence of payments $z_0, z_1, \ldots, z_T$ meeting the strict definition of an investment project. To wit, where a single outlay phase (an initial sequence of negative or zero elements beginning with z_0) is followed by a single payoff phase (a sequence of positive or zero elements). In other words, where there is only a *single alternation of signs*. For example, the payments streams $(z_0, z_1, z_2) = (-5, 1, 5)$ or $(-1, -5, 8)$ meet this condition, whereas the stream $(z_0, z_1, z_2) = (-1, 5, -2)$ does not.[10]

[8] *Mathematical footnote:* For the two-date case, by assumption $z_0 + \dfrac{z_1}{1+r} > 0$. And by definition $z_0 + \dfrac{z_1}{1+\rho} \equiv 0$. Thus, $\rho > r$.

[9] *Mathematical footnote:* For the special case just described:

$$c_0 + \frac{z}{r} > 0 \quad \text{and} \quad c_0 + \frac{z}{\rho} > 0$$

Once again, it must be that $\rho > r$.

> **CONCLUSION**
> **For independent projects, if the payments stream has only a single alternation of signs (at the point in time where the investment phase gives way to the payoff phase), then the Present-Value Rule ("Adopt if $V_0 > 0$") is equivalent to the Internal-Rate Rule ("Adopt if $\rho > r$").**

Discrepancies between the two rules arise in two classes of cases: (1) independent projects that violate the single-alternation condition, and (2) interdependent projects of all types.

Consider first the project defined by the twice-alternating payments stream $(z_0, z_1, z_2) = (-1, 5, -6)$. Calculating the internal rate ρ leads to *two* solutions: $\rho = 100\%$ and $\rho = 200\%$.[11] Attempting to apply the decision rule "Adopt if $\rho > r$," should we use $\rho = 100\%$ or 200%, or is neither of these two appropriate? As another example, consider the payments stream $(z_0, z_1, z_2) = (-1, 3, -2.5)$. Here there are *no* solutions for ρ in real numbers. How then can we compare ρ and r?[12]

The difficulty is immediately explained if we think in terms of Present Value. Figure 14.7 shows, for a number of possible payments sequences (projects), how V_0 changes as the discount rate varies upward from $r = -100\%$. Project *I*, whose payments sequence is $(-1, 0, 4)$, is an investment project in the strict sense; its V_0 therefore declines throughout as r rises. As follows from the definition of the Rate of Return

[10] *Mathematical footnote:* In the simplest situation, a *single* negative z_0 would be followed by a series of positive elements up to z_T. Using the same method as in the preceding footnote, by assumption:

$$z_0 + \frac{z_1}{1+r} + \frac{z_2}{(1+r)^2} + \ldots + \frac{z_T}{(1+r)^T} > 0$$

That is, Present Value V_0 is positive. And by definition of the IRR:

$$z_0 + \frac{z_1}{1+\rho} + \frac{z_2}{(1+\rho)^2} + \ldots + \frac{z_T}{(1+\rho)^T} = 0$$

Comparing these two equations, it is immediately evident that $\rho > r$.

(If the outlay phase extends for several dates before the payoff phase begins, the proof is somewhat more difficult but the same principle applies.)

[11] *Mathematical footnote:* $0 = -z_0 + \dfrac{z_1}{1+\rho} + \dfrac{z_2}{(1+\rho)^2}$ is a quadratic equation. Such an equation may have two, one, or zero solutions among the real numbers.

[12] These two examples involve something stronger than alternation of signs in the benefit stream. Notice that in each case the *summed magnitude of the negative elements* exceeds the sum of the positive elements. It has been proved mathematically that the ROR calculation will lack a unique result only under this condition. However, such situations may be quite realistic. A mining project might involve an initial outlay (z_0 is negative), followed by one or more years of positive z_1, z_2, z_3, and so on. Once the mine is exhausted, however, very large costs (negative z_T) may be incurred in closing it down at the terminal date T.

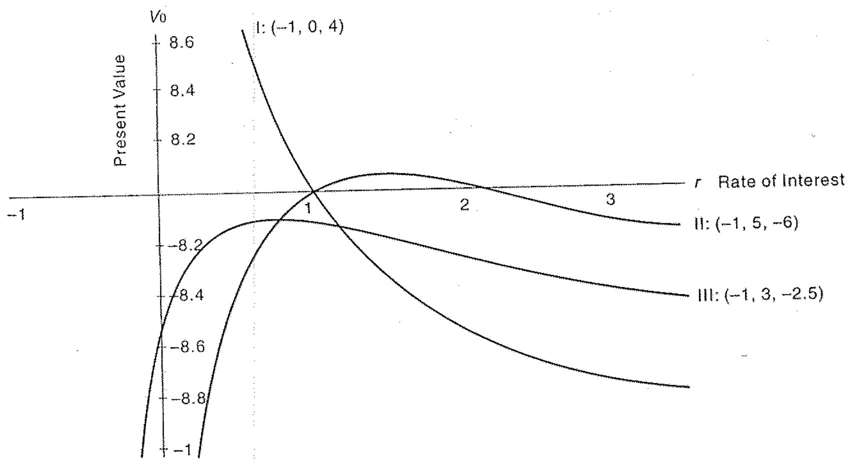


FIGURE 14.7

Present Values and Rates of Return for Three Projects

As the payments for project *I* show only a single sign alternation (from negative to positive), it is a simple investment project. Its Present Value is a declining function of the interest rate *r*. Projects *II* and *III* each involve more than sign alternation; their Present Values first rise and then fall as *r* increases. Project *I* has a unique Rate of Return $\rho = 100\%$ at the intersection of its V_0 curve with the horizontal axis. Project *II* has two such intersections at $\rho = 100\%$ and 200%; project *III* has none. Thus, evaluating projects by a comparison of ρ with *r* can be ambiguous or even impossible.

concept, its ρ is represented by the point where the $V_0(I)$ curve cuts the horizontal axis. Project *II*, with sequence $(-1, 5, -6)$, has positive V_0 (and therefore warrants adopting) for all *r between* 100 percent and 200 percent, but otherwise Present Value is negative and the project should not be adopted. The two algebraic solutions for ρ correspond to the two intersections of $V_0(II)$ with the horizontal axis. Finally, project *III*, with sequence $(-1, 3, -2.5)$ has negative V_0 for *all r*; under the Present-Value Rule, it should never be adopted.[13]

Thus, for independent projects, the Present-Value Rule is clearly the more fundamental. Among other things, it tells us when the alternative ROR Rule will or will not be valid.

An even more serious problem with the ROR investment criterion arises in considering *interdependent* projects. Taking the extreme case of mutually exclusive projects, it is not even clear what the ROR Rule would then be. Should the investor adopt

[13]This statement is true if we consider only interest rates *r* that are constant over time. The project $(-1, 3, -2.5)$ may have positive V_0 if r_1 and r_2 differ. Using equation (14.9), it may be verified that $V_0 > 0$ if, for example, $r_1 = 100\%$ and $r_2 = 200\%$.

the mutually exclusive project that has *highest* ρ? That could be a serious mistake. The project with highest ρ might be of such small scale as to provide only a tiny increment to wealth; an alternative larger-scale project with smaller ρ might generate a much greater wealth increment.

EXERCISE 14.8

(a) For the projects tabulated in Exercise 14.6, calculate the Rates of Return for *M*, *N*, and *MN*. (b) If *M* and *N* are mutually exclusive options, which has higher ρ? Is that project the same as the one with higher Present Value V_0? (c) If *MN* is also available as a third mutually exclusive option, which of the three possibilities *M*, *N*, and *MN* has the highest ρ? Again, is that the same as the one with highest Present Value V_0?

Answer (a) Substituting the tabulated payoffs for *M*, equation (14.12) becomes

$$0 \equiv -100 + \frac{125}{1+\rho}.$$

The solution is $\rho(M) = 25\%$. Similar computations lead to the results $\rho(N) = 80\%$, $\rho(MN) = 50\%$. (b) As between *M* and *N*, the latter has the higher ρ. *N* also was shown in Exercise 14.6 to have higher Present Value. So here there is no disagreement. (c) When *MN* is considered as well, $\rho(N)$ remains the highest of the three. However, Exercise 14.6 showed that *MN* is the option with highest Present Value V_0. So here the ROR criterion does not correctly rank the projects.

We must conclude, therefore, that the ROR Rule ("Adopt if $\rho > r$") is not a reliable guide for investment decision. That does not mean the rate of return is mathematically incorrect as a *concept*, only that it needs to be used with great caution in choosing among investment projects.

Nevertheless, both in business practice and in economic analysis one often sees rates of return calculated and possibly even used for investment choices. Why? Apart from sheer ignorance, there are several reasons.

It may be useful to have a division of labor. A company could employ *project specialists* to filter through the investment options by calculating the ROR associated with each. These project appraisers would not need to know the discount rate *r*, which could be provided instead by *finance specialists* acquainted with the terms on which the company can obtain project funding. Finally, top management can make the decision in the light of the information on ρ and *r* provided by the two sets of subordinates. While there is some risk of error, if most projects are independent and if their payoffs do not involve serious sign alternations, the administrative convenience of such a division of labor may warrant its use. Still, caution is certainly indicated.

In economic analysis as well, once again it is often convenient to have a criterion such as ρ that describes investment options without necessarily having to pay attention to the rate of interest needed for calculating Present Value. An economist might use ρ as a criterion to show how the desirability of certain options, such as acquiring a college education, has varied over time or among different countries—without attempting to take account of possible historical or geographical differences in interest rates. Once again, however, caution is called for. On the basis of ROR comparisons alone, it would not be safe to say that college education was a better investment in 1980 compared to 1990, or in country *X* than in country *Y*.

Example 14.6

RATE OF RETURN TO EDUCATION—INTERNATIONAL COMPARISONS

In 1985 George Psacharopoulos[a] reported on a massive continuing survey of rates of return on educational investments in some 60 countries. The table illustrates some of the results obtained.

Social Rates of Return to Education

REGION	PRIMARY	SECONDARY	HIGHER
Africa	26%	17%	13%
Asia	27	15	13
Latin America	26	18	16

Source: Psacharopoulos (1985), table 1, p. 586.

The data refer to the most recent years available, mainly in the 1970s. (These tabulated "social" rates of return are typically lower than the "private" rates of return that would enter into individuals' calculations, because government subsidies tend to raise the private profitability of education.) What is rather surprising is the similarity of the rates within each column, considering the huge and diverse regions represented. The picture of declining ROR for more advanced education makes sense, following the general principle of diminishing returns.

Comment

Investment in education probably meets the single-alternation condition justifying use of the ROR rule; that is, costs tend to be followed by benefits without further sign reversals. However, it would be incorrect to infer, for example, that because primary education shows a 26 percent rate of return ρ in both Africa and Latin America, it is an equally attractive investment for both regions. First of all, the interest rates needed for the $\rho > r$ comparison may differ. Also, conceivably, primary education in Africa and Latin America might involve rather different *scales* of investment.

[a] George Psacharopoulos, "Returns to Education: A Further International Update and Implications," *Journal of Human Resources* 20 (Fall 1985).

14.5 REAL VERSUS MONETARY INTEREST: ALLOWING FOR INFLATION

This chapter has so far dealt only with the *real* rate of interest. Following the usual practice in microeconomics, we have looked behind the "veil of money" to think in terms of real goods, in this case, present corn versus future corn. In practice, however, people almost always deal in terms of *monetary* rates of interest. Suppose a bank advertises that it pays 8 percent. That means if you make a deposit of \$100 you can withdraw

\$108 at the end of the year. But if the cost of living is rising, the \$108 after a year will not buy you as much as \$108 at the beginning of the year. Perhaps the end-of-year \$108 will buy only as much as \$103 would have bought at the beginning of the year. If so, although 8 percent is the *monetary* rate of interest, the *real* rate of interest is only about 3 percent.

The real rate of interest is what we have previously been calling simply *the* rate of interest—the extra amount of future corn that must be offered in the market in exchange for current corn. Rewriting equation (14.1), the real interest rate between date 0 and date 1 is again:

$$1 + r_1 \equiv -\frac{\Delta c_1}{\Delta c_0} \tag{14.13}$$

The monetary interest rate between date 0 and date 1, which we will symbolize as r_1', is the *premium on current money over future money*. Thus, r_1' is the extra amount of future money that must be offered in exchange for current money.

$$1 + r_1' \equiv -\frac{\Delta m_1}{\Delta m_0} \tag{14.14}$$

To show the relationship between r_1 and r_1', we need to introduce the concept of the *price level*: the amount of money required at any date to purchase real goods. There will be a current and a future price level:

$$P_0^m \equiv -\frac{\Delta m_0}{c_0} \quad \text{and} \quad P_1^m \equiv -\frac{\Delta m_1}{\Delta c_1} \tag{14.15}$$

Let us now write out the identity:

$$\frac{\Delta m_1}{\Delta m_0} \equiv \frac{\Delta m_1}{\Delta c_1}\frac{\Delta c_1}{\Delta c_0}\frac{\Delta c_0}{\Delta m_0} \tag{14.16}$$

Substituting from the preceding equations:

$$1 + r_1' \equiv \frac{P_1^m}{P_0^m}(1 + r_1)$$

Let us define a_1, *the anticipated rate of price inflation* between date 0 and date 1:

$$1 + a_1 \equiv \frac{P_1^m}{P_0^m} \tag{14.17}$$

It follows immediately that:

$$1 + r_1' \equiv (1 + a_1)(1 + r_1)$$

Or, simplifying:

$$r_1' = r_1 + a_1 + a_1 r_1 \tag{14.18}$$

Accordingly, the monetary rate of interest equals the real rate of interest plus the anticipated rate of price inflation, plus the cross product of the latter two. When r_1 and

a_1 remain in their usual range of percentage points, the cross-product term can, to a good approximation, be ignored. Furthermore, the shorter the period of compounding the better the approximation. For continuously compounded interest, the cross product drops out entirely and we have exactly:[14]

$$r'_1 = r_1 + a_1 \tag{14.19}$$

This equation is known as the "Fisher hypothesis."[15] An obvious implication is that, if people expect inflation to be high, the money interest rate r'_1 will be high.

PROPOSITION: **The money rate of interest equals the real rate of interest plus the anticipated rate of price inflation.**

Example 14.7 REAL AND MONETARY RATES OF INTEREST IN THE UNITED KINGDOM

The British Treasury offers savers the opportunity to purchase either ordinary bonds paying fixed *monetary* rates of interest (r') or else inflation-adjusted bonds. In effect, these latter bonds offer fixed *real* rates of interest r.

Using the difference between the rates on the two types of bond at any moment as a measure of anticipated inflation a at that date, G. Thomas Woodward[a] examined 14 different bond maturities from 1982 to 2024. For each maturity, he used data from April 1982 to August 1990 to statistically estimate equations in the form:

$$r' = H + Ka$$

If the Fisher hypothesis is valid, then H should correspond to the *real* rate of interest over the period, assuming it was about constant (otherwise, H would be an average of the real rates). Also, K should equal 1. For technical statistical reasons, however, the tests had to be run in "first-difference" form, precluding any direct estimate of H. Thus, the only issue was how close the estimated K was to 1.

Generally, for the 14 different maturities, the estimates of K were fairly close to 1. However, except for the near maturities, they tended to be on the high side—in the neighborhood of 1.1 or 1.2. The author considered whether this divergence might be due to income taxes. (If taxes have to be paid on interest earnings, the effective difference between r' and r will not be so great as appears in the crude data.) Adjusted for taxes, most of the indicated K values were quite close to 1, except that some of the near maturities were somewhat less. (As a possible explanation, monetary theory indicates that very short-maturity instruments serve as money substitutes, and therefore are held in part for liquidity rather than for pure investment purposes.) Overall, the author concluded, the Fisher hypothesis is supported by the British evidence.

[a] G. Thomas Woodward, "Evidence of the Fisher Effect from U.K. Indexed Bonds," *Review of Economics and Statistics* 74 (May 1992).

[14] *Mathematical footnote:* If i is an annually compounded interest rate, a unit investment at date 0 will grow to $1 + i$ units at the end of one year. With quarterly compounding, the end-of-year terminal value will be $(1 + i/4)^4$. Generalizing, for any compounding frequency f per year, terminal value will be $(1 + i/f)^f$. Defining $h \equiv \frac{f}{i}$, this becomes $[(1 + \frac{1}{h})^h]^i$. For continuous compounding, we let f approach infinity, which means that h does so as well. As $h \to \infty$, the expression within brackets becomes e, the base of the natural logarithms. Thus, at the end of a year, with continuous compounding the terminal value becomes e^i. In terms of continuously compounded rates for the monetary rate of interest r'_1, the real rate of interest r_1, and the anticipated rate of inflation a_1:

$$e^{r'_1} \equiv e^{r_1} e^{a_1}$$

Taking logarithms, $r'_1 = r_1 + a_1$ follows directly.

[15] Named after the American economist Irving Fisher (1867–1947). Actually, not just this one hypothesis but the entire analysis of intertemporal choice in this chapter derives from Fisher's classic formulation in *The Theory of Interest* (Macmillan, 1930).

The analysis in this section has significant implications for ongoing debates in macroeconomics. According to one view, expansion of the money supply tends to *reduce* monetary interest rates. If the government unexpectedly pays some of its bills with newly printed money, people will find themselves with unexpectedly large current cash balances. Provided they do not anticipate having correspondingly larger future cash balances, they should be willing to trade larger amounts of current money for claims on future money. A higher $\Delta m_0/\Delta m_1$, or, equivalently, a lower $\Delta m_1/\Delta m_0$ (higher or lower in absolute value, of course), means that the monetary interest rate r'_1 in equation (14.14) must fall. The opposed view is that expansion of the money supply tends to *raise* the monetary rate of interest. An increase in current money may lead people to believe that the government has embarked on a course of action that will make future money balances larger still. With general anticipations of higher $\Delta m_1/\Delta m_0$, the monetary interest rate r'_1 will rise. In short, an expansion of the money supply lowers the monetary rate r'_1 if it is believed to be a unique event, but continuing monetary expansion may generate inflationary expectations that raise r'_1.

The main lesson to be learned from this discussion is that high monetary rates of interest do not necessarily imply high *real* yields to investors. In fact, taking inflation and taxes into account, the experience of investors over the past half century has been unimpressive.

Example 14.8 NOMINAL AND REAL YIELDS, 1926–1987

Roger G. Ibbotson and Rex A. Sinquefield[a] examined the yields of various financial instruments over the period 1926–1987. In the table, column 2 shows the arithmetic mean of the compounded annual *monetary* (nominal) returns received by holders of various types of securities over the 55-year period 1926–1981. Column 3 shows the inflation-adjusted or *real* average annual returns. Finally, column 4 shows the standard deviation σ of the real returns, a statistical measure of risk. Generally speaking, higher average returns are highly correlated with riskiness. Their low risk partially explains why the real return has been so low for government issues, both short-term and long-term.

Nominal and Real Yields per Annum, 1926–1987

	AVERAGE NOMINAL YIELD	AVERAGE REAL YIELD	σ OF REAL YIELD
U.S. Treasury bills	3.5%	0.5%	4.4%
Long-term government bonds	4.6	1.7	10.2
Long-term corporate bonds	5.2	2.3	10.0
Common stocks	12.0	8.8	21.2
Small-company common stocks	17.7	14.2	35.2

Source: Ibbotson and Sinquefield (1989), pp. 72, 74.

Comment

The historical record for investors becomes significantly worse if we allow for income taxes, and in particular the combination of taxes and inflation. Taxes and inflation interact to the disadvantage of investors, because in most countries income taxes are levied upon the *nominal* rather than the real return. Suppose someone purchases shares of common stock for \$100, receives \$3 in dividends, and sells the stock for \$110 at the end of the year. The nominal before-tax return is 13 percent. Suppose, however, that the rate of inflation is 10 percent, and the stockholder's marginal tax rate is 25 percent. The stockholder will pay 75¢ in tax on the dividends, and \$2.50 on the capital gain, leaving \$113.00 − \$3.25 = \$109.75 at the end of the year. But with 10 percent inflation, the purchasing power of this \$109.75 is only $\frac{\$109.75}{1.10} = \$99.77$, so the real yield has been negative. Taking taxes into account, U.S. investors have incurred negative real yields over much of the past half-century. We should not be surprised, therefore, to learn that the fraction of income devoted to saving in the United States has become, in the opinion of many observers, disturbingly low.

[a]Roger G. Ibbotson and Rex A. Sinquefield, *Stocks, Bonds, Bills, and Inflation: Historical Returns (1926–1987)*. The Research Foundation of the Institute of Chartered Financial Analysts, 1989.

14.6 THE MULTIPLICITY OF INTEREST RATES

Up to now the text has usually spoken of "the" interest rate. But many different interest rates coexist in the market at any moment of time. One important distinction, between real and monetary rates, has just been discussed and explained. However, varying interest rates are observed even *within* each of these categories. Example 14.8 revealed great divergences in the historical yields of different types of financial instruments, among them Treasury bills, long-term bonds, and common stocks. Or consider dealings with your bank. A savings account might pay around 4 percent per annum. Should you want to borrow, however, the same bank might offer you a rate of 7 percent on a long-term mortgage, 12 percent on a commercial loan, or 15 percent to finance a consumer purchase.

What explains these differences? As suggested in Example 14.8, divergences among interest rates can be due to disparities in *riskiness*. We need to look into this more closely, however.

"Risk" has more than one meaning in common usage, and in particular it is necessary to distinguish between *default risk* and *variability risk*.

A bank might make a one-year \$1,000 loan at an interest rate of 10 percent. The bank cannot be certain of repayment: the borrower may default on interest or principal, or both. This *default risk* is obviously always undesirable from the point of view of the lender (investor).

In contrast to bank loans or bonds, common stocks and many other financial instruments do not carry any explicit quoted promise of repayment. The investor must contemplate a range of possible outcomes: dividends may or may not be paid, the stock price may rise or fall. Estimating the probabilities of the various outcomes, each investor will anticipate an "expected" (average) return, together with larger or smaller variability around this average. In economic analysis the term "risk" standing alone is understood to refer to the magnitude of this *variability risk*.

Viewed as of now (date 0), suppose there are $s = 1, 2, \dots, S$ possible states or events that might affect the net benefit at date 1, each with associated probability π_s. (In accordance with the laws of probability, the π_s must sum to 1.) If we denote the date 1 benefit in each possible state as z_{1s}, then the average is the statistical "expectation" $\hat{z}_1$:

$$\hat{z}_1 \equiv \pi_1 z_{11} + \pi_1 z_{12} + \dots + \pi_S z_{1S} \tag{14.20}$$

For assessing variability a convenient measure is the *standard deviation* σ:

$$\sigma(z_1) \equiv [\pi_1(z_{11} - \hat{z}_1)^2 + \pi_2(z_{12} - \hat{z}_1)^2 + \dots + \pi_S(z_{1S} - \hat{z}_1)^2]^{1/2} \tag{14.21}$$

If standard deviation is held constant, higher expectation $\hat{z}_1$ will surely always be desired. It is not quite so obvious whether, holding $\hat{z}_1$ constant, people will generally prefer higher or lower standard deviation $\sigma(z_1)$. In other words, does the market reflect preference for or aversion to variability risk? While in the abstract the issue might remain somewhat in doubt, the evidence is that risk aversion rather than risk preference dominates in financial markets. Example 14.8 showed that over the period 1926–1987 investors were willing to hold U.S. Treasury bills (low variability risk) even though they yielded only 3.5% per annum, whereas to hold common stocks (high variability risk) they required a yield of 12.0%.

In view of the evidence for risk aversion in financial markets, it might however seem puzzling that *gambling*—deliberately seeking risk—remains an important economic activity. Without necessarily providing a full answer, the following points should be noted: (1) Apart from "pathological" cases, people generally gamble in ways that put in jeopardy only a small fraction of their wealth, for example, small lottery bets. Such gambling can be regarded as a kind of consumption good, buying a thrill at low cost. The financial markets, in contrast, reflect the fact that investors avoid risk in making big decisions about their overall asset portfolios. (2) Even a risk-averse person will accept some gambles, provided the subjective expectation of return is sufficiently high. For a horse-racing expert, betting at the track may yield a high expectation of return.[16]

[16]And someone who receives a revelation from on high, as to which horse will win, may think he's onto a sure thing, not a gamble at all!

As a possibly related point, Example 14.9 shows that investments in works of art—though quite risky—have not historically been associated with high rates of return.

Example 14.9

ART APPRECIATION

Art as an investment is subject to various risks including fire, theft, and the possibility of forgery or mistaken attribution—in addition to variability of prices when the time comes to resell. James E. Pesando[a] examined the variability risk as well as the rate of return on investment in modern prints, using 1977–1992 data for repeat sales of the same prints at auction. There were 27,961 repeat sales for the 28 artists covered. The table reveals that the average 1.51 percent real return on prints was lower even than the return on 180-day Treasury bills—despite the fact that investing in prints is far riskier (as measured by the standard deviation of return).

Real Returns per Annum, 1977–1992

	MEAN	*STANDARD DEVIATION*
Prints	1.51%	19.94%
Stocks	8.14	22.47
U.S. government bonds	2.54	21.83
180-day Treasury bills	2.23	3.43

Source: Pesando (1993), table 2, p. 1080.

Comment

The probable explanation: since investors obtain other benefits from owning works of art, in particular the pleasure of viewing and displaying their art collections, they do not require as high a financial return.

[a]James E. Pesando, "Art as an Investment: The Market for Modern Prints," *American Economic Review* 83 (December 1993).

In contrast with Example 14.8, Example 14.9 indicates that in this period U.S. government bonds (but not 180-day Treasury bills) had relatively high variability risk. In fact, the reported standard deviation for these bonds exceeds the σ for prints and even approaches the σ for common stocks. The main reason is that, owing mainly to changing anticipations of inflation, monetary interest rates fluctuated considerably during the period. As seen earlier in the chapter, Present Values of future payments are highly sensitive to changes in the rate of interest, and so the market values of longer-term government bonds fluctuated considerably from year to year.

Two other elements, related to riskiness, also help explain the multiplicity of observed interest rates:

1. *Transaction costs:* Interest rates often reflect an element of transaction cost. This is especially the case where risks are involved, in which case costly inquiry into creditworthiness may be required. Consumption loans typically carry high interest rates because it is troublesome to investigate many small borrowers and to enforce penalties for default.
2. *Term:* We saw earlier that at any moment of time there is a term structure of interest rates. Today's short-term rate r_1 may differ from the "forward" short-term rates r_2, r_3, and so on. Or, as this is more usually expressed, the short-term rate r_1 may differ from the longer-term rates R_2, R_3, and so on as defined in Table 14.1.

Interest rates almost always rise as term increases. In June 1996, for example, U.S. Treasury issues maturing within three to six months typically were paying around 5.3 percent to investors, whereas issues maturing in 30 years yielded 7.0 percent. In financial parlance, there is a "rising yield curve."

Rising yield curves mainly reflect investors' desire for *flexibility*. Because placing funds in a short-term loan or investment leaves investors in a relatively flexible position, they are willing to accept a low rate of return. In contrast, they will probably require a higher expected return in exchange for being "locked in" to a very long term investment. (Thanks to the possibility of resale, investors are not *literally* locked in to a long-term investment such as a 20-year corporate bond. However, in attempting to sell off the unmatured bond, they may find that whatever forces are making them want to sell are also making potential purchasers reluctant to buy.)

It is risk that explains the need for flexibility. Even if a long-term investment itself were riskless, so that the promised or expected return is quite certain, unpredictable possible future changes in one's own circumstances might make a person avoid long-term commitments.

An Application: The Discount Rate for Project Analysis

That investments vary in riskiness has important implications for project analysis (as discussed earlier in section 14.4). Dealing for simplicity with independent projects only, both the Present-Value Rule ("Accept if $V_0 > 0$") and the Rate of Return Rule ("Accept if $\rho > r$") require selection of an appropriate interest or discount rate r. Which of the multiplicity of coexisting interest rates is the appropriate one?

It seems fairly straightforward that *the discount rate employed in evaluating a project should correspond to its variability risk.* If the project under consideration is no riskier than a Treasury bond, it would be appropriate to discount its anticipated returns at the relatively low interest rate carried by such government obligations. If, as is more likely the case, a project has about the same riskiness as the firm's other ongoing activities, then the discount rate should be the rate of return that the financial markets require for providing funds to the company (its overall "cost of capital").[17]

One type of mistake is very common in project analysis. Suppose a company—say, General Motors—is considering an independent project whose riskiness is typical of its activities overall. According to the foregoing discussion, the expected project returns should be discounted at GM's overall "cost of capital," say, 10 percent. Imagine that, at that rate, the project fails the Present-Value Rule; that is, $V_0 < 0$. Now suppose one of GM's financial analysts says: "We don't really have to sell new stock or issue new bonds to finance this project. If we were instead to mortgage one of our downtown office buildings, with that collateral a bank would be happy to provide funds at a much lower rate, say 6 percent. Since at 6 percent the project has positive Present Value, let's go ahead with it."

The error here is like attempting to pull yourself up by your own bootstraps. A low-interest loan might well be obtained by putting an office building up as collateral. However, that collateral is no longer available to protect the holders of *other* existing GM securities, which will therefore become riskier and thus tend to fall in value. Perhaps the simplest way to look at this is to realize that the value of all of GM's assets, taken together, must equal the value of its stock and liabilities taken together.[18] If a project is adopted that fails the Present-Value test using a discount rate corresponding to the additional risk the project contributes to the firm's overall activities, the value of GM as an ongoing concern will fall.

EXERCISE 14.9

X Corporation has a simple capital structure, consisting solely of common stock. Its expected annual profit, assumed to continue indefinitely, is \$100 per annum with standard deviation \$50. The capital market views this pattern as warranting a discount rate of $r = 10\%$. Thus, *X*'s stock, calculated by using equation (14.11″) in the form $P = \hat{z}/r$, is worth $\frac{\$100}{0.10} = \$1{,}000$.

Management is considering a new project which would cost \$200 and provide additional net profit of \$20 per annum with standard deviation \$10. Making the simplifying assumption of perfect correlation of the new project with old operations, after adoption the overall standard deviation of net profit would be \$60. (a) Is the project worth adopting, if financed with a new issue of common stock? (b) What would happen if the project were financed instead by new mortgage debt, issued on a riskless basis at 6 percent?

[17]Modern finance theorists have developed a concept called "beta" to measure the riskiness of corporate securities. Beta involves not only the standard deviation σ of the security itself, but also its correlation with the overall market. Suppose company *X*'s stock has high σ, so that its return is quite variable, but its variability is negatively correlated with general stock price movements. Then for the representative investor whose portfolio can be regarded as a cross section of the entire stock market, stock *X* is actually risk-reducing—holding it offsets the riskiness of the market as a whole. Consequently, its stock price P_X should be relatively high and its expectation of return low. The returns on most stocks will obviously be *positively* correlated with the market as a whole; owing to the increased risk, their expected returns would have to be correspondingly high. See F. Black, M. C. Jensen, and M. Scholes, "The Capital-Asset Pricing Model: Some Empirical Tests," in *Studies in the Theory of Capital Markets*, ed. M. C. Jensen (New York: Praeger, 1972).

[18]This proposition is known as the Modigliani-Miller theorem. See F. Modigliani and M. H. Miller, "The Cost of Capital, Corporation Finance, and the Theory of Investment," *American Economic Review* 48 (June 1958).

Answer (a) Because $\hat{z}/\sigma(z) = \frac{\$120}{\$60}$ is in the same ratio as the previous $\frac{\$100}{\$50}$, new common stockholders should be willing to provide the additional \$200 of new funds at the same 10 percent cost of capital. The company overall is now worth $\frac{120}{0.10} = \$1{,}200$, of which the new stock accounts for \$200. The old shareholders' stock is still worth \$1,000. Since $V_0 = 0$, the project is just on the borderline of warranting adoption. (b) The company overall is still worth \$1,200. Because by assumption the riskless bonds are worth \$200 at 6 percent, the old shares are still worth \$1,000. It is true that for the old shareholders the expected annual net profit $\hat{z}$ rises by 8 percent (from \$100 to \$100 + \$20 − \$12 = \$108), but these shareholders now face a 20 percent increase in variability $\sigma(z)$, from \$50 to \$60. Thus, the expected profit $\hat{z}$ will be discounted at a higher rate. In fact, the new discount rate will be 10.8 percent, because $\frac{\$108}{0.108} = \$1{,}000$.

The principle that the discount rate should reflect the riskiness that the project contributes to overall operations has important implications for government investments as well. Like GM but even more so, the U.S. government could acquire funds for a risky activity while still borrowing on a riskless ("full faith and credit") basis. However, if the rate of return on a government project is not sufficient to reflect its riskiness, adopting it will reduce the overall wealth of the citizenry.[19]

14.7 THE FUNDAMENTALS OF INVESTMENT, SAVING, AND INTEREST

What are the major forces that explain why investment and saving have been high at certain times and in some geographical areas and at other times and places have been low? Similarly for interest rates: when and where are they high, and when and where are they low?

Interest, saving, and investment all relate to comparisons between present and future; therefore, time is the crucial consideration. The three fundamentals are *time preference, time endowment,* and *time productivity.*

Time Preference The more impatient people are, the more they prefer current consumption. Think in terms of a "representative individual" in an economy. The two panels of Figure 14.8 illustrate the implications of steep indifference curves (high time preference) and flat indifference curves (low time preference). In each case, the representative individual's preference map is juxtaposed against a typical Production-Possibility Curve *QQ*. For simplicity the endowment point *E* is assumed to lie on the horizontal axis.

In a representative-individual model, the equilibrium is at the tangency of *QQ* with the highest attainable indifference curve. Furthermore, because from equation (14.1) the absolute slope of the budget line equals $1 + r_1$, the slope of this tangency determines the interest rate.[20] For the high time preference pciture in panel (a), it is

[19]J. Hirshleifer, "Investment Decision Criteria: Private Decisions" and "Investment Decision Criteria: Public Decisions," in *The New Palgrave Dictionary of Money and Finance* (Macmillan, 1992).

[20]Figure 14.8 has a certain similarity to the Robinson Crusoe diagram of Figure 14.4. However, whereas in Figure 14.4 the tangency R^* was an *optimum* position for Crusoe, here the corresponding point $Q^* = C^*$ is the *equilibrium* of an economy with identical individuals. The crucial difference is that in a Crusoe situation there is no budget line. In a representative-individual picture, however, a budget line can be drawn through the tangency point. Anyone can trade but, everyone being alike, at the equilibrium price ratio no trade actually takes place.

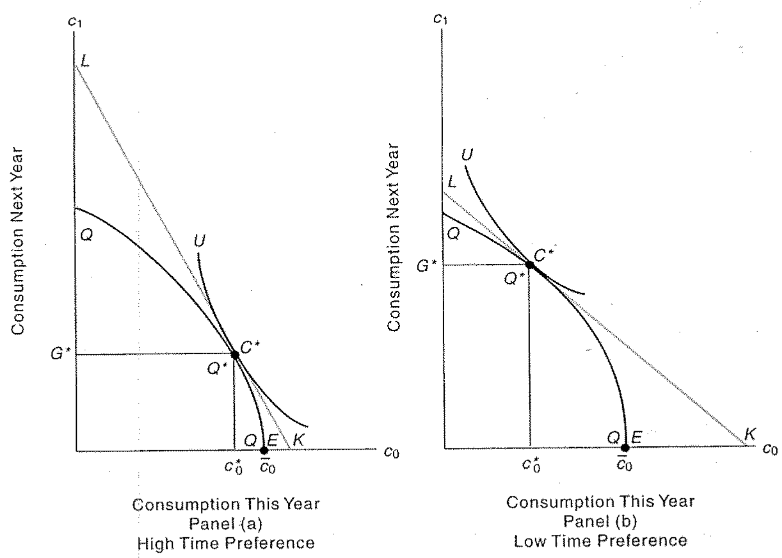


FIGURE 14.8

The Effects of Time Preference on Investment and Interest: Representative-Individual Model

In a representative-individual model the equilibrium is determined by the tangency of the Production-Possibility Curve QQ with the highest attainable indifference curve. (Although a budget line KL exists, because everyone is alike no trading takes place.) In panel (a) time preference is high (the indifference curves are steep); thus, the equilibrium interest rate is also high and little investment takes place. In panel (b) time preference is low (the indifference curves are flat); the equilibrium interest rate is therefore low and a great deal of investment takes place.

evident that the scale of saving and investment $\bar{c}_0 - c_0^*$ will be low and the interest rate r_1 (steepness of the equilibrium budget line) high. The reverse holds for the low time preference picture in panel (a).

Low time preference, willingness to invest even at a low interest rate, is associated with personal characteristics such as farsightedness, strong family ties, ability to defer enjoyment, and the like. The later years of the Roman Empire were characterized by a decline in such "puritanical" attitudes (shift toward high time preference), and interest rates accordingly rose. Arguably, a similar change in values has taken place in recent decades, as is evidenced by lower saving rates and rising real interest rates.

Even more fundamentally, time preference is linked to biological factors. Personal mortality is surely a discouragement to saving, moderated by the fact that offspring provide a vicarious way of surviving past one's own life span. In recent times these considerations have operated in opposite directions. Rising life spans have encouraged saving, while smaller family sizes have discouraged it.

Time Endowment The effects of differing time endowments are pictured in Figure 14.9. First, suppose the representative endowment E is again on the horizontal axis. The solution $Q^* = C^*$ is as before at the tangency of the Production-Possibility Curve QQ with the highest attainable indifference curve. The absolute slope of the equilibrium budget line through $Q^* = C^*$ determines the interest rate r_1, and the amount of investment is shown by the horizontal distance between E and Q^*.

A change in time endowment toward the future is pictured by an upward shift of the endowment point to the position E′. If *time productivity* remains the same, the technological possibilities for transforming current income into future income are unchanged. Geometrically, the new Production-Possibility Curve $Q'Q'$ through E' will be vertically parallel to QQ. The new optimum is $Q'^* = C'^*$. Without extended discussion, it is evident from the geometry that the new equilibrium interest rate r_1' is higher than r_1 (the new equilibrium budget line has steeper slope). Furthermore, the scale of investment (the horizontal distance between E' and Q'^*) will be less.

As an illustration in the opposite direction, consider a community struck with disaster. A catastrophe usually damages goods relatively close to consumption more drastically than it injures the basic productive powers of the economy. A drought or a hurricane, for example, may destroy crops while leaving long-term productive fundamentals—fertility of the soil, mineral resources, and human skills—unimpaired. Because present incomes are affected more seriously than anticipated future incomes, the endowment point will shift not toward the north as in Figure 14.9 but toward the west

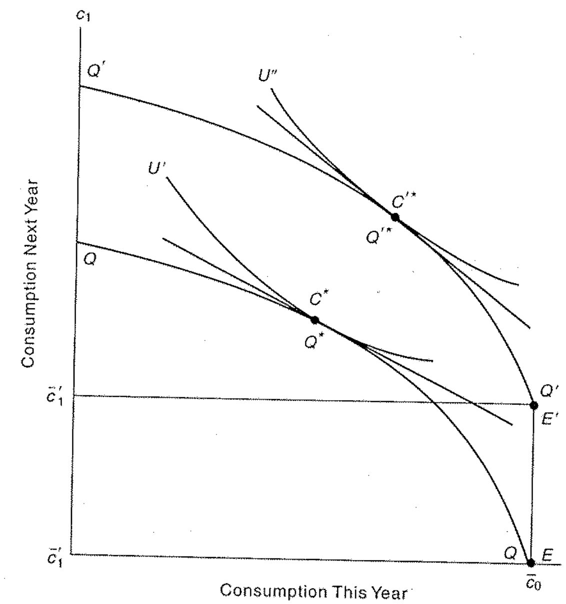


FIGURE 14.9

The Effects of Time Endowment on Investment and Interest: Representative-Individual Model

Here the endowment shifts vertically from E to E', owing to an increase in the future-dated element from $\bar{c}_1 = 0$ to c_1'. Assuming the technological possibilities for transforming current income into future income remain unchanged, the Production-Possibility Curve QQ also shifts vertically upward to the position $Q'Q'$. The effect is to raise the interest rate (as shown by the slopes of the budget lines that can be drawn through Q and Q') and decrease the scale of investment.

instead. It can easily be verified that the scale of investment will tend to fall while the interest rate will tend to rise.

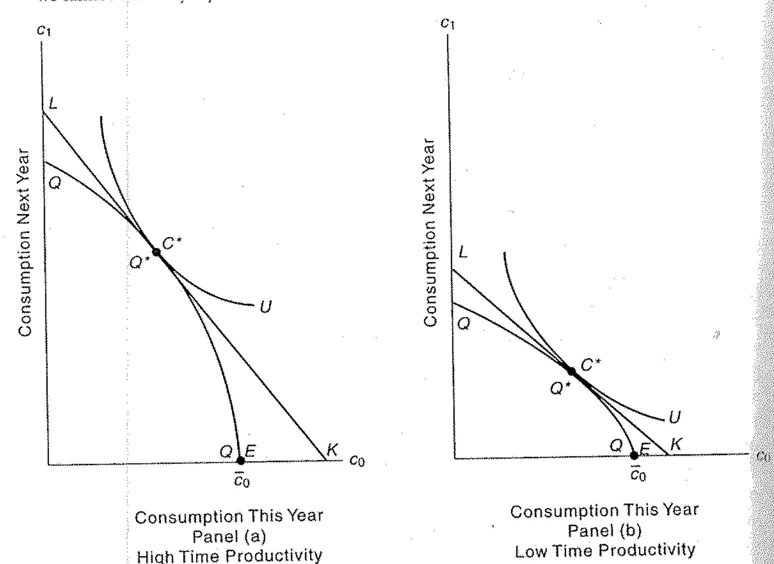
Time Productivity In the two panels of Figure 14.10, higher and lower time productivity of investment are represented by steeper and flatter Production-Possibility Curves QQ. With high time productivity, interest rates will tend to be high; with low time productivity, they will be low. This difference has been observed in comparisons between newer and more productive versus older and more mature communities. For example, interest rates historically have been higher in America than in Britain, and higher in California than in New England.

With regard to the scale of investment, there are countervailing effects. There is of course less inducement to invest when marginal time productivity is low (flat QQ curve). On the other hand, a flatter QQ curve means there will necessarily be smaller provision for the future. The relative poverty of the future acts to *increase* the motive to invest, even if the marginal payoff is relatively small. Overall, then, with low time productivity there will be lesser provision for the future, but we cannot say whether or not the scale of investment will be less.

FIGURE 14.10

The Effects of Time Productivity on Investment and Interest: Representative-Individual Model

In panel (a) time productivity is high (the Production-Possibility Curve QQ is steep), so the equilibrium interest rate is also high. In panel (b) time productivity is low (the QQ curve is relatively flat); the equilibrium interest rate is therefore low. At the equilibrium $Q^* = C^*$ in panel (b), although the consumption provided for the future is certainly less than in panel (a), we cannot definitely say whether the scale of investment is also less.



Two other factors supplement the fundamentals described above:

Degree of Isolation The interest rate in a locality that is financially linked with other regions cannot diverge too far from the more normal rates in the outside world. If such a divergence ever appeared, investments and loans would flow from the low-interest area to the high-interest area. The differences historically observed between interest rates in Britain and America, or between New England and California, have therefore been much smaller than would have been the case had the communities in question been more completely isolated from one another.

Risk Riskiness is correlated with futurity. Today already exists, tomorrow is somewhat uncertain, and the farther future is increasingly shrouded in mist. Given *risk aversion*, a future endowment that is more uncertain (has higher perceived variability risk) is like a future endowment that is effectively smaller. Thus, the riskiness of the future acts as an impoverishment of the future, increasing the "precautionary" motivation to save (saving for a rainy day).

Example 14.10 *SAVING AND UNEMPLOYMENT INSURANCE*

The risk of unemployment provides a major motive for precautionary saving. However, this motive is weakened to the extent that government unemployment insurance (UI) replaces lost income.

Between 1984 and 1990, UI programs replaced about 45 percent of workers' lost earnings, for up to 26 weeks. A study by Eric M. Engen and Jonathan Gruber[a] analyzed the hypothetical effects of changes in this replacement proportion. An increase from 45 percent to 55 percent would, they calculated, reduce per capita holdings of financial assets by between \$109 and \$290. At the same time, for a typical unemployment spell of about 13 weeks, the higher replacement proportion on average provides an additional \$547 of income. Therefore, they concluded, as a first approximation each dollar provided under UI crowds out between 20¢ and 53¢ of individual saving.

However, they agreed, saving might be motivated not by fear of a single unemployment episode but of repeated spells of unemployment. If the reduced level of financial assets were attributed to expecting to receive \$547 of UI payments two times rather than only once, the estimated crowding-out effect per dollar would be about halved. As another qualification, the estimates applied only to effects upon financial assets. Conceivably, UI might also reduce saving reflected by other forms of wealth, such as pension entitlements or home ownership. However, it appeared that financial assets were the only ones substantially affected by UI.

As additional support for their main conclusion, the authors found the crowding-out effect to be larger for workers who face higher risks of unemployment. The crowding-out effect was also larger for single workers, who lack the safety net provided by spousal earnings.

[a]Eric M. Engen and Jonathan Gruber, "Unemployment Insurance and Precautionary Saving," working paper no. 5252, National Bureau of Economic Research, 1995.

SUMMARY

Saving is consuming less than one's income; *investment* is a productive activity that transforms potential current income into future income. Individuals' choices as to present consumption versus future consumption interact in the market to determine the price ratios between current and future real goods. In the simplest two-date case, the ratio P_0/P_1 can also be written as $1 + r_1$, which defines the real rate of interest r_1 between date 0 (the present) and date 1 (one year from now).

In a hypothetical world of pure exchange (where investment opportunities are absent), each individual's wealth—the market value of one's endowment—is fixed. Within this constraint each person borrows or lends to achieve an optimum time pattern of consumption C^*. Since aggregate investment is zero, saving by some people (lending) is necessarily balanced by the dissaving of others (borrowing). The intersection of the aggregate supply curve of lending and the aggregate demand curve for borrowing determines P_0/P_1 and therefore the equilibrium interest rate r_1^*, together with the equilibrium amount of borrowing and lending $B^* = L^*$.

In a world of production and exchange, where real investment opportunities exist, the individual maximizes wealth by choosing a *production optimum* Q^* (determining the level of personal investment) together with a utility-maximizing *consumption optimum* C^* (determining the level of saving). If a person's investment exceeds saving, the difference is made up by borrowing from other savers; in the reverse case, the difference is lent out to other investors. The intersection of the aggregate supply curve of saving and the aggregate demand curve for investment determines the market equilibrium rate of interest r_1^* and the aggregate amount of saving and investment $S^* = I^*$; at this interest rate, aggregate borrowing also equals aggregate lending: $B^* = L^*$. The difference between the magnitudes of $S^* = I^*$ and $B^* = L^*$ represents the amount of "self-financed" investment.

This analysis generalizes easily to any number of dates, so as to determine the time sequences of consumption, saving and investment, and lending and borrowing—together with the "term structure" of short-term and long-term interest rates.

If markets for intertemporal claims ("capital markets") are perfect, the Separation Theorem holds. If the Separation Theorem is at least approximately valid, a manager who maximizes the wealth of the firm will be making the correct production decisions for all the owners individually, regardless of their possibly differing time preferences. This is what makes possible the formation of large business firms.

A project or set of projects that increases wealth has positive Present Value (PV). In the simple two-period case, with cash flows z_0 and z_1, Present Value V_0 is defined as:

$$V_0 \equiv z_0 + \frac{z_1}{1 + r}$$

If a single project is independent of the others considered, it should be adopted if V_0 is positive, rejected if V_0 is negative. If projects are mutually exclusive, the one with largest V_0 should be chosen. As the most general rule, the *overall set of projects* that maximizes Present Value should be adopted.

The definition of Present Value can be generalized to the multiperiod stream of payments $z_0, z_1, \ldots, z_T$, using either the sequence of short-term interest rates $r_1, r_2, \ldots, r_T$ or long-term rates $R_1, R_2, \ldots, R_T$. If the interest rate is expected to remain at the constant level r over time, the Present Value formula takes the simple form:

$$V_0 \equiv z_0 + \frac{z_1}{1 + r} + \ldots + \frac{z_T}{(1 + r)^T}$$

An alternative criterion sometimes used in project evaluation is the Rate of Return (ROR), which is intended to measure the growth rate of funds invested in a project. The ROR is defined as ρ in:

$$0 \equiv z_0 + \frac{z_1}{1 + \rho} + \ldots + \frac{z_T}{(1 + \rho)^T}$$

It would be definitely incorrect to choose among mutually exclusive projects or sets of projects on the basis of which has the highest ρ—because then a high-ρ but small-scale project could displace a larger project that, despite lower ρ, generates a greater wealth increment V_0. For independent projects, however, the ROR Rule "Adopt if $\rho > r$" leads to the same answers as the PV Rule "Adopt if $V_0 > 0$"—*provided* there is only a single sign alternation in the sequence of payments. (That is, if an initial outlay period is succeeded by a later payoff period.)

In selecting an appropriate interest rate r for use either in calculating V_0 or for comparison with the ROR measure ρ, the major difficulty is to allow properly for risk. Each firm has a "cost of capital" that represents the financial markets' evaluation of the riskiness of its anticipated annual payoffs z. Measuring the anticipated returns by the mathematical "expectation" $\hat{z}$ and variability risk by the standard deviation $\sigma(z)$, the cost of capital will reflect the ratio $\hat{z}/\sigma(z)$. Each project should be evaluated in the light of its impact upon the firm's overall $\hat{z}$ and $\sigma(z)$. If a project leaves the firm's ratio $\hat{z}/\sigma(z)$ unaffected—in other words, if its riskiness is typical of the existing activities of the business—then the appropriate discount rate would be the same as the firm's overall cost of capital.

The *real* interest rate r_1 is the extra market value of current real goods over future real goods. The money interest rate r_1' is the extra market value of current money over future money. The relation between the real interest rate and the money interest rate depends on the anticipated rate of price-level inflation a_1:

$$r_1' = r_1 + a_1$$

The explanation is that someone who borrows money, in order to repay in money, must offer the lender an additional return to cover the expected fall in the real value of money.

The fundamental determinants of the real interest rate and the scale of investment in a community include *time preference* (more urgent desires for current goods tend to reduce investment and raise interest rates), *time endowment* (anticipations of higher future income raise interest rates and reduce investment), and *time productivity* (better productive opportunities raise both investment and interest rates). Levels of *riskiness* also affect both investment and interest. Nations and regions that differ in these respects tend to be characterized also by dissimilar levels of interest rates. However, differences in interest rates are moderated by international financial markets, which channel investible funds from low-interest to high-interest areas.

QUESTIONS

For Review

1. Explain the analogy between the intertemporal optimum of the consumer (choice between current consumption C_0 and future consumption C_1) and the optimum of the consumer at a moment of time (choice between consumption of commodity X and commodity Y).

†2. Which statement is correct? Explain.
   a. The annual rate of interest is the ratio P_0/P_1, the price of a current consumption claim divided by the price of a consumption claim dated one year in the future.
   b. The annual rate of interest is the *premium* on the value of current relative to one-year future claims, as given by the identity $P_0/P_1 \equiv 1 + r_1$.

3. What is wealth? How is it related to current and future incomes?

†4. In market intertemporal equilibrium under pure exchange, aggregate borrowing B equals aggregate lending L. What can be said about the equilibrium of saving S and investment I? Is the amount $I = S$ necessarily smaller or larger than the amount $B = L$?

†5. Turning to equilibrium in a production economy, aggregate saving must equal aggregate investment. What can be said about borrowing and lending?

6. What is the Separation Theorem? What is its importance? What would tend to happen if it were not applicable?

†7. a. What is the Present-Value Rule?
   b. Will this rule always lead decision makers to correct choices of projects when the Separation Theorem holds?
   c. What if the Separation Theorem does not hold?

8. Explain the relation between the rate of interest r as defined in the equation $r \equiv P_0/P_1 - 1$ and as defined in the equation $r \equiv z/V_0$.

9. What is the *money* rate of interest, and how is it related to the *real* rate of interest?

For Further Thought and Discussion

1. In a two-period preference diagram, picture the following endowment situations. Indicate whether the person is likely to be a borrower or a lender.
   a. A young woman with an elderly, wealthy, loving uncle in Australia.
   b. A farmer whose crop has been destroyed by hurricane.
   c. A sugar beet farmer who has just learned that this year's sugarcane crop has been destroyed by hurricane.
   d. A 35-year-old star baseball player.

†2. "Saving need not equal investment for any single individual, but the two must be equal for the market as a whole." Is this necessarily true in equilibrium? Would it be true in a disequilibrium situation, as might result from a floor or ceiling upon interest rates?

†3. In a newly settled country, resources are likely to have great potential but are as yet undeveloped.
   a. Would you expect the real interest rate to be high or low?
   b. Comparing situations in which the new country is or is not in close contact with the rest of the world, in which situation will the interest rate be higher?
   c. In which situation will more investment take place? Explain.

†4. One country is "stagnant" (i.e., has little investment and economic growth) because productive opportunities yielding a good return on investment are lacking. Another stagnant country has excellent investment opportunities, but has little investment because the citizens' time preferences are very high. Which country would tend to have a high, and which a low, real interest rate? Explain.

†Answers to these questions appear at the end of the book.

5. Money interest rates throughout the world have generally been higher since World War II than they were in the prewar period. Indicate which of the following might provide part of the explanation, and analyze.
   a. Higher rates of time preference (changes in tastes).
   b. Higher rates of time productivity (changes in investment opportunities).
   c. Lower ratios of current to anticipated future incomes (increased relative scarcity of current endowments).
   d. Higher rates of inflation (changes in anticipations as to monetary policies of governments).

†6. a. Are negative rates of interest impossible?
   b. Is there a limit upon how negative the interest rate can be?

7. "Annual income twenty pounds, annual expenditure nineteen pounds, result happiness. Annual income twenty pounds, annual expenditure twenty-one pounds, result misery" (Mr. Micawber in Charles Dickens's *David Copperfield*). Is this sound economics? Analyze.

†8. An issue of *Consumer Reports* magazine asked whether homebuyers might advantageously finance purchases of appliances by additions to their home mortgages. The alternatives considered were (1) purchase of appliances through a retail store for \$675, financed by a two-year contract at 15 percent interest; and (2) purchase of the same appliances for \$450 through the home builder, financed by a mortgage add-on 27-year contract at 7.75 percent interest. The article contended that the first option was superior. The justification offered was that, for the two-year 15 percent contract, the total of interest-plus-principal payments would add up to only \$785—whereas, for the 27-year mortgage add-on contract at 7.75 percent, the total payments would eventually sum to \$1,075. Analyze.

†9. During World War II it was necessary to decide how much of military expenditures were to be financed by taxation and how much by government borrowing. Some economists argued that financing the war by borrowing would "shift the burden to future generations." Is this correct? How would you go about determining how much of the cost of a war is borne by the current generation versus future generations?