tution effect (the change in quantity demanded when the level of utility is fixed) and an income effect (the change in the quantity demanded with the level of utility changing but the relative price of good X unchanged). We denote the change in X that results from a unit change in the price of X holding by $\partial X/\partial P_X|_{U=U^*}$. Thus the total change in X resulting from a unit change in P_X is

$$dX/dP_X = \partial X/\partial P_X|_{U=U^*} + (\partial X/\partial I)(\partial I/\partial P_X) \quad (A4.17)$$

The first term on the right-hand side of equation (A4.17) is the substitution effect (because utility is fixed), and the second term is the income effect (because income increases).

From the consumer's budget constraint, $I = P_X X + P_Y Y$, we know by differentiation that

$$\partial I/\partial P_X = X \quad (A4.18)$$

Suppose for the moment that the consumer owned goods X and Y. Then equation (A4.18) would tell us that when the price of good X increases by $1, the amount of income the consumer can obtain by selling the good increases by $X. In our theory of the consumer, however, the consumer does not own the good. As a result, equation (A4.18) tells us how much additional income the consumer would need to leave him as well off after the price change as before. For this reason, it is customary to write the income effect as negative (reflecting a loss of purchasing power) rather than positive. Equation (A4.17) then appears as follows:

$$dX/dP_X = \partial X/\partial P_X|_{U=U^*} - X(\partial X/\partial I) \quad (A4.19)$$

In this new form, called the *Slutsky equation*, the first term represents the substitution effect, the change in demand for good X obtained by keeping utility fixed. The second term is the income effect, the change in purchasing power resulting from the price change times the change in demand resulting from that change in purchasing power.

EXERCISES

1. Which of the following utility functions are consistent with convex indifference curves, and which are not?
   a. $U(X,Y) = 2X + 5Y$
   b. $U(X,Y) = (XY)^{.5}$
   c. $U(X,Y) = \text{Min}(X,Y)$, where Min is the minimum of the two values of X and Y.

2. Show that two utility functions given below generate the identical demand functions for goods X and Y:
   a. $U(X,Y) = \log(X) + \log(Y)$
   b. $U(X,Y) = (XY)^{.5}$

3. Assume that a utility function is given by Min (X,Y), as in Exercise 1c. What is the Slutsky equation that decomposes the change in the demand for X in response to a change in its price? What is the income effect? What is the substitution effect?

CHAPTER 5

CHOICE UNDER UNCERTAINTY

So far we have assumed that prices, incomes, and other variables are known with certainty. However, many of the choices that people make involve considerable uncertainty. For example, most people borrow to finance large purchases, such as a house or a college education, and plan to pay for the purchase out of future income. But for most of us, future incomes are uncertain. Our earnings can go up or down; we can be promoted, demoted, or even lose our jobs. Or if we delay buying a house or investing in a college education, we risk having its price rise in real terms, making it less affordable. How should we take these uncertainties into account when making major consumption or investment decisions?

Sometimes we must choose how much risk to bear. What, for example, should you do with your savings? Should you invest your money in something safe, such as a savings account, or something riskier but potentially more lucrative, such as the stock market? Another example is the choice of a job or even a career. Is it better to work for a large, stable company where job security is good but the chances for advancement are limited, or to join (or form) a new venture, which offers less job security but more opportunity for advancement?

To answer questions such as these, we must be able to quantify risk so we can compare the riskiness of alternative choices. We therefore begin this chapter by discussing measures of risk. Afterwards, we will examine people's preferences toward risk. (Most people find risk undesirable, but some people find it more undesirable than others.) Next, we will see how people can deal with risk. Sometimes risk can be reduced—by diversification, by buying insurance, or by investing in additional information. In other situations (e.g., when investing in stocks or bonds), people must choose the amount of risk they wish to bear.

5.1 DESCRIBING RISK

To describe risk quantitatively, we need to know all the possible outcomes of a particular action and the likelihood that each outcome will occur.[1] Suppose, for example, that you are considering investing in a company that is exploring for offshore oil. If the exploration effort is successful, the company's stock will increase from $30 to $40 a share; if not, it will fall to $20 a share. Thus, there are two possible future outcomes, a $40 per share price and a $20 per share price.

Probability

Probability refers to the likelihood than an outcome will occur. In our example, the probability that the oil exploration project is successful might be ¼, and the probability that it is unsuccessful ¾. Probability is a difficult concept to formalize because its interpretation can depend on the nature of the uncertain events and on the beliefs of the people involved. One *objective* interpretation of probability relies on the frequency with which certain events tend to occur. Suppose we know that of the last 100 offshore oil explorations 25 have succeeded and 75 have failed. Then the probability of success of ¼ is objective because it is based directly on the frequency of similar experiences.

But what if there are no similar past experiences to help measure probability? In these cases objective measures of probability cannot be deduced, and a more *subjective* measure is needed. Subjective probability is the perception that an outcome will occur. This perception may be based on a person's judgment or experience, but not necessarily on the frequency with which a particular outcome has actually occurred in the past. When probabilities are subjectively determined, different people may attach different probabilities to different outcomes and thereby make different choices.[2] For example, if the search for oil were to take place in an area where no previous searches had ever occurred, I might attach a higher subjective probability than you to the chance that the project will succeed because I know more about the project, or because I have a better understanding of the oil business and can therefore make better use of our common information. Either different information or different abilities to process the same information can explain why subjective probabilities vary among individuals.

[1]Some people distinguish between uncertainty and risk along the lines suggested by the economist Frank Knight some 60 years ago. *Uncertainty* can refer to situations in which many outcomes are possible but their likelihoods are unknown. *Risk* then refers to situations in which we can list all possible outcomes, and we know the likelihood that each outcome will occur. We will always refer to risky situations but will simplify the discussion by using uncertainty and risk interchangeably.

[2]In any case, the probable outcomes must be *mutually exclusive*, in the sense that one and only one actual outcome will occur in the future. As a result, the probabilities associated with each possible outcome will sum to one.

Whatever the interpretation of probability, it is used in calculating two important measures that help us describe and compare risky choices. One measure tells us the *expected value* and the other the *variability* of the possible outcomes.

Expected Value

The *expected value* associated with an uncertain situation is a weighted average of the *payoffs* or values associated with all possible outcomes, with the probabilities of each outcome used as weights. The expected value measures the *central tendency*, i.e., the payoff that we would expect on average. Our offshore oil exploration example has two possible outcomes: Success yields a payoff of $40 per share, while Failure yields a payoff of $20 per share. Denoting "probability of" by Pr, the expected value in this case is given by

$$\text{Expected Value} = \text{Pr(Success)}(\$40/\text{share}) + \text{Pr(Failure)}(\$20/\text{share})$$
$$= (1/4)(\$40/\text{share}) + (3/4)(\$20/\text{share}) = \$25/\text{share}$$

More generally, if there are two possible outcomes having payoffs X_1 and X_2, and the probabilities of each outcome are given by Pr_1 and Pr_2, then the expected value E(X) is

$$E(X) = Pr_1X_1 + Pr_2X_2 \quad (5.1)$$

Variability

Suppose you are choosing between two part-time sales jobs that have the same expected income ($1500). The first job is based entirely on commission—the income earned depends on how much you sell. The second job is salaried. There are two equally likely incomes under the first job—$2000 for a good sales effort and $1000 for one that is only modestly successful. The second job pays $1510 most of the time, but you would earn $510 in severance pay if the company goes out of business. Table 5.1 summarizes these possible outcomes, their payoffs, and their probabilities.

TABLE 5.1 INCOME FROM SALES JOBS

	Outcome 1		Outcome 2	
	Probability	Income ($)	Probability	Income ($)
Job 1: Commission	.5	2000	.5	1000
Job 2: Fixed salary	.99	1510	.01	510