

$$\frac{\partial X}{\partial P} = \frac{\partial X}{\partial P_1} + \dots + \frac{\partial X}{\partial P_n} \quad (\text{A4.17})$$

The first term on the right-hand side of equation (A4.17) is the substitution effect (because utility is fixed), and the second term is the income effect (because income increases). D.V. & P.Y. know by definition.

From the consumer's budget constraint, $I = P_x X + P_y Y$, we know $\partial I / \partial X$ cause income and substitution effects.

$$\partial V / \partial P = -X$$

Suppose for the moment that the consumer owned goods X and Y. Then equation (A4.18) would tell us that when the price of good X increases by \$1, the amount of income the consumer can obtain by selling the good increases by \$X. In our theory of the consumer, however, the consumer does not own the good. As a result, equation (A4.18) tells us how much additional income the consumer would need to leave him as well off after the price change as before. For this reason, it is customary to write the income effect as negative (reflecting a loss of purchasing power) rather than positive. Equation (A4.17) then

$$dX/dP_Y = \partial X / \partial P_X|_{U=U^*} - X(\partial X / \partial I) \quad (\text{A4.1})$$

In this new form, called the *Slutsky equation*, the first term represents the substitution effect, the change in demand for good X obtained by keeping utility fixed. The second term is the income effect, the change in purchasing power resulting from the price change times the change in demand resulting from that change in purchasing power.

CHOICE UNDER
UNCERTAINTY

So far we have assumed that prices, incomes, and other variables are known with certainty. However, many of the choices that people make involve considerable uncertainty. For example, most people borrow to finance large purchases, such as a house or a college education, and plan to pay for the purchase out of future income. But for most of us, future incomes are uncertain. Our earnings can go up or down; we can be promoted, demoted, or even lose our jobs. Or if we delay buying a house or investing in a college education, we risk having its price rise in real terms, making it less affordable. How should we take these uncertainties into account when making major consumption or investment decisions?

Sometimes we must choose how much risk to bear. What, for example, should you do with your savings? Should you invest your money in something safe, such as a savings account, or something riskier but potentially more lucrative, such as the stock market? Another example is the choice of a job or even a career. Is it better to work for a large, stable company where job security is good but the chances for advancement are limited, or to join (or form) a new venture, which offers less job security but more opportunity for advance-

To answer questions such as these, we must be able to quantify risk so we can compare the riskiness of alternative choices. We therefore begin this chapter by discussing measures of risk. Afterwards, we will examine people's preferences toward risk. (Most people find risk undesirable, but some people find it more undesirable than others.) Next, we will see how people can deal with risk. Sometimes risk can be reduced—by diversification, by buying insurance, or by investing in additional information. In other situations (e.g., when investing in stocks or bonds), people must choose the amount of risk they wish to bear.

and Y_i :

1. Which of the following utility functions are consistent with convex indifference curves, and which are not?

 - $U(X,Y) = 2X + 5Y$
 - $U(X,Y) = (XY)^{.5}$
 - $U(X,Y) = \text{Min}(X,Y)$, where Min is the minimum of the two values of X and Y.

2. Show that two utility functions given below generate the identical demand functions for good

Whatever the interpretation of probability, it is used in calculating two important measures that help us describe and compare risky choices. One measure tells us the *expected value* and the other the *variability* of the possible outcomes.

5.1 DESCRIBING RISK

To describe risk quantitatively, we need to know all the possible outcomes of a particular action and the likelihood that each outcome will occur.¹ Suppose, for example, that you are considering investing in a company that is exploring for offshore oil. If the exploration effort is successful, the company's stock will increase from \$30 to \$40 a share; if not, it will fall to \$20 a share. Thus, there are two possible future outcomes, a \$40 per share price and a \$20 per share price.

Probability

Probability refers to the likelihood that an outcome will occur. In our example, the probability that the oil exploration project is successful might be $\frac{1}{4}$, and the probability that it is unsuccessful $\frac{3}{4}$. Probability is a difficult concept to formalize because its interpretation can depend on the nature of the uncertain events and on the beliefs of the people involved. One *objective* interpretation of probability relies on the frequency with which certain events tend to occur. Suppose we know that of the last 100 offshore oil explorations 25 have succeeded and 75 have failed. Then the probability of success of $\frac{1}{4}$ is objective because it is based directly on the frequency of similar experiences.

But what if there are no similar past experiences to help measure probability? In these cases objective measures of probability cannot be deduced, and a more *subjective* measure is needed. Subjective probability is the perception that an outcome will occur. This perception may be based on a person's judgment or experience, but not necessarily on the frequency with which a particular outcome has actually occurred in the past. When probabilities are subjectively determined, different people may attach different probabilities to different outcomes and thereby make different choices.² For example, if the search for oil were to take place in an area where no previous searches had ever occurred, I might attach a higher subjective probability than you to the chance that the project will succeed because I know more about the project, or because I have a better understanding of the oil business and can therefore make better use of our common information. Either different information or different abilities to process the same information can explain why subjective probabilities vary among individuals.

¹Some people distinguish between uncertainty and risk along the lines suggested by the economist Frank Knight some 60 years ago. *Uncertainty* can refer to situations in which many outcomes are possible but their likelihoods are unknown. Risk then refers to situations in which we can list all possible outcomes, and we know the likelihood that each outcome will occur. We will always refer to risky situations but will simplify the discussion by using uncertainty and risk interchangeably.

²In any case, the probable outcomes must be *mutually exclusive*, in the sense that one and only one actual outcome will occur in the future. As a result, the probabilities associated with each possible outcome will sum to one.

The *expected value* associated with an uncertain situation is a weighted average of the payoffs or values associated with all possible outcomes, with the probabilities of each outcome used as weights. The expected value measures the *central tendency*, i.e., the payoff that we would expect on average. Our offshore oil exploration example has two possible outcomes: Success yields a payoff of \$40 per share, while Failure yields a payoff of \$20 per share. Denoting "probability of" by Pr , the expected value in this case is given by

$$\begin{aligned} \text{Expected Value} &= \Pr(\text{Success})(\$40/\text{share}) + \Pr(\text{Failure})(\$20/\text{share}) \\ &= (\frac{1}{4})(\$40/\text{share}) + (\frac{3}{4})(\$20/\text{share}) = \$25/\text{share} \end{aligned}$$

More generally, if there are two possible outcomes having payoffs X_1 and X_2 , and the probabilities of each outcome are given by Pr_1 and Pr_2 , then the expected value $E(X)$ is

$$E(X) = Pr_1 X_1 + Pr_2 X_2 \quad (5.1)$$

Variability

Suppose you are choosing between two part-time sales jobs that have the same expected income (\$1500). The first job is based entirely on commission—the income earned depends on how much you sell. The second job is salaried. There are two equally likely incomes under the first job—\$2000 for a good sales effort and \$1000 for one that is only modestly successful. The second job pays \$1510 most of the time, but you would earn \$510 in severance pay if the company goes out of business. Table 5.1 summarizes these possible outcomes, their payoffs, and their probabilities.

TABLE 5.1 INCOME FROM SALES JOBS

	Outcome 1	Outcome 2	
Probability	Income (\$)	Probability	Income (\$)
Job 1: Commission	.5	.5	2000
Job 2: Fixed salary	.99	.01	1510

Note that the two jobs have the same expected income because $.5(\$200) + .5(\$100) = .99 (\$1510) + .01 (\$510) = \$1500$. But the variability of the possible payoffs is different for the two jobs. This variability can be usefully analyzed by a measure that presumes that large differences (whether positive or negative) between actual payoffs and the expected payoff, called *deviations*, signal greater risk. Table 5.2 gives the deviations of actual incomes from the expected income for the example of the two sales jobs.

TABLE 5.2 DEVIATIONS FROM EXPECTED INCOME (\$)

	Outcome 1	Deviation	Outcome 2	Deviation
Job 1	\$2000	\$500	\$1000	\$500
	1510	10	510	990

In the first commission job, the average deviation is \$500, which is obtained by weighting each deviation by the probability that each outcome occurs. Thus,

$$\text{Average Deviation} = .5(\$500) + .5(\$500) = \$500$$

For the second fixed-salary job, the average deviation is calculated as follows:

$$\text{Average Deviation} = .99(\$10) + .01(\$990) = \$19.80$$

The first job is thus substantially more risky than the second because its average deviation of \$500 is much greater than the average deviation of \$19.80 for the second job.

In practice one usually encounters two closely related but slightly different measures of variability. The *variance* is the average of the *squares* of the deviations of the payoffs associated with each outcome from their expected value. The *standard deviation* is the square root of the variance. Table 5.3 gives some of the relevant calculations for our example.

The average of the squared deviations under Job 1 is given by

$$\text{Variance} = .5(\$250,000) + .5(\$250,000) = \$250,000$$

TABLE 5.3 CALCULATING VARIANCE (\$)

	Outcome 1	Deviation Squared	Outcome 2	Deviation Squared	Variance
Job 1	\$2000	\$250,000	\$1000	\$250,000	\$250,000
	1510	100	510	980,100	9,900

Probability

0.2

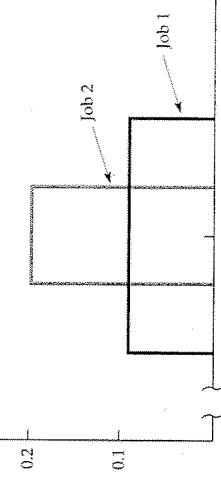


FIGURE 5.1 Outcome Probabilities for Two Jobs.

The distribution of payoffs associated with Job 1 has a greater spread and a greater variance than the distribution of payoffs associated with Job 2. Both distributions are flat because all outcomes are equally likely.

The standard deviation is therefore equal to the square root of \$250,000, or \$500. Similarly, the average of the squared deviations under Job 2 is given by

$$\text{Variance} = .99(\$100) + .01(\$980,100) = \$9900$$

The standard deviation is the square root of \$9,900, or \$99.50. Whether we use variance or standard deviation to measure risk (it's really a matter of convenience)—both provide the same ranking of risky choices, the second job is substantially less risky than the first. Both the variance and the standard deviation of the incomes earned are lower.³

The concept of variance applies equally well when there are many outcomes rather than just two. Suppose, for example, that the first job yields incomes ranging from \$1,000 to \$20,000 in increments of \$100 that are all equally likely. The second job yields incomes from \$1300 to \$1700 (again in increments of \$100) that are also all equally likely. Figure 5.1 shows the alternatives graphically.

You can see from Figure 5.1 that the first job is riskier than the second. The “spread” of possible payoffs for the first job is much greater than the spread of payoffs for the second. And the variance of the payoffs associated with the first job is greater than the variance associated with the second.

³In general, when there are two outcomes with payoffs X_1 and X_2 , each occurring with probability P_{X_1} and P_{X_2} , and $E(X)$ is the expected value of the outcomes, the variance is given by

$$\sigma^2 = P_{X_1}[(X_1 - E(X))^2] + P_{X_2}[(X_2 - E(X))^2]$$

The standard deviation, which is the square root of the variance, is written as σ .

Probability

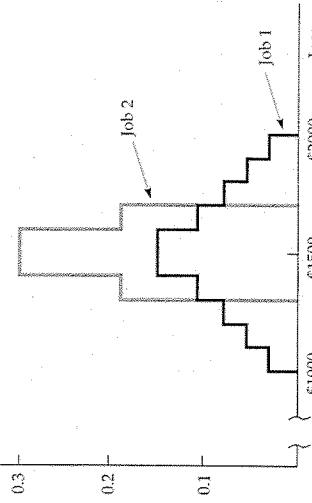


FIGURE 5.2 Unequal Probability Outcomes. The distribution of payoffs associated with Job 1 has a greater spread and a greater variance than the distribution of payoffs associated with Job 2. Both distributions are peaked because the extreme payoffs are less likely than those near the middle of the distribution.

In this particular example, all payoffs are equally likely, so the curves describing the payoffs under each job are flat. But in many cases, some payoffs are more likely than others. Figure 5.2 shows a situation in which the more extreme payoffs are the least likely. Again, the salary from Job 1 has a greater variance. From this point on we will use the variance of payoffs to measure the variability of risky situations.

Decision Making

Suppose you are choosing between the two sales jobs described in our original example. Which job would you take? If you dislike risk, you will take the second job. It offers the same expected income as the first but with less risk. But suppose we add \$100 to each of the payoffs in the first job, so that the expected payoff increases from \$1500 to \$1600. Table 5.4 gives the new earning and the squared deviations.

The jobs can then be described as follows:

Job 1:	Expected Income = \$1600	Variance = \$250,000	Variance = \$9900
Job 2:	Expected Income = \$1500	Variance = \$250,000	Variance = \$980,100

TABLE 5.4 INCOMES FROM SALES JOBS—MODIFIED (\$)

	Outcome 1	Deviation Squared	Outcome 2	Deviation Squared
Job 1	\$2100	\$250,000	\$1100	\$250,000
Job 2	1510	100	510	980,100

- Job 1 offers a higher expected income but is substantially riskier than Job 2. Which job is preferred depends on you. An aggressive entrepreneur may opt for the higher expected income and higher variance, but a more conservative person might opt for the second. To see how people might decide between incomes that differ in both expected value and in riskiness, we need to develop our theory of consumer choice further.

5.2 PREFERENCES TOWARD RISK

We used a job example to describe how people might evaluate risky outcomes, but the principles apply equally well to other choices. In this section we concentrate on consumer choices generally, and on the utility that consumers obtain from choosing among risky alternatives. To simplify things, we'll consider the consumption of a single commodity—the consumer's income, or more appropriately, the market basket that the income can buy. We assume that all consumers know all probabilities, and (for much of this section) that payoffs are now measured in terms of utility rather than dollars.

Figure 5.3a shows how we can describe a woman's preferences toward risk. The curve OB , which gives her utility function, tells us the level of utility (on the vertical axis) that she can attain for each level of income (measured in thousands of dollars on the horizontal axis). The level of utility increases from 10 to 16 to 18 as income increases from \$10,000 to \$20,000. But note that marginal utility is diminishing, falling from 10 when income increases from 0 to \$10,000, to 6 when income increases from \$10,000 to \$20,000, to 2 when income increases from \$20,000 to \$30,000.

Now suppose she has an income of \$15,000 and is considering a new but risky sales job that will either double her income to \$30,000 or cause it to fall to \$10,000. Each possibility has a probability of .5. As Figure 5.3a shows, the utility level associated with an income of \$10,000 is 10 (at Point A), and the utility level associated with a level of income of \$30,000 is 18 (at B). The risky job must be compared with the current job, for which the utility is 13 (at C).

To evaluate the new job, she can calculate the expected value of the resulting income. Because we are measuring value in terms of the woman's utility, we

must calculate the *expected utility* she can obtain. The expected utility is the sum of the utilities associated with all possible outcomes, weighted by the probability that each outcome will occur. In this case, expected utility is

$$E(u) = (\frac{1}{2})u(\$10,000) + (\frac{1}{2})u(\$30,000) = 0.5(10) + 0.5(18) = 14$$

The new risky job is thus preferred to the original job because the expected utility of 14 is greater than the original utility of 13.

The old job involved no risk—it guaranteed an income of \$15,000 and a utility level of 13. The new job is risky, but it offers the prospect of both a higher expected income (\$20,000) and, more important, a higher expected utility. If the woman wished to increase her expected utility, she would take the risky job.

Different Preferences Toward Risk

People differ in their willingness to bear risk. Some are risk averse, some risk loving, and some risk neutral. A person who prefers a certain given income to a risky job with the same expected income is described as being *risk averse*. (Such a person has a diminishing marginal utility of income.) Risk aversion is the most common attitude toward risk. To see that most people are risk averse most of the time, note the vast number of risks that people insure against. Most people not only buy life insurance, health insurance, and car insurance, but also seek occupations with relatively stable wages.

Figure 5.3a applies to a woman who is risk averse. Suppose she can have a certain income of \$20,000, or a job yielding an income of \$30,000 with probability .5 and an income of \$10,000 with probability .5 (so that the expected income is \$20,000). As we saw, the expected utility of the uncertain income is 14, an average of the utility at point A (10) and the utility at B (18), and is shown by E. Now we can compare the expected utility associated with the risky job to the utility generated if \$20,000 were earned without risk. This utility level, 16, is given by D in Figure 5.3a. It is clearly greater than the expected utility associated with the risky job.

A person who is *risk neutral* is indifferent between earning a certain income and an uncertain income with the same expected income. In Figure 5.3c the utility associated with a job generating an income of either \$10,000 or \$30,000 with equal probability is 12, as is the utility of receiving a certain income of \$20,000.⁴

⁴When people are risk neutral, the marginal utility of income is constant, so the income they earn can be used as an indicator of well-being. A government policy that doubled people's incomes would then also double their utility. At the same time, government policies that alter the risks that people face, without changing their expected incomes, would not affect their well-being. Risk neutrality allows one to avoid the complications that might be associated with the effects of governmental actions on the riskiness of outcomes.

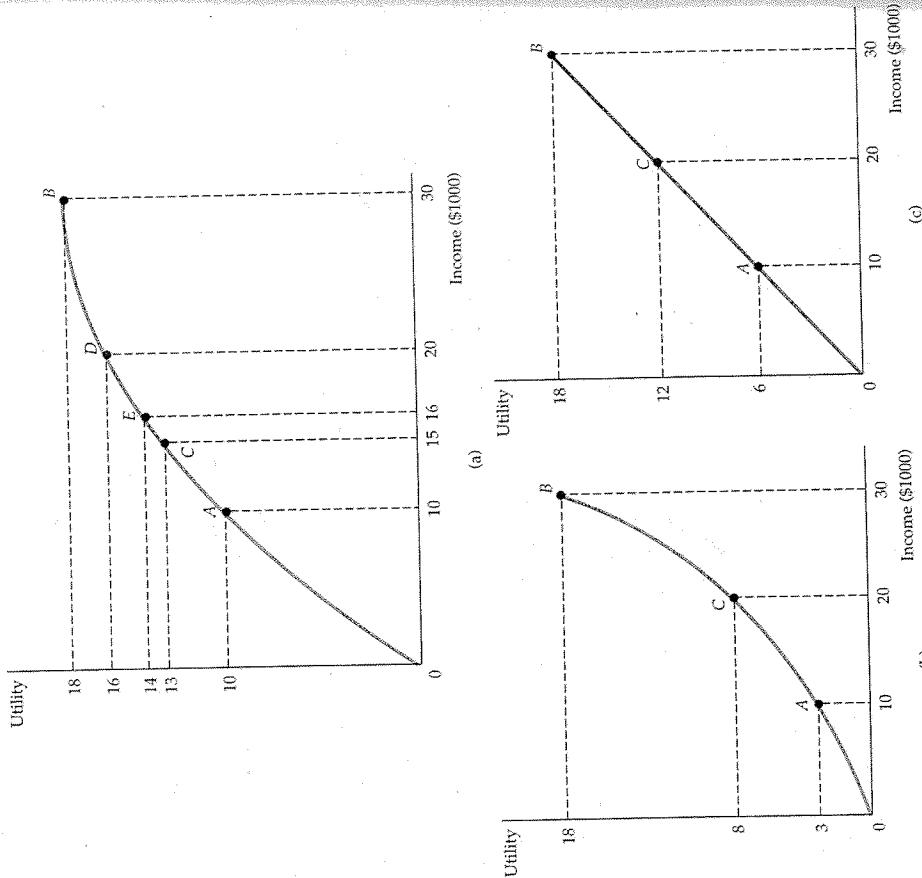


FIGURE 5.3 Risk Aversion. People may differ in their preferences toward risk. In part (a) a consumer's marginal utility diminishes as income increases. The consumer is risk averse because she would prefer a certain income of \$20,000 (with a utility of 16) to a gamble with a .5 probability of \$10,000 and a .5 probability of \$30,000 (and expected utility of 14). In part (b) the consumer is risk loving, because she would prefer the same gamble (with expected utility of 10.5) to the certain income (with a utility of 8). Finally, in part (c) the consumer is risk neutral and is indifferent between certain events and uncertain events with the same expected income.

Figure 5.3b shows the third possibility—*risk loving*. In this case the expected utility of an uncertain income that can be \$10,000 with probability .5 or \$30,000 with probability .5 is *higher* than the utility associated with a certain income of \$20,000. Numerically,

$$E(u) = .5u(\$10,000) + .5u(\$30,000) = .5(3) + .5(18) = 10.5 > u(\$20,000) = 8$$

The primary evidence for risk loving is that many people enjoy gambling. Some criminologists might also describe certain criminals as risk lovers, especially when a robbery is committed that has a relatively high prospect of apprehension and punishment. These special cases aside, few people are risk loving, at least with respect to major purchases or large amounts of income or wealth.⁵

The *risk premium* is the amount of money that a risk-averse person would pay to avoid taking a risk. The magnitude of the risk premium depends in general on the risky alternatives that the person faces. To determine the risk premium, we have reproduced the utility function of Figure 5.3a in Figure 5.4. Recall that an expected utility of 14 is achieved by a woman who is going to

take a risky job with an expected income of \$20,000. This is shown graphically by drawing a horizontal line to the vertical axis from point *F*, which bisects straight line *AB* (thus representing an average of \$10,000 and \$30,000). But the utility level of 14 can also be achieved if the woman has a *certain* income of \$16,000, as shown by dropping a vertical line from point *E*. Thus, the risk premium of \$4000, given by line segment *EF*, is the amount of income (\$20,000 minus \$16,000) she would give up to leave her indifferent between the risky job and the safe one.

How risk averse a person is depends on the nature of the risk involved and on the person's income. Generally, risk-averse people prefer risks involving a smaller variability of outcomes. We saw that when there are two outcomes, an income of \$10,000 and an income of \$30,000, the risk premium is \$4000. Now consider a second risky job, involving a .5 probability of receiving an income of \$40,000 and a .5 probability of getting an income of \$0. The expected value of this risky job is also \$20,000, but the expected utility is only 10:

$$\text{Expected utility} = .5u(\$0) + .5u(\$40,000) = 0 + .5(19) = 10$$

Since the utility associated with having a certain income of \$20,000 is 16, the woman loses 6 units of utility if she is required to accept the job. The risk premium in this case is equal to \$10,000 because the utility of a certain income of \$10,000 is 10. She can afford to give up \$10,000 of her \$20,000 expected income to have the certain income of \$10,000 and will have the same level of expected utility. Thus the greater the variability, the more a person is willing to pay to avoid the risky situation.

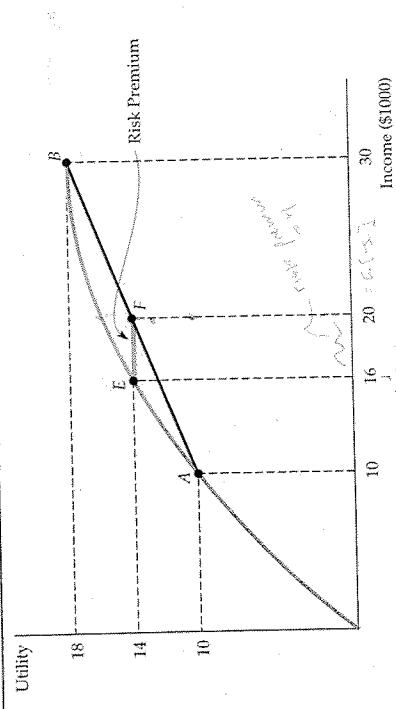


FIGURE 5.4 Risk Premium. The risk premium, *EF*, measures the amount of income an individual would give up to leave her indifferent between a risky choice and a certain one. Here, the risk premium is \$4000 because a certain income of \$16,000 gives her the same expected utility (14) as the uncertain income that has an expected value of \$20,000.

⁵People may be averse to some risks and act like risk lovers with respect to others. This issue was treated by Milton Friedman and Leonard J. Savage in "The Utility Analysis of Choices Involving Risk," *Journal of Political Economy* (1948): 279–304.

⁶This example is based on Kenneth R. MacCrimmon and Donald A. Wehrung, "The Risk In-Basket," *Journal of Business* 57 (1984): 367–387.

tions executives could opt to delay a choice, to collect information, to bargain, or to delegate a decision, so as to avoid taking risks or to modify the risks that they would take later.

The study found that executives vary substantially in their preferences toward risk. Roughly 20 percent of those answering indicated that they were relatively neutral toward risk, while 40 percent opted for the more risky alternatives, and 20 percent were clearly risk averse (20 percent did not respond). More important, executives (including those who chose risky alternatives) made substantial efforts to reduce or eliminate risk, usually by delaying decisions and by collecting more information.

In general, risk can arise where the expected gain is either positive (e.g., a chance for a larger reward versus a small one) or negative (e.g., a chance for a large loss or for no loss). The study found that executives differ in their preferences toward risk, depending on whether the risk involved gains or losses. In general, those executives who liked risky situations did so when losses were involved. (Perhaps they were willing to gamble against a large loss in the hope of breaking even.) However, when the risks involved gains, the same executives were more conservative, opting for the less risky alternatives.⁷

EXAMINE 5.1 DETERMING CRIME

Fines may deter certain types of crimes, such as speeding, double-parking, tax evasion, and air polluting better than incarceration.⁸ The party choosing to violate the law in these ways has good information and can reasonably be assumed to be behaving rationally.

Other things being equal, the greater the fine, the more a potential criminal will be discouraged from engaging in the crime. If it were costless to catch criminals and if the crime imposed a calculable cost of \$1000 on society, we might choose to catch all violators and impose a fine of \$1000 on each. That would discourage people from engaging in the activity, if the benefit of the activity to them were less than the fine.

In practice, however, it is very costly to catch lawbreakers. Therefore, we save on administrative costs by imposing relatively high fines but allocating resources, so that the probability of a violator's being apprehended is sub-

⁷Once we develop a deeper understanding of people's attitudes toward risk, we can explain why some people treat the risk of a small gain in income very differently from the risk of a small loss, for example. Prospect theory, developed by psychologists Daniel Kahneman and Amos Tversky, helps to explain this phenomenon. See "Rational Choice and the Framing of Decisions," *Journal of Business* 59 (1986): S251-S278, and "Prospect Theory: An Analysis of Decision under Risk," *Econometrica* 47 (1979): 263-292.

⁸This discussion indirectly on Gary S. Becker, "Crime and Punishment: An Economic Approach," *Journal of Political Economy* (Mar./Apr. 1968): 169-217. See also Mitchell Polinsky and Steven Shavell, "The Optimal Tradeoff Between the Probability and the Magnitude of Fines," *American Economic Review* 69 (Dec. 1979): 880-891.

stantially less than one. Thus the size of the fine that needs to be imposed to discourage criminal behavior depends on the risk preferences of the potential violators. In general, the more risk averse a person is, the smaller the fine that must be imposed to discourage him or her, as the following example demonstrates.

Suppose that a city wants to deter people from double-parking. By double-parking, a typical resident saves 35 in terms of his own time available to engage in activities that are more pleasant than searching for a parking space. If the driver is risk neutral and if it were costless to catch violators, a fine of just over \$5, say, \$5.01, would need to be assessed every time he double-parked. This would ensure that the net benefit of double-parking to the driver (\$5 benefit less the \$5.01 fine) would be less than zero, so that he would choose to obey the law. In fact, all potential violators whose benefit was less than or equal to \$5 would be discouraged, while a few whose benefit was greater than \$5 would violate the law (they might have to double-park in an emergency).

Heavy monitoring is expensive but fortunately may not be necessary. The

same deterrence effect can be obtained by assessing a fine of \$50 and catching

only one in ten violators (or perhaps a fine of \$500 with a one-in-100 chance

of being caught). In each case the expected penalty is \$5 ($\$50 \times .1$) or $\$500 \times .01$). A policy of a high fine and a low probability of catching a violator is likely to reduce enforcement costs.

The fines to be assessed need not be large. If drivers were substantially risk averse, a much lower fine could be used because they would be willing to forgo the activity in part because of the risk associated with the enforcement process. In the previous example, a \$25 fine with a .1 probability of catching the violator might discourage most people from violating the law.

5.3 REDUCING RISK

Sometimes consumers choose risky alternatives that suggest risk-loving rather than risk-averse behavior, as the recent growth in state lotteries shows. Nonetheless, in the face of a broad variety of risky situations, consumers are generally risk averse. In this section we describe three ways by which consumers commonly reduce risks: diversification, insurance, and obtaining more information about choices and payoffs.

Diversification

Suppose that you are risk averse and try to avoid risky situations as much as possible. You plan to take a part-time job selling appliances on a commission basis. You have a choice as to how to spend your time—you can sell only air