Econ 422
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Midterm Exam Solutions

I. Intertemporal Consumption and Investment Decisions (25 points, 5 points each)

Dinah lives in a two period Fisherian world. Her utility function for consumption streams can be written as $U(C_0, C_1) = C_0^{0.45} C_1^{0.55}$ hence her marginal rate of substitution is

$$MRS = -\frac{U_0}{U_1} = -\frac{.45C_1}{.55C_0}$$

Her endowment of present and future resources is (100,000, 206,000). She can borrow or lend at a market real interest rate of 3 percent.

a. What is her wealth?

$$W = W_0 + \frac{W_1}{1+r} = 100,000 + \frac{206,000}{1.03} = 300,000$$

b. Write the equation for her budget constraint.

$$C_0 + \frac{C_1}{1.03} = 300,000$$

c. Derive the marginal rate of substitution.

$$U_0 = 0.45C_0^{-0.55}C_1^{0.55},\ U_1 = 0.55C_0^{0.45}C_1^{-0.45}$$
$$\Rightarrow -\frac{U_0}{U_1} = \frac{0.45C_0^{-0.55}C_1^{0.55}}{0.55C_0^{0.45}C_1^{-0.45}} = \frac{0.45C_1}{0.55C_0}$$

d. What is her optimal consumption in the present and in the future?

Optimal consumption is found by first setting MRS = -(1 + r):

$$-\frac{0.45C_1}{0.55C_0} = -1.03 \Rightarrow C_1 = \frac{0.55}{0.45}(1.03)C_0 = 1.259C_0$$

Then use the budget constraint to solve for C_0:

$$C_0 + \frac{1.259C_0}{1.03} = 300,000 \Rightarrow C_0 = \frac{300,000}{2.222} = 135,000$$

Then use $C_1 = 1.259*C_0$ to give

$$C_1 = 169{,}950$$

e. What financial transaction is required to attain the optimal consumption stream? Indicate what is exchanged for what, when, and indicate the amounts involved.

The endowment at time zero is 100,000 and consumption is 135,000. C0 is greater than the endowment the individual must borrow against the future endowment of 206,000 to finance consumption today. The amount borrowed is 35,000. The amount to be paid back in period 1 is 35,000(1.03) = 36,050. The amount remaining for future consumption is 206,000 – 36,050 = 169,950.

II. Present Value Computations (25 points, 5 points each)

1. At age 20 you put $2,000 into an IRA account held in the form of a Vanguard Index 500 mutual fund. At age 60, as you begin to think seriously about retiring, you find that the account is worth $192,865.48. What has been the compound annual rate of return on this account? (5 points)

$$r = \left(\frac{192{,}865.48}{2{,}000}\right)^{1/40} - 1 = 0.121$$

2. What the price of a coupon bond with a 5% annual coupon, semi-annual coupon payments, face value of $1000, and a yield to maturity of 5%? (5 points)

Since the yield-to-maturity is equal to the annual coupon rate, the bond price is equal to its face value = $1,000

3. Suppose you just started a new job as a university professor making $80,000 per year, and you have the opportunity to put part of your salary in a retirement account. Consider the following scenarios: (15 points, 5 points each)

a. You save 10% of your salary every year for 30 years starting next year. Assume your salary does not grow over time. What is the future value of your savings if you can earn 5% per year on your savings? What is the present value of your savings if the annual discount rate is 5%?

Here you use the future value and present value of an annuity formulas:

$$FV = \left(\frac{C}{r}\right)\left((1+r)^T - 1\right),\ PV = \left(\frac{C}{r}\right)\left(1 - \frac{1}{(1+r)^T}\right)$$

*You save 0.10*80,000 = $8,000 every year for 30 years and the interest rate is 5%. This is a finite annuity. Using the above formulas gives*

$$FV = \left(\frac{8{,}000}{0.05}\right)\left((1.05)^{30} - 1\right) = \$531{,}510.78$$

$$PV = \left(\frac{8{,}000}{0.05}\right)\left(1 - \frac{1}{(1.05)^{30}}\right) = \$122{,}979.61$$

b. You save 10% of your salary every year for 30 years starting next year. Additionally, assume now that the university matches 100% of your savings each year. That is, the university contributes $1 for every $1 you save. Assume your salary does not grow over time. What is the future value of your savings if you can earn 5% per year on your savings? What is the present value of your savings if the annual discount rate is 5%?

*Now the annual savings is double: 0.20*80,000 = 16,000. The FV and PV are now*

$$FV = \left(\frac{16{,}000}{0.05}\right)\left((1.05)^{30} - 1\right) = \$1{,}063{,}021.56$$

$$PV = \left(\frac{16{,}000}{0.05}\right)\left(1 - \frac{1}{(1.05)^{30}}\right) = \$245{,}959.22$$

c. You save 10% of your salary every year for 30 years starting next year. Additionally, assume now that the university matches 100% of your savings each year. That is, the university contributes $1 for every $1 you save. Furthermore, assume your salary grows at 2% per year. What is the future value of your savings if you can earn 5% per year on your savings? What is the present value of your savings if the discount rate is 5%? (Hint: you might find it easier to first compute the present value, and then compute the future value directly from the present value).

Now we have a finite growing annuity. The initial annuity payment is 16,000 and the growth rate is 0.02. The PV of the growing finite annuity is based on the formula

$$PV = \left(\frac{C}{r-g}\right)\left(1 - \left(\frac{1+g}{1+r}\right)^{T}\right) = \left(\frac{16{,}000}{0.05 - 0.02}\right)\left(1 - \left(\frac{1.02}{1.05}\right)^{30}\right) = \$309{,}808.95$$

The future value of this is then

$$FV = PV(1+r)^{T} = \$309{,}808.95(1.05)^{30} = \$1{,}338{,}976.42$$

III. Bond Pricing and the Term Structure of Interest Rates (25 points, 5 points each)

The following is a list of prices for zero coupon bonds of various maturities:

Maturity (years)	Price of zero coupond bond
1	$943.40
2	$898.47
3	$847.62
4	$792.16

Please answer the following questions:

a. Calculate the spot rates associated with each bond

The spot rates are computed using

$$ {}_0r_1 = \left(\frac{1000}{943.40}\right) - 1 = 0.0600 $$

$$ {}_0r_2 = \left(\frac{1000}{898.47}\right)^{1/2} - 1 = 0.0550 $$

$$ {}_0r_3 = \left(\frac{1000}{847.62}\right)^{1/3} - 1 = 0.0567 $$

$$ {}_0r_4 = \left(\frac{1000}{792.16}\right)^{1/4} - 1 = 0.0600 $$

b. Calculate the implied 1-year forward rates, ${}_{t-1}f_t$, for t=2, 3, 4?

The implied forward rates are computed using

$$ {}_1f_2 = \frac{(1+{}_0r_2)^2}{1+{}_0r_1} - 1 = 0.0500 $$

$$ {}_2f_3 = \frac{(1+{}_0r_3)^3}{(1+{}_0r_2)^2} - 1 = 0.0600 $$

$$ {}_3f_4 = \frac{(1+{}_0r_4)^4}{(1+{}_0r_1)^3} - 1 = 0.0700 $$

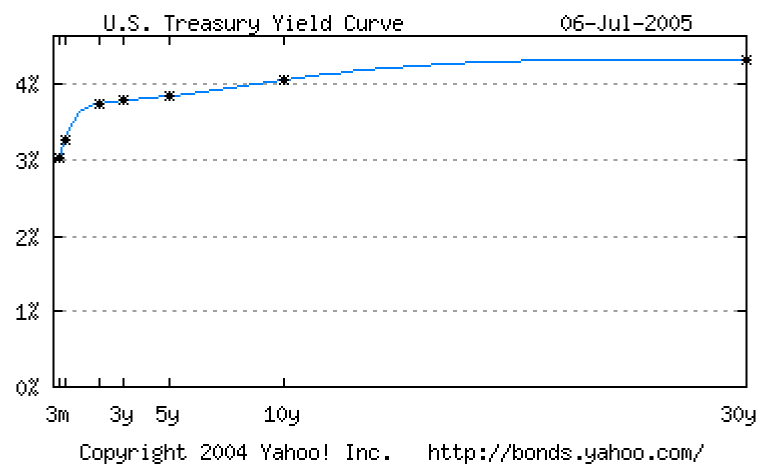
c. Suppose there is a parallel shift in the yield curve so that all spot rate increase by 0.01. Which zero coupon bond will experience the biggest percentage change in price?

Since $\frac{dP}{P} \approx \frac{-T}{(1+r)} dr$ *it is clear that the 4 year bond will experience the biggest percentage change in price.*

d. Can the price of a zero coupon bond maturing in 5 years be $800? Why or why not? Justify your answer.

No. This means that you would be paying more today for $ received in 5 years than you would for $ received in 4 years. No rational person would do this.

e. The figure below shows the current Treasury yield curve (taken this morning from the Yahoo! Bonds page).



If the expectations hypothesis of the term structure holds, what does the information in the yield curve say about the course of future interest rates?

The yield curve is upward sloping, and so the 1 year implied forward rates are higher than today's 1 year spot rate. However, the yield curve flattens at about 20 year so the implied forward rates are constant after about 20 years.

IV. Valuing Stocks (25 points, 5 points each)

1. IBM's stock is expected to pay a dividend of $2.15 at the end of the year, and then the dividend is expected to grow at 11.2% per year forever. The required rate of return (market capitalization rate) on IBM stock is 15.2% per year.

a. What is the current price of IBM stock?

Using the formula for a growing perpetuity with D = 2.15, g = 0.112 and r = 0.152 gives

$$P_0 = \frac{D}{r-g} = \frac{2.15}{0.152 - 0.112} = 53.75$$

b. What is the expected price of IBM's stock at the beginning of next year?

Using the formula for a growing perpetuity with D = 2.15(1.112)=2.3908, g = 0.112 and r = 0.152 gives

$$P_1 = \frac{D(1+g)}{r-g} = \frac{2.15(1.112)}{0.152 - 0.112} = 59.77$$

Notice that $P_1 = (1+g)P_0 = (1.112)53.75 = 59.77$.

c. If an investor were to buy IBM stock now and sell it after receiving the $2.15 dividend a year from now, what is the expected capital gain in percentage terms? What is the dividend yield, and what is the total rate of return on the investment?

$$\text{capital gain } = \frac{P_1 - P_0}{P_0} = \frac{59.77 - 53.75}{53.75} = .112$$

$$\text{dividend yield } = \frac{D}{P_0} = \frac{2.15}{53.75} = .04$$

total return = capital gain + dividend yield $= .112 + .04 = .152$

2. The probability distribution for next year's price of Yahoo! stock is given by

Price P_1	$90	$100	$110	$120
Probability	0.1	0.2	0.5	0.2

a. Compute the expected value and variance of next year's price.

$$E[P_1] = 90(.1) + 100(.2) + 110(.5) + 120(.2) = 108$$

$$E[P_1^2] = 90^2(.1) + 100^2(.2) + 110^2(.5) + 120^2(.2) = 11740$$

$$V(P_1) = E[P_1^2] - \left(E[P_1]\right)^2 = 11740 - (108)^2 = 76$$

c. Compute the expected rate of return if the current price is $100 and you expect to receive a $2 dividend at time 1.

Return	(90+2-100)/100 = -.08	(100+2-100)/100 = .02	(110+2-100)/100 = .12	(120+2-100)/100 = .22
Probability	0.1	0.2	0.5	0.2

$$E[R] = (-.08)(.1) + (.02)(.2) + (.12)(.5) + (.22)(.2) = .1$$

Alternatively, you may compute E[R] directly from E[P_1] using

$$R = \frac{P_1 + DIV - P_0}{P_0} \Rightarrow E[R] = \frac{E[P_1] + DIV - P_0}{P_0} = \frac{108 + 2 - 100}{100} = 0.10$$

V. Capital budgeting (10 points)

The Zephyr Company has to choose between two machines which do the same job but have different lives: the A machine provides service for three years, while the B machine provides service for four years. The two machines have the following costs, expressed in real terms: (Neither has a scrap value at the end of its life.)

Year	Machine A	Machine B
0	$20,000	$25,000
1	$5,000	$4,000
2	$5,000	$4,000
3	$5,000	$4,000
4		$4,000

Assuming that the opportunity cost of capital is 6 percent in real terms and ignoring taxes, which machine would you choose? Explain and show your work.

We discussed two solutions to this type of problem. The first solution is based on computing the equivalent annual cost (EAC) for each project. This is based on first computing the PV of the costs for the two projects, and then computing the fixed annual payment (EAC) that has the same PV of cost. For PV of the costs for the two projects are

$$PVC_A = 20,000 + \frac{5,000}{1.06} + \frac{5,000}{(1.06)^2} + \frac{5,000}{(1.06)^3} = 33365.06$$

$$PVC_B = 25,000 + \frac{4,000}{1.06} + \frac{4,000}{(1.06)^2} + \frac{4,000}{(1.06)^3} + \frac{4,000}{(1.06)^4} = 38860.42$$

The EAC are computed using the finite annuity formula

$$EAC = \frac{PVC}{PVA(r,T)}, \; PVA(r,T) = \frac{1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

For project A, PVA(r, T) is

$$PVA(0.06,3) = \frac{1}{0.06}\left(1 - \frac{1}{(1.06)^3}\right) = 2.6730$$

For project B, PVA(r, T) is

$$PVA(0.06,4) = \frac{1}{0.06}\left(1 - \frac{1}{(1.06)^4}\right) = 3.4651$$

Therefore, the EAC values for project A and B are

$$EAC_A = \frac{33365.06}{2.6730} = 12482.20$$

$$EAC_B = \frac{38860.42}{3.4651} = 11214.79$$

Since the EAC for project B is less, choose project B.

The second method is based on replicating the projects enough times so that they cover the same number of periods and then computing the PV of each project. Here, machine A must be replaced 3 times (in years 3, 6 and 9) and machine B must be replaced 2 times (in years 4 and 8). Under this approach the PV of costs are computed using

$$PV_A = 33,365.06 + \frac{33,365.06}{(1.06)^3} + \frac{33,365.06}{(1.06)^6} + \frac{33,365.06}{(1.06)^9} = 104,648.79$$

$$PV_B = 38,860.42 + \frac{38,860.42}{(1.06)^4} + \frac{38,860.42}{(1.06)^8} = 92,023.02$$

As with the equivalent annual cost approach, machine B is cheaper.