Econ 422
Midterm Solutions

I. Intertemporal Consumption and Investment Decisions (32 points)

a. To determine the interest rate, r, we use the money market line and solve

$$4(1 + r) = 5 \Rightarrow 1 + r = 5/4 = 1.25 \Rightarrow r = 0.25 \text{ or } 25\%$$

b. Optimal investment occurs where the MRT = -(1 + r). This tangency point occurs when 2.6 – 1.6 = 1 M is invested.

c. The investment of 1 M returns 3 M next year.

d. The total rate of return on the investment is

$$(3 - 1)/1 = 2 \text{ or } 200\%$$

e. The marginal rate of return is 25% = slope of the investment possibilities frontier at the tangency point.

f. The PV and NPV of the investment project is

$$PV = 3/1.25 = 2.4 \text{ M}$$
$$NPV = 2.4 - 1 = 1.4 \text{ M}$$

g. The optimal allocation of consumption at times 0 and 1 is determined by the tangency point of the IC with the budget constraint. From the figure, we see that optimal consumption at time 0 is 1 M and optimal consumption at time 1 is 3.75 M.

h. The owner lends 1.6 – 1 = 0.6 M (at r = 25%) today and receives 0.6*(1.25) = 0.75 M next period. Consumption next period is then consists of the 3M investment return plus the 0.75 M money market return (= 3.75 M).

II. Present Value Calculations

1. For an investment of $1000 today, the Washington State Employees Credit Union is offering to pay you $1600 at the end of 8 years.

a. To find the annually compounded rate of interest we solve

$$(1 + r)^8 = 1600/1000 = 1.6 \Rightarrow r = (1.6)^{1/8} - 1 = 0.0605 = 6.05\%$$

b. With semi-annual compounding, we solve

$$(1 + r/2)^{2*8} = 1.6 \Rightarrow r = 2*((1.6)^{1/16} - 1) = 0.0596 \text{ or } 5.96\%$$

c. With continuous compounding, we solve

exp(r*8) = 1.6 => r*8 = ln(1.6) => r = ln(1.6)/8 = 0.0588 or 5.88%

2. Suppose Washington Mutual grants you a $300,000, 30 year mortgage with a fixed annual interest rate of 10%.

a. To determine the payment on the mortgage we use our present value of a finite annuity formula. However, we must be careful to use the correct interest rate in the annuity formula. Here, the periodic (monthly) interest rate is

r = 0.10/12 = 0.0083

Therefore, we solve

300,000 = PMT*A(0.0083,30*12,$1) = PMT*113.95
=> PMT = 300,000/113.95 = 2,632.73

where A(0.0083,30*12,$1) = (1/0.0083)*(1 – $(1/1.0083)^{360}$) = 113.95

b. When market rates fall to 5%, the PV of the 10% mortgage changes because the monthly payments are fixed at 2,632.73. The new periodic rate is r = 0.05/12 = 0.00416. The new PV is

PV = 2,632.73* A(0.00416,30*12,$1) = 2,632.73*1185.44 = 488,213

Which is larger than the original mortgage amount of 300,000.

c. The bank is definitely better off when mortgage rates fall. To see this, if Washington Mutual had to issue a new 300,000 mortgage at 5% instead of 10%, the monthly payments would only be

PMT = 300,000/1185.44 = 1,617.78

This is considerably lower than the 2,632.73 from the 10% mortgage. The home buyer is worse off because she could be financing the home with a lower monthly payment from the new mortgage.

III. Bond Pricing and the Term Structure of Interest Rates (16 points)

a. To determine the implicit 1 period forward rates, we use the equation

$$ {}_{t-1}f_t = \frac{(1+r_t)^t}{(1+r_{t-1})^{t-1}} - 1 $$

where ${}_{t-1}f_t$ is the 1 period implied forward rate starting at time t-1. In this problem, we have

$$
{}_1f_2 = \frac{(1.11)^2}{1.10} - 1 = 0.12
$$
$$
{}_2f_3 = \frac{(1.12)^3}{1.11^2} - 1 = 0.14
$$

We may think of ${}_1f_2$ as the market's best guess of the 1 period spot rate between periods 1 and 2, and ${}_2f_3$ as the market's best guess of the 1 period spot rate between periods 2 and 3.

b. Assuming the expectations hypothesis holds means that the implied forward rates are in fact unbiased expectations of future spot rates (expectation of the forecast error is zero). The implied forward rates from part a. give us predictions of future 1 period spot rates. That is, E[${}_1f_2$] = ${}_1r_2$ and E[${}_2f_3$] = ${}_2r_3$. To determine the term structure next year, we need a guess of the 2 year spot rate to occur next year: ${}_1r_3$. To get this rate we take a geometric average of the two 1 period forward rates:

$$
(1+{}_1r_3)^2 = (1+{}_1f_2)(1+{}_2f_3)
$$
$$
\Rightarrow {}_1r_3 = \left[(1+{}_1f_2)(1+{}_2f_3)\right]^{1/2} - 1 = \left[(1.12)(1.14)\right]^{1/2} - 1 = 0.13
$$

c. Using the general PV formula, the price of a 12% coupon bond with face value $100 is given by

$$
PV = \frac{12}{1.10} + \frac{12}{1.11^2} + \frac{12+100}{1.12^3} = \$100.4
$$

d. The yield to maturity on the bond solves the equation

$$
\frac{12}{1+y} + \frac{12}{(1+y)^2} + \frac{12+100}{(1+y)^3} - \$100.4 = 0
$$

IV. Valuing Stocks (8 points)

a. Using the infinite growing annuity formula for a stock price gives

$$
P = \frac{DIV}{r-g} = \frac{1}{0.07-0.03} = \frac{1}{0.04} = 25
$$

b. This problem can be solved in two ways. The exact answer follows from valuing the stock at the new growth rate and then computing the percentage change in price

$$P^{new} = \frac{DIV}{r-g} = \frac{1}{0.07-0.031} = \frac{1}{0.039} = 25.641$$
$$\%\Delta P = (P^{new} - P)/P = (25.641-25)/25 = 0.0256 = 2.56\%$$

An approximate answer uses the derivative of P with respect to g:

$$\frac{dP}{dr} = -P(r-g)^{-1} \Rightarrow \frac{dP}{P} = -(r-g)^{-1}dr$$
$$= -(0.07-0.04)^{-1}(0.031-0.03) = -25(0.001) = 0.025 = 2.5\%$$

V. The NPV and IRR Rules for Investment Analysis (20 points)

a. The NPV rule for accepting a project is: accept the project if NPV > 0. The NPV rule for ranking mutually exclusive projects is: take the project with the highest NPV

b. The IRR rule for accepting a project is: accept the project if IRR > r, where r is the relevant discount rate. The IRR rule for ranking mutually exclusive projects is: accept the project with the highest IRR.

c. The IRR is the rate at which NPV = 0. From the figure, the IRR for project A is about 26% and the IRR for project B is about 24%.

d. If the discount rate is r = 6%, the NPV of project B is higher than the NPV of project A (and both NPVs > 0) so the NPV rule says take project B. The IRRs for both projects are greater than 6%. However, since the IRR for project A is greater than the IRR for project B, the IRR rule says do project A.

e. If the discount rate is 22%, the NPV of project A is now greater than the NPV of project B (and both NPVs > 0), so the NPV rule says take project A. The IRR for both projects are greater than 22%. However, since the IRR for project A is greater than the IRR for project B, the IRR rule says do project A.