Econ 422
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Final Exam Solutions

This is a closed book exam. However, you are allowed one page of notes (double-sided).
Answer all questions. For the numerical problems, if you make a computational error you
may still receive full credit if you provide the correct formula for the problem. There are
25 questions, and each question is worth 4 points. Total points = 100. There is one extra
credit problem worth 15 points. You have 2 hours and 10 minutes to complete the exam.
Good luck.

I. Portfolio Theory (28 points, 4 points each)

Consider portfolios of three assets: Amazon stock (stock A), Boeing stock (stock B) and
T-bills (risk-free asset). Assume the following information

$$E[R_A] = 0.20, E[R_B] = 0.10$$
$$SD(R_A) = 0.30, SD(R_B) = 0.20$$
$$CORR(R_A, R_B) = 0.2$$
$$r_f = 0.03$$

Transfer the diagram below to your blue book and use it to answer the following
questions.
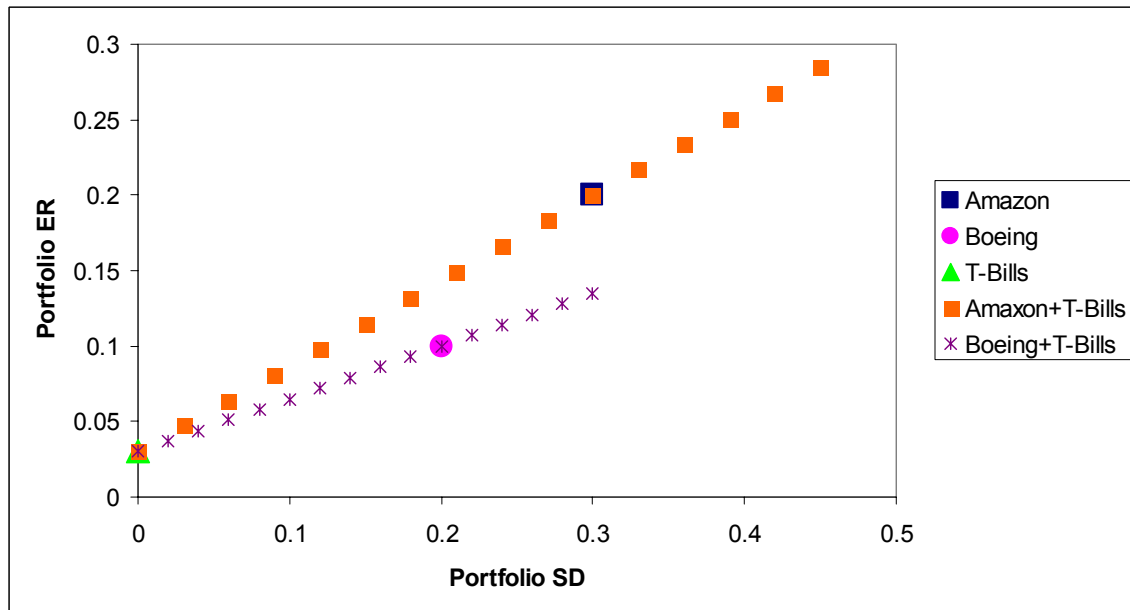
a. Let x_A denote the share of wealth in Amazon stock and $1-x_A$ denote the share of wealth in T-Bills. Using the information in the diagram, sketch the portfolio expected return and standard deviation values for portfolios of T-bills and Amazon stock for values of x_A between 0 and 1.5.

b. Let x_B denote the share of wealth in Boeing stock and $1-x_B$ denote the share of wealth in T-Bills. Using the information in the diagram, sketch the portfolio expected return and standard deviation values for portfolios of T-bills and Boeing stock for values of x_B between 0 and 1.5.

c. What are the values of Sharpe's slope for Amazon and Boeing stock? Using Sharpe's slope, which stock provides better investment opportunities when combined with T-Bills?
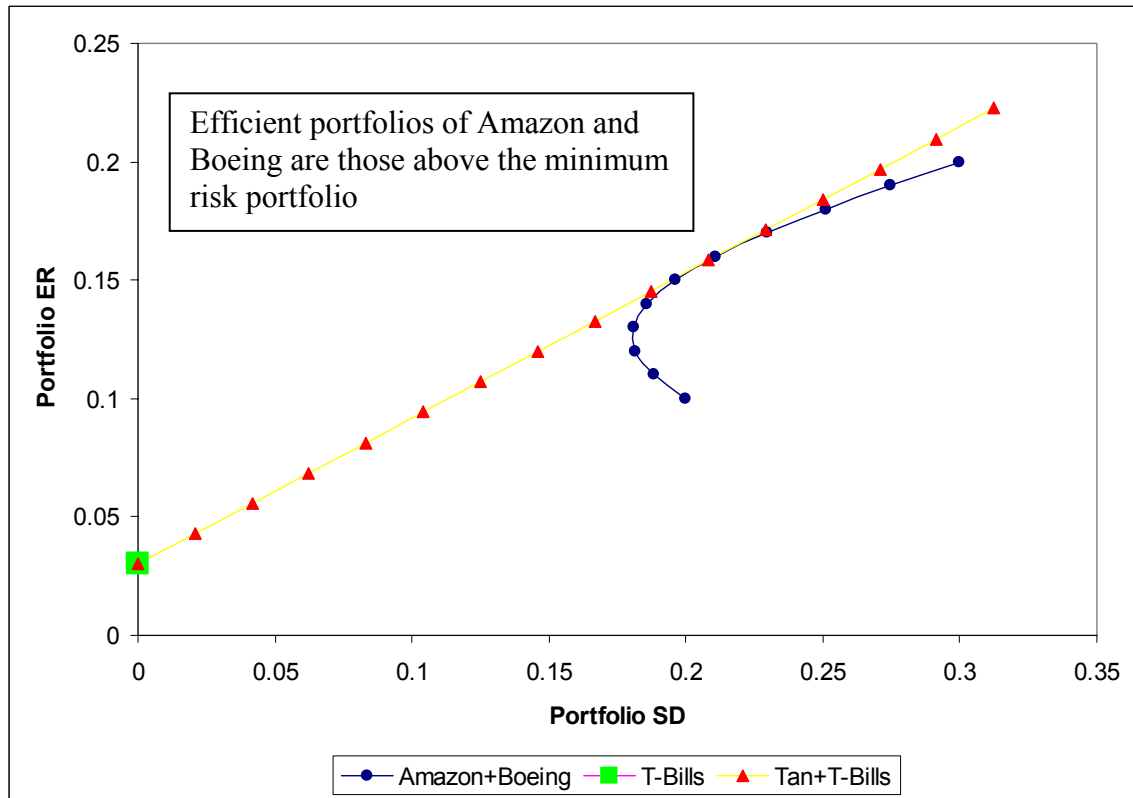
$$\text{Sharpe's slope} = \frac{E[R]-r_f}{SD(R)}$$

Amazon: $\dfrac{0.20-0.03}{0.30} = 0.567$

Boeing: $\dfrac{0.20-0.03}{0.30} = 0.35$

Amazon and T-Bills provides the better investment opportunities when combined with T-Bills.

Transfer the diagram below to your blue book and use it to answer the following questions

d. The curved line in the diagram represents expected return and standard deviation values for portfolios of Amazon and Boeing stock. Using this curved line, indicate the location of the *efficient* portfolios of Amazon and Boeing stock.

e. Graph the expected return and standard deviation values for efficient portfolios consisting of T-Bills, Amazon stock and Boeing stock.

f. Consider the portfolio of T-Bills, Amazon stock and Boeing stock with $x_A = 0.25$, $x_B = 0.50$, $x_f = 0.25$. Compute the expected return of this portfolio.

$$E[R_p] = (0.25)(0.20) + (0.50)(0.10) + (0.25)(0.03) = 0.1075$$

g. Consider the portfolio of T-Bills, Amazon stock and Boeing stock with $x_A = 0.25$, $x_B = 0.50$, $x_f = 0.25$. Compute the variance and standard deviation of this portfolio. (Hint: $\text{cov}(R_A, r_f) = \text{cov}(R_B, r_f) = 0$. Write out the 3×3 covariance matrix.)

The covariance matrix is

$$\Sigma = \begin{pmatrix} 0.09 & 0.012 & 0 \\ 0.012 & 0.04 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$V(R_p) = (0.25)^2(0.09) + 2(0.25)(0.50)(0.12) + (0.50)^2(0.04) = 0.018625$$

$$SD(R_p) = 0.136473$$

II. (16 points, 4 points each)

Insightful Corporation, a local software company, is trying to decide which of two mutually exclusive projects to do. The nominal cash flows associated with the projects are described in the table below:

Project/year	0	1	2
A	-50	50	50
B	-70	60	70

Project A has risk that is comparable to the company's current assets. Project B has risk that is *twice* that of the company's current assets. The current annual risk-free interest rate is 3 percent, and the expected annual market risk premium is 7 percent. The company has a debt/equity ratio of 0.2, and the beta for the company's stock is 1.5 while the beta for its debt is 0.4.

a. What is the beta for the company's current assets?

$$\beta_A = \left(\frac{D}{D+E}\right)\beta_D + \left(\frac{E}{D+E}\right)\beta_E$$

$$\frac{D}{E} = 0.2 \Rightarrow D = 0.2E \Rightarrow \frac{D}{D+E} = \frac{0.2E}{0.2E+E} = \frac{0.2}{1.2} = 0.167$$

$$\frac{E}{D+E} = 1 - 0.167 = 0.833$$

$$\beta_A = (0.167)(0.4) + (0.833)(1.5) = 1.317$$

b. What are the betas for the two projects?

The beta for project A is the asset beta, and the beta for project B is twice the beta for project A:

$$\beta_A = 1.317, \ \beta_B = 2(1.317) = 2.633$$

c. Using the CAPM, what are the appropriate discount rates (risk adjusted expected returns) for the two projects?

The CAPM equation for the risk adjusted return is

$$E[R] = r_f + \beta(E[R_M] - r_f)$$
$$E[R_A] = 0.03 + 1.317(0.07) = 0.122$$
$$E[R_B] = 0.03 + 2.633(0.07) = 0.214$$

d. Compute the NPV for the two projects. Which project should Insightful choose?

$$NPV_A = -50 + \frac{50}{1.122} + \frac{50}{(1.122)^2} = 34.26$$

$$NPV_B = -70 + \frac{60}{1.214} + \frac{70}{(1.214)^2} = 26.88$$

Do project A.

III. Market Efficiency (12 points, 4 points each)

1. State the efficient markets hypothesis, and name the three types of market efficiency.

Market prices of assets reflect all currently available information and provide a fair valuation. The three forms of market efficiency are: weak form, semi-strong form and strong form.

2. "If the efficient-market hypothesis is true, a pension fund manager might as well select a portfolio by throwing darts at the financial assets listed in the Wall Street Journal." Explain why this is not so.

A pension fund manager has certain target return goals as well certain risk control goals. Throwing darts at the stock page may generate a diversified portfolio, but there is no way to control for the expected return or risk of the resulting portfolio.
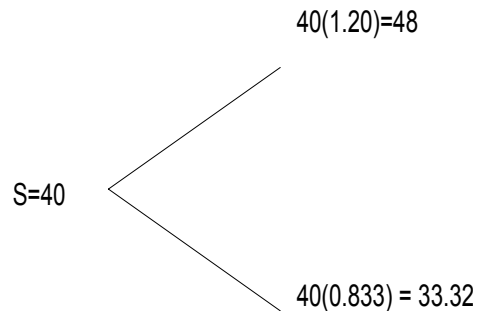
3. A marketing statement for a mutual fund company is "Our Spartacus equity fund outperformed the S&P 500 three out of the last four years. Why would you invest in the S&P 500 when you can do better by buying our fund?" Explain why this statement may be misleading.

Without knowing the risk characteristics of the Spartacus equity fund, it is hard to judge its performance relative to the S&P 500 index. If the Spartacus fund is loaded with a bunch of high beta stocks, then it is expected to do better than the S&P 500 index. An investor who is willing to tolerate higher risk may well be better off buying the Spartacus fund, but an investor who is risk averse may be better off investing in the S&P 500 index and T-Bills.
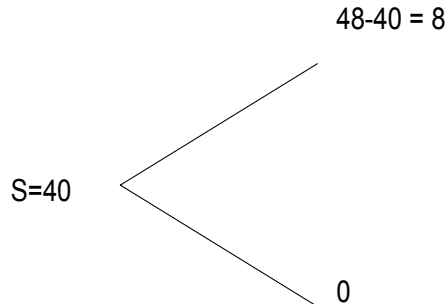
IV. Options (20 points, 4 points each part)

1. The stock of Heavy Metal (HM) changes only once a month: with equal probability either it goes up by 20 percent or it falls by 16.7 percent. Its price now is $40. The interest rate is 12.7 percent per year, or about 1 percent per month. Consider a one-month call option of HM stock with an exercise price of $40.

a. Graph the tree diagram showing the current stock price of HM and the two possible values of the stock price next month.

```
                                    40(1.20)=48
                                   /
                                  /
              S=40
                                  \
                                   \
                                    40(0.833) = 33.32
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b. At the expiration date of the option, what is the value of the call if the stock price goes up and what is the value if the stock price goes down?

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                                    48-40 = 8
                                   /
                                  /
              S=40
                                  \
                                   \
                                    0
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c. What is the current value of the call option?

The current value of the call option may be computed in two ways: (1) by creating a replicating portfolio of the stock and a risk free bond; (2) by valuing the call using risk neutral probabilities.

Risk neutral probabilities are computed by noting that the expected return on any asset must be equal to the risk free rate

$$E[R] = p(0.20) + (1-p)(-0.167) = 0.01 \Rightarrow p = 0.482, \ 1-p = 0.518$$

Given the risk neutral probabilities, the Call option may be valued using the formula

$$C = \frac{E[Payoff]}{1+r_f} = \frac{pC^{up} + (1-p)C^{down}}{1+r_f}$$

$$= \frac{0.482(8) + 0.518(0)}{1.01} = 3.8201$$

The replicating portfolio has the form

$$x_S S + x_B B$$

where S is the stock price and B is the PV of a bond that pays $1 in one month. The replicating portfolio mimics the payoff on the call option in one month:

$$x_S 40 + x_B = 8$$
$$x_S 33.32 + x_B = 0$$

Solving for xs and xb gives

$$x_S = 0.545, \; x_B = -18.158$$

Since the final value of the replicating portfolio matches the final value of the call, to prevent arbitrage the current values of these two assets must be the same. The current value of the replicating portfolio is

$$C = 0.545(40) - 18.158/1.01 = 3.8201$$

d. If you know the current value of the call option, how can you determine the current value of a put with the same exercise price and expiration date as the call? (Hint: I am not asking you to compute the numerical value of the put option)

Use put-call parity: $P + S = C + PV(X) \Rightarrow P = C + PV(X) - S$

f. What happens to the current value of the call option if the stock price increase next period stays at 20 percent per year, but that the stock price fall doubles to 25.4 percent per year? (Hint: draw the new stock price tree and compute the new payoffs to the call option)

It goes up to 5.10253

2. Graph the payoffs at expiration of the following derivative securities based on the price of the S&P 500 index. (8 points, 4 points each)

a. A long position in a futures contract at a price of X.

b. The combination of a long position in a call option and a short position in a put option, where both options have an exercise price of X.

V. Hedging (16 points, 4 points each)

1. You own a $1 million portfolio of aerospace stocks with a beta of 1.2 and an R^2 of 0.3. You are very enthusiastic about aerospace but uncertain about the prospects for the overall stock market.

a. Explain how you could hedge out your market exposure by selling the market short. How much would you sell?

Here the hedge ratio is 1.2. We should short $1.2 million in a market index to offset our long position of $1 million in the portfolio of aerospace stocks.

b. What is the beta of your hedge portfolio that is long in aerospace stocks and short in the market?

The beta of the hedge portfolio is zero: the hedge portfolio is market neutral

c. How confident are you about the quality of the hedge against the market? Justify your answer.

Since the R^2 of the underlying regression used to compute beta is only 0.3, the portfolio of aerospace stocks does not move that closely with the market; i.e., only 30% of the movement of the aerospace stocks is explained by the market. The market does not seem to provide a very good hedge.

d. How in practice would you go about "selling the market"?

Here we would need to short a some type of index fund that represents the market. We could not use a mutual fund because it is not possible to short mutual funds. However, we could use an exchange traded fund that mimicks the S&P 500. It is possible to short such an asset. We could also look at selling a futures contract on the S&P 500 index.

VI. Extra Credit (15 points, 5 points each)

Consider an individual whose utility of wealth function is U(W)=ln(W), i.e. the person's utility associated with any given level of wealth is equal to the natural log of the wealth level. If the person chooses occupation A, his or her wealth is given by the following wealth distribution:

Wealth: 1,000,000   2,000,000
Probability: 0.8   0.2

If the person chooses occupation B, his or her wealth is given by the following wealth distribution:

Wealth: 1,100,000   1,300,000
Probability: 0.5   0.5

a. Which occupation will the person choose and why? Or will the person be indifferent to the alternatives? Explain.

The individual will choose the occupation that maximizes the expected utility of wealth. The expected utilities for the two occupations are

$$E[U(W_A)] = 0.8\ln(1,000,000) + 0.2\ln(2,000,000) = 13.954$$
$$E[U(W_B)] = 0.5\ln(1,100,000) + 0.2\ln(1,300,000) = 13.994$$

Since occupation B gives the higher expected utility, choose B.

b. Compute the certainty equivalent wealth and risk premium for occupation A.

The certainty equivalent wealth is the amount of wealth with certainty that gives the same expected utility as the gamble: $U(W^c) = E[U(W_a)]$. Therefore, we solve

$$\ln(W^c) = 13.954 \Rightarrow W^c = \exp(13.954) = 1,148,698.355$$

c. Repeat parts (a) and (b) using the utility function U(W) = W. Now, the expected utility is the same as the expected value of wealth

$$E[W_a] = 0.8(1,000,000) + 0.2(2,000,000) = 1,200,000$$
$$E[W_b] = 0.5(1,100,000) + 0.5(1,300.000) = 1,200,000$$

The individual is now indifferent between the two occupations. Here the certainty equivalent wealth is the same as the expected wealth.