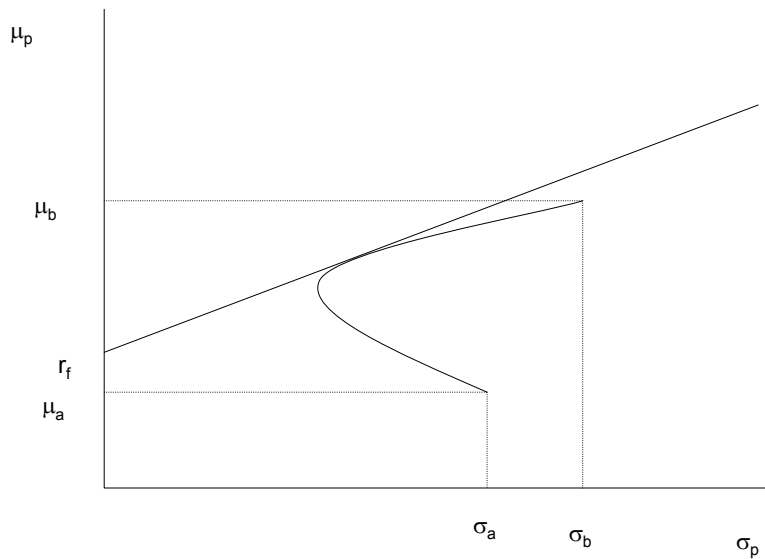


Econ 422  
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Summer 2004  
Final Exam Solutions

This is a closed book exam. However, you are allowed one page of notes (double-sided). Answer all questions. For the numerical problems, if you make a computational error you may still receive full credit if you provide the correct formula for the problem. There are 25 questions, and each question is worth 4 points. Total points = 100. You have 2 hours and 10 minutes to complete the exam. Good luck.

I. Portfolio Theory (24 points, 4 points each)

Consider the problem of allocating wealth between two risky assets and a risk free asset (T-bill) under the assumption that investors only care about maximizing portfolio expected return and minimizing portfolio variance. Transfer the graph below to your blue book or exam sheet and use it to answer the following questions.



- Which point on the graph gives the portfolio that has 100% invested in asset A, and which point on the graph gives the portfolio that has 100% invested in asset B
- Which point on the graph gives the portfolio of assets A and B that has the smallest variance.
- Mark on the graph the set of efficient portfolios that only contain assets A and B.
- Mark on the graph the set of efficient portfolios that are combinations of T-bills and the risky assets.
- Draw an indifference curve for an investor who is very risk averse, and indicate the optimal combination of T-bills and risky assets.
- Draw an indifference curve for an investor who is not very risk averse, and indicate the optimal combination of T-bills and risky assets.

## II. Random Variables and Portfolio Theory (28 points, 4 points each)

- The probability distribution for next year's price of XYZ stock is given by

Price: $P_1$	90	100	110	120
Probability	0.1	0.2	0.5	0.2

- Compute the expected value of next year's price.

$$E[P_1] = 90(.1) + 100(.2) + 110(.5) + 120(.2) = 108$$

- Compute the variance and standard deviation of next year's price.

$$E[P_1^2] = 90^2(.1) + 100^2(.2) + 110^2(.5) + 120^2(.2) = 11740$$

$$\text{var}(P_1) = E[P_1^2] - E[P_1]^2 = 11740 - (108)^2 = 76$$

$$SD(P_1) = \sqrt{76} = 8.717$$

- Compute the expected rate of return if the current price is \$100 and you expect to receive a \$2 dividend at time 1.

$$r = \frac{P_1 + D_1 - P_0}{P_0}$$

$$E[r] = (-.08)(.1) + (.02)(.2) + (.12)(.5) + (.22)(.2) = .1$$

- Two companies have stock for which the expected returns and return standard deviations are forecast as:

Company	A	B
Expected Return	0.20	0.20
Standard Deviation	0.30	0.30

Consider a portfolio made up with 60% in asset A and 40% in asset B.

a. What is the expected return on the portfolio?

$$R_p = .6R_A + .4R_B$$

$$E[R_p] = .6(.20) + .4(.20) = .20$$

b. What is the variance and standard deviation of the portfolio return if the correlation between the returns on asset A and B is 1?

$$\text{var}(R_p) = x_A^2\sigma_A^2 + x_B^2\sigma_B^2 + 2x_Ax_B\rho_{AB}\sigma_A\sigma_B$$

$$\text{var}(R_p) = (.6)^2(.3)^2 + (.4)^2(.3)^2 + 2(.6)(.4)(.3)(.3) = .09$$

$$SD(R_p) = \sqrt{.09} = .3$$

c. What is the variance and standard deviation of the portfolio return if the correlation between the returns on asset A and B is 0?

$$\text{var}(R_p) = x_A^2\sigma_A^2 + x_B^2\sigma_B^2 + 2x_Ax_B\rho_{AB}\sigma_A\sigma_B$$

$$\text{var}(R_p) = (.6)^2(.3)^2 + (.4)^2(.3)^2 = .0468$$

$$SD(R_p) = \sqrt{.0468} = .2163$$

d. Since the two stocks' expected returns and standard deviations are the same, does it make any sense to combine them in a portfolio, or would you be just as well off holding either one of the stocks separately? Explain.

If the two stocks are not perfectly positively correlated, combining them into a portfolio will generate the same expected return but will also reduce the portfolio variance. Therefore, it is always better to hold a portfolio of the two assets instead of holding each asset individually.

III. (18 points, 6 points each)

Insightful Corporation is trying to decide which of two mutually exclusive projects to do. The nominal cash flows associated with the projects are described in the table below:

Project/Period	0	1	2
A	-50	50	50
B	-70	60	70

The project has risk that is comparable to that of the company's current assets. The current risk-free interest rate is 5 percent, and the expected market risk premium is 8 percent. The company has a debt/equity ratio of 0.2, and the beta for the company's stock is 1.5 while the beta for its debt is 0.4.

a. What is the beta for the company assets?

$$\beta_A = \beta_D \left( \frac{D}{D+E} \right) + \beta_E \left( \frac{E}{D+E} \right)$$

$$\frac{D}{E} = 0.2 \Rightarrow D = 0.2E \Rightarrow \frac{D}{D+E} = \frac{0.2E}{1.2E} = \frac{1}{6}$$

$$\frac{E}{D+E} = 1 - \frac{D}{D+E} = \frac{5}{6}$$

$$\beta_A = .4(.167) + 1.5(.833) = 1.3167$$

b. Using the CAPM, what is the expected return on the company assets?

$$E[r] = r_f + \beta_A(E[R_M] - r_f)$$

$$= .05 + 1.3167(.08) = .1553$$

c. Compute the NPV for the two projects. Which project should Insightful choose?

$$NPV_A = -50 + \frac{50}{1.553} + \frac{50}{(1.553)^2} = 30.736$$

$$NPV_B = -70 + \frac{60}{1.553} + \frac{70}{(1.553)^2} = 34.375$$

Choose project B.

IV. Options (20 points, 5 points each part)

1. One-year European call and European put options on Amazon stock, both with an exercise price of \$100, currently sell for \$10 and \$8 respectively. The risk free, annually compounded interest rate is 6 percent. What is the value of the company's stock?

Using Put-Call parity:

$$S = C - P + PV(X) = 10 - 8 + X / (1.06) = 96.33962$$

2. Option traders often refer to straddles. Here is an example of a straddle:

Buy a call with an exercise price of \$100 and simultaneously buy a put with an exercise price of \$100.

a. Draw a position diagram for the straddle, showing the payoff at expiration from the investor's net position.

b. The strategy is a bet on variability. Explain briefly the nature of the bet.

The straddle pays off if the stock price moves up or down. Therefore, the straddle will payoff if the volatility of the stock is high. Hence, the straddle is a bet on there being high stock return volatility.

3. The Black-Scholes option pricing formula for a European call option on a non-dividend paying stock depends of 5 variables. Describe these 5 variables, and indicate the direction of the marginal impact of increasing each variable on the call price.

The Black-Scholes option pricing formula depends on

- S = current stock price; positive marginal impact
- X = exercise price, negative marginal impact
- T = time to maturity in years, positive marginal impact
- Rf = continuously compounded risk-free rate, positive marginal impact
- $\sigma$  = standard deviation of continuously compounded return on stock, positive marginal impact.

V. Miscellaneous (10 points, 5 points each)

1. At age 20 you put \$2000 into an IRA account held in the form of a Vanguard Index 500 mutual fund. At age 60, as you begin to think seriously about retiring, you find that the account is worth \$192,865.48. What has been the average annual rate of return on this account?

$$r_A = \left( \frac{192,865.48}{2,000} \right)^{1/40} - 1 = .121$$

2. What are the three forms of market efficiency? Briefly describe how you could test the hypothesis that the stock market is weak form efficient?

The three forms of market efficiency are

1. Weak form: cannot predict future prices based on past history of prices
2. Semi-strong form: cannot predict future prices based on all publicly available information
3. Strong-form: cannot predict future prices based on all information, public and/or private.

To test weak form efficiency, one can look to see if current returns are correlated with past returns. Note: must look at returns and not prices.

Extra Credit Question (10 points):

Here is an example of a Butterfly strategy:

Simultaneously buy one call with an exercise price of \$100, sell two calls with an exercise price of \$110, and buy one call with an exercise price of \$120.

a. Draw a position diagram for the butterfly, showing the payoff at expiration from the investors net position.

b. The strategy is a bet on variability. Explain briefly the nature of the bet.

The butterfly pays off if the stock price does not move outside of the exercise price bands. This will occur if the stock return variance is very small. Therefore, the butterfly is a bet on the stock return variance being very small.