

Investment Decision Making

Econ 422: Investment, Capital & Finance
University of Washington
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Investment Decisions

- Fisher Model Criteria
 - Production or Real Investment chosen to maximize Wealth
(= present discounted stream of consumption)
 - Our **Net Present Value (NPV)** calculations calculate the increment in Wealth associated with given projects
- If projects are mutually exclusive, choose the one with highest NPV. If multiple projects are feasible, rank according to NPV and select top ones first.

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Implementing the NPV Rule

1. Determine the expected cash flows for the project (negative and positive)
2. Compute the NPV for the project as follows:

$$NPV = C_0 + \sum C_t / (1+r_{0,t})^t \quad \text{for } t = 1 \text{ to } T$$

(Note: $C_0 = -I_0$ typically)

3. Rely on the term structure for discount rates when needed
4. **NPV Rule: Undertake project if its NPV > 0**

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Competitors/Alternatives to the NPV Rule

- **Payback rule--misleading**
 1. Calculate the time for a project to payback or recover the initial investment cost (break-even analysis)
 2. Compare projects based on payback time
 - Ignores value of all future cash flows beyond payback
 - Provides equal weight to cash flows before the cutoff date, i.e., sequential timing matters rather than including time value of money

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Competitors/Alternatives to the NPV Rule

- **Average return on book value-- inappropriate**
 - Book value = historic or accounting cost
 - Book rate of return = book income from project ÷ book assets of project
 - Cash flows ≠ book income
 - Fails to discount properly—averaging not necessarily appropriate

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Competitors/Alternatives to the NPV Rule

- **Internal Rate of Return (IRR)**
 - Commonly used
 - Sometimes equivalent to NPV Rule
 - Sometimes requires ad hoc adjustment
- **Real Option Methodology** (discuss in Options segment)
 - Introduces stages and more flexibility

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Calculating IRR

Recall **NPV Rule: NPV > 0**. Note NPV calculation depends on r .

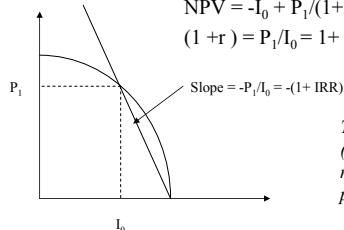
IRR Method— Determine discount rate such that NPV of project = 0. Select projects with IRR > r .

Example: Let I_0 = amount of investment made today, P_1 = return on the investment next period.

The IRR is that r which makes $NPV(r) = 0$:

$$NPV = -I_0 + P_1/(1+r) = 0$$

$$(1+r) = P_1/I_0 = 1 + \text{IRR}$$



The slope of the transformation curve (MRT) at a given point represents the marginal IRR for a small incremental project in the neighborhood of the point.

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IRR Rules

- IRR is the discount rate for which $NPV = 0$; therefore, accept those projects for which IRR exceeds the discount rate:

IRR Rule: Choose projects with IRR > r

- *The IRR rule interpreted: When the internal rate of return for the project exceeds what you would receive by lending, you will increase wealth by making the investment—transforming current resources into future resources via direct investment rather than lending.*

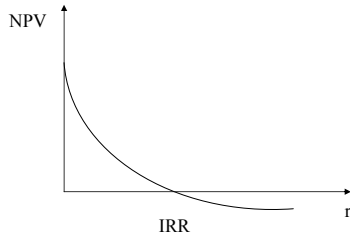
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Graphical Representation of IRR

IRR is r such that:

$$NPV = C_0 + \sum C_t/(1+r)^t = 0 \quad t = 1, \dots, T$$

NPV is **usually** a decreasing function of r .



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Example: Calculating IRR

Suppose an investment project has the following cash flows: -4, 5 at time periods 0 and 1. Find the IRR.

$$NPV(\text{IRR}) = -4 + 5/(1+\text{IRR}) = 0. \text{ Now solve for IRR:}$$

$$\Rightarrow 4 = 5/(1+\text{IRR})$$

$$\Rightarrow (1+\text{IRR}) = 5/4$$

$$\Rightarrow \text{IRR} = 5/4 - 1 = 1/4 = 0.25$$

Suppose the appropriate discount rate is $r = 0.20$. Then

$$NPV(0.20) = -4 + 5/(1.20) = 1.67 > 0$$

$$\text{Note: } \text{IRR} = 0.25 > r = 0.20 \Rightarrow NPV(r) > 0$$

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Example: Calculating IRR

Suppose instead the investment project has the following cash flows: -3, 2, 2. What is the IRR?

$$NPV(r) = -3 + 2/(1+r) + 2/(1+r)^2 = 0$$

Recall for Quadratic Equation:

$$ax^2 + bx + c = 0$$

Multiplying through by $(1+r)^2$

$$-3(1+r)^2 + 2(1+r) + 2 = 0$$

Quadratic Formula:

$$x = [-b \pm (b^2 - 4ac)^{1/2}] / 2a$$

Using the quadratic formula:

$$(1+r) = -2 \pm (4 - (4 * -3 * 2))^{1/2} / (2 * -3)$$

$$(1+r) = [-2 \pm (28)^{1/2}] / (2 * -3)$$

$$(1+r) = [-2 \pm (5.2915)] / (-6)$$

Multiple solutions possible!

$$(1+r) = [-2 + (5.2915)] / (-6)$$

$$(1+r) = [-2 - (5.2915)] / (-6)$$

$$(1+r) = [-2 + (5.2915)] / (-6)$$

$$(1+r) = [-2 - (5.2915)] / (-6)$$

$$1+r = -0.54858$$

$$1+r = 1.21525$$

$$r = -1.54858$$

$$r = 0.21525$$

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General Case of Solving for IRR

For a project with finite cash flows: $C_0, C_1, C_2, \dots, C_T$

$$NPV = C_0 + \sum C_t/(1+r)^t = 0 \quad t = 1, \dots, T$$

When $T > 2$ you need to solve numerically.

IRR rule: Accept project if $\text{IRR} > r$.

Notice any similarities?

Recall calculating Yield to Maturity involved solving for r such that:

$$P_0 = \sum C/(1+r)^t + F/(1+r)^T \quad t = 1, \dots, T$$

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Pitfalls of IRR Methodology

Practical problems encountered with the application of IRR

Methods:

- Multiple solutions arising from multiple roots or no solution
- No ability to incorporate term structure of interest rates
- Confusion with reverse cash flows (borrowing)

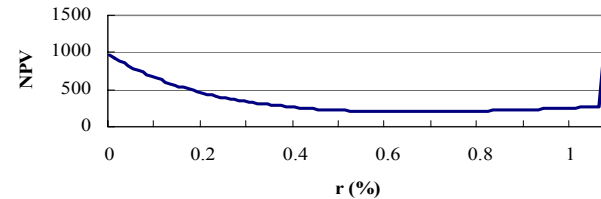
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No Solution

Some projects by nature of the cash flows have nonnegative NPVs such that there is no IRR, i.e., no r such that $NPV = 0$:

$$NPV = 2000 - 6000/(1+r) + 5000/(1+r)^2$$

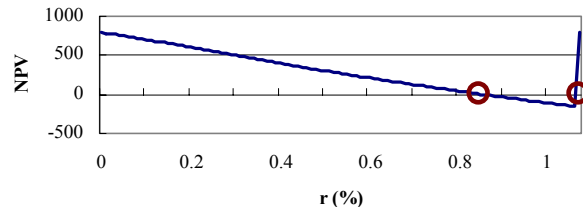
$NPV > 0$ all interest rates



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Multiple Solutions

- NPV equation for a T period stream of cash flows is a polynomial in r of order T.
- Changes of signs in the stream of cash flows can cause multiple IRRs



For $r < 0.855$ project has $NPV > 0$; for r between 0.855 and 1.06 project should not be undertaken, but undertaken for $r > 1.06$

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No Ability to Handle Variation in r

- NPV uses term varying discount rates when appropriate:

$$NPV = C_0 + \sum C_t / (1 + r_t) \quad t = 1, \dots, T$$

That is, NPV can make use of a non-flat term structure

- IRR is predicated on a **fixed** rate of return.

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Reverse Cash Flows & IRR

Suppose your parents lend you money to purchase your first car. The relevant discount rate is 10%. You will make annual payments to them in return. Your parents receive the following cash flows (a simplification):

$$\{-\$1,000, \$474.75, \$474.75, \$474.75\}$$

Solving numerically, IRR = 20% which exceeds $r = 10\%$. Your parents accept this transaction. The NPV for $r = 10\%$ is

$$\$180.57 > 0.$$

Both IRR and NPV suggest your parents provide you the loan.

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Reverse Cash Flows & IRR

Now consider your IRR. You receive the following cash flows:

$$\{\$1,000, -\$474.75, -\$474.75, -\$474.75\}$$

The IRR is again 20%. NPV to you is

$$-\$180.57$$

which suggests you do not accept loan terms based on NPV rule. The IRR and NPV rule are only consistent if in the presence of reverse/negative cash flows (borrowing) IRR rule is modified to accepting projects if

$$r > \text{IRR}.$$

→ IRR does not provide consistent decision rule

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IRR: Mutually Exclusive Projects Ranked Incorrectly

Consider three mutually exclusive projects: A, B, C with the following cash flow, IRR and NPVs

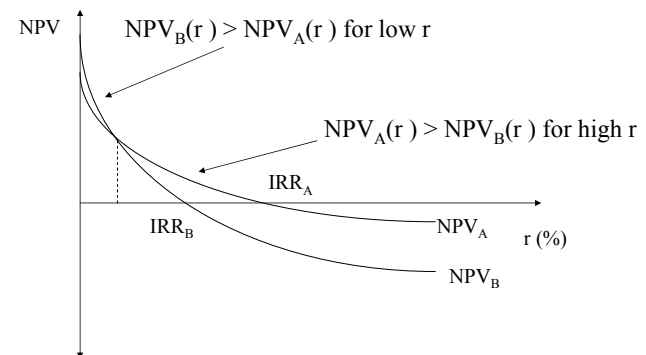
Project	Time	0	1	2	IRR	NPV(10%)
A		-100	70	70	25.7	\$21.49
B		-120	70	97	23.7	\$23.80
C		-20	0	27	16.2%	\$2.31

Based on IRR criteria, Project A would be undertaken. Based on a NPV criteria, Project B would be undertaken.

Note: To maximize wealth, project B should be undertaken.

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NPV versus IRR cont.



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Applying NPV to Make Investment Decisions

What to discount?

1. Only cash flows = \$ in - \$ out
2. Incremental cash flows matter; i.e., focus on the incremental or additional cash flows of the firm if the project is adopted versus if the project is not adopted
3. Treat inflation consistently

Consistent Treatment of Inflation

Discount real (constant dollars) cash flows by a real discount rate. Cash flows in real terms: C_0, C_1, \dots, C_T and real discount rate r_R

$$NPV = C_0 + \sum C_t / (1+r_R)^t \quad t = 1, \dots, T$$

Discount nominal cash flows by a nominal discount rate.

Consider cash flows in real terms: C_0, C_1, \dots, C_T ; real discount rate r_R , expected inflation rate π , nominal discount rate r_N :

Nominal Cash flow: $C_t(1+\pi)^t$

Nominal discount rate: $(1+r_N) = (1+r_R)*(1+\pi)$

$$NPV = C_0 + \sum C_t(1+\pi)^t / (1+r_R)^t (1+\pi)^t \quad t = 1, \dots, T$$

$$NPV = C_0 + \sum C_t(1+\pi)^t / (1+r_N)^t \quad t = 1, \dots, T$$

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Example: Real versus Nominal Treatment

- It doesn't matter which—nominal or real—that you use, you just need to be consistent:

NPV analysis in real terms:

Real cash flow: $\{-100, 50, 50, 50\}$; $r_R = 9\%$

$$\begin{aligned} NPV &= -100 + 50/(1.09) + 50/(1.09)^2 + 50/(1.09)^3 \\ &= 26.5 \end{aligned}$$

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Example: Real versus Nominal Treatment

NPV analysis in nominal terms:

Same real cash flows: $\{-100, 50, 50, 50\}$; $r_R = 9\%$; expected inflation of 10%

$$\begin{aligned} NPV &= -100 + 50(1+\pi)/(1+r_N) + 50(1+\pi)^2/(1+r_N)^2 \\ &\quad + 50(1+\pi)^3/(1+r_N)^3 \end{aligned}$$

$$\begin{aligned} NPV &= -100 + 50(1.1)/[(1.09)(1.1)] \\ &\quad + 50(1.1)^2/[(1.09)^2(1.1)^2] \\ &\quad + 50(1.1)^3/[(1.09)^3(1.1)^3] = 26.5 \end{aligned}$$

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Evaluating Investments with Unequal Life Spans

- NPV calculations may not always in and of themselves be compatible.
- Two useful approaches to use when administering NPV analysis
 - Equivalent annual cost
 - Common life span cost

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Example: Choice of Durability

You are the owner of a manufacturing firm and you need to make a decision to purchase a new machine for production. There are two options—machine A and machine B. Using your finance knowledge, you have been entrusted with making the purchasing decision. In comparing the two machines, you note the following:

1. Machine A is more expensive, but lasts longer and is cheaper to operate
2. Machine B is cheaper, but has a shorter life and is more expensive to operate
3. Both machines provide the same revenue or benefits in each year of operation

Machine\Time	Machine Costs			
	0	1	2	3
A	15	4	4	4
B	10	6	6	N/A

You need to make a choice on the basis of the relative cost of the two machines.

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Choice of Durability, cont.

Machine\Time	Machine Costs			
	0	1	2	3
A	15	4	4	4
B	10	6	6	N/A

Using PV Analysis, assuming a 6% discount rate:

$$\text{PV of Machine A costs} = \text{PVC}_A = 15 + 4/(1.06) + 4/(1.06)^2 + 4/(1.06)^3 = 25.69$$

$$\text{PV of Machine B costs} = \text{PVC}_B = 10 + 6/(1.06) + 6/(1.06)^2 = 21.00$$

Our PV Analysis suggests that Machine B is cheaper, but it will not provide services in period 3. Machine A provides services in period 3.

Our PV analysis is not really helpful here. What is the relevant horizon over which to evaluate these two machines

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Equivalent Annual Cost Approach

- Assume that when a machine wears out it will be routinely replaced so you can think of continuous production.
- In this approach, what is the annualized cost (cost pro-rata/per annum) associated with buying and operating each machine?

Idea: The present value of the machine cost is the same as an annuity of the equivalent annual cost over the life of the machine.

$$\text{PVC} = \text{PVA}(r, T) * \text{EAC}$$

where

$$\text{PVA}(r, T) = \text{annual finite annuity paying } 1\$ \text{ for } T \text{ years} = (1/r)[1 - 1/(1+r)^T]$$

$$\text{EAC} = \text{equivalent annual cost annuity payment}$$

Solving for EAC gives

$$\text{EAC} = \text{PVC}/\text{PVA}(r, T)$$

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Equivalent Annual Cost Approach

With $r = 0.06$ and $T = 3$

$$PVA(0.06, 3) = (1/0.06)[1 - 1/(1.06)^3] = 2.673012$$

And with $r = 0.06$ and $T = 2$

$$PVA(0.06, 2) = (1/0.06)[1 - 1/(1.06)^2] = 1.833393$$

Therefore, with $PVC_A = 25.69$ and $PVC_B = 21.00$

$$EAC_A = 25.69/2.673012 = 9.61$$

$$EAC_B = 21.00/1.833393 = 11.45$$

Based on Equivalent Annual Cost, Machine A is cheaper.

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Common Life Span Approach

Idea: If replacements are continued to a common lifetime then the benefit streams will match and we can compare the present value of costs.

In our example, with lifetimes of 3 years and 2 years, use a common horizon of 6 years. Machine A will be replaced once, Machine B will be replaced twice. Discount cost stream back to today:

$$PVC_A (\text{replace 1 time}) = 25.69 + 25.69/(1.06)^3 = 47.26$$

$$PVC_B (\text{replace 2 times}) = 21.00 + 21.00/(1.06)^2 + 21.00/(1.06)^4 = 56.32$$

Over the six year common horizon, Machine A is cheaper.

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Replacement Decision

Suppose you are considering replacing an older machine. Your current machine has operating cost of 8. A new machine has the following costs:

Machine\Time	0	1	2	3
New	15	5	5	5

Using a discount rate of 10%, $PVC_{\text{New}} = 27.43$

$$EAC_{\text{New}} = PVC_{\text{New}}/PVA(r, T) = 27.43/PVA(0.1, 3) = 27.43/2.486852 = 11.03 > 8$$

The annual cost for the new machine which includes the purchase and operating costs exceeds the cost of utilizing the existing machine. Do not replace existing machine.

Note: The original cost of the existing machine does not factor into your decision as this cost is sunk or irretrievable.

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