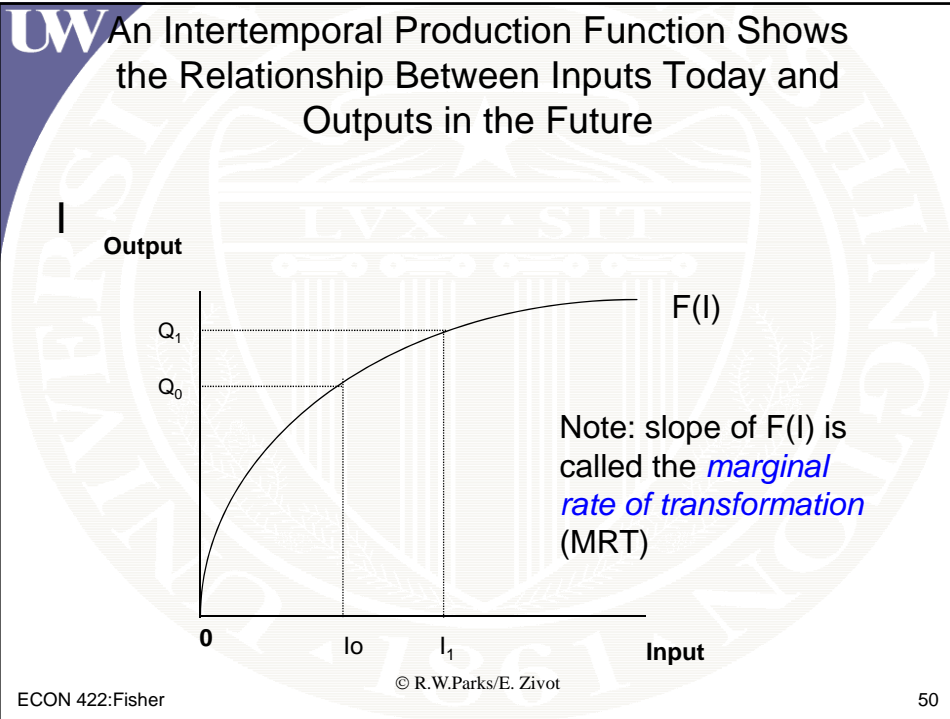# Determinants of the Level of Real Interest Rates
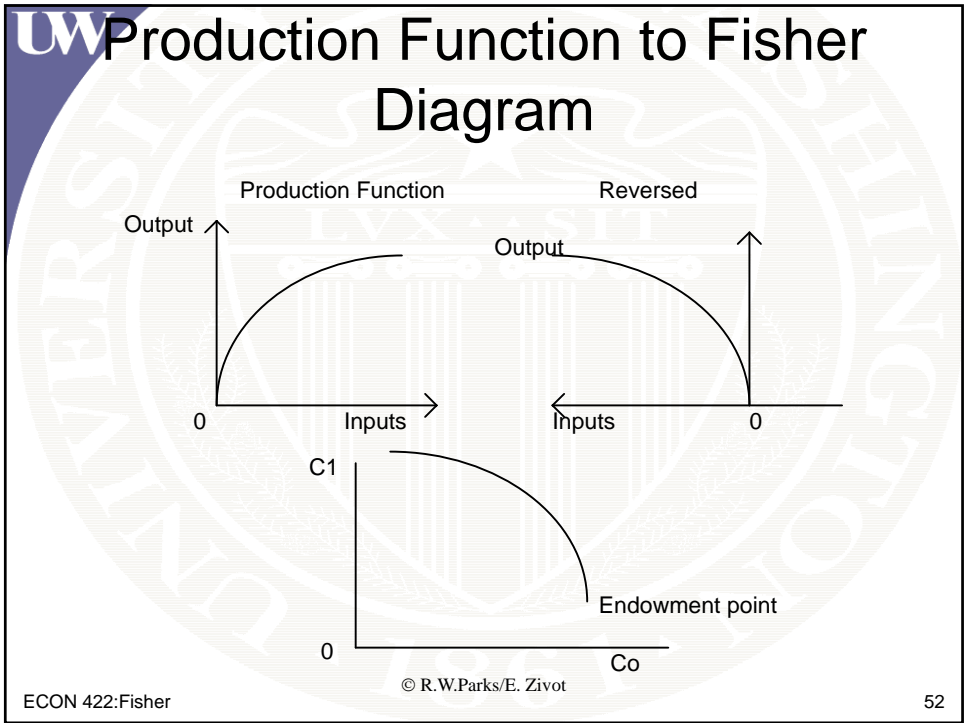
- Societal Preferences
  - » The more present oriented are societal preferences, the higher the market r
    - – Shifts borrowing curve out
- Societal Endowments
  - » The more present oriented are societal endowments, the lower the market r
    - – Shifts lending curve out

- Productive Opportunities [see later]

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II. Intertemporal Production (Real Investment) and Exchange: Outline

A. Individual optima

B. The Fisher Separation Theorem

C. Market equilibrium

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An Intertemporal Production Function Shows the Relationship Between Inputs Today and Outputs in the Future

I

Output

Q_1

Q_0

$F(I)$

Note: slope of F(I) is called the *marginal rate of transformation* (MRT)

0   Io I_1 **Input**

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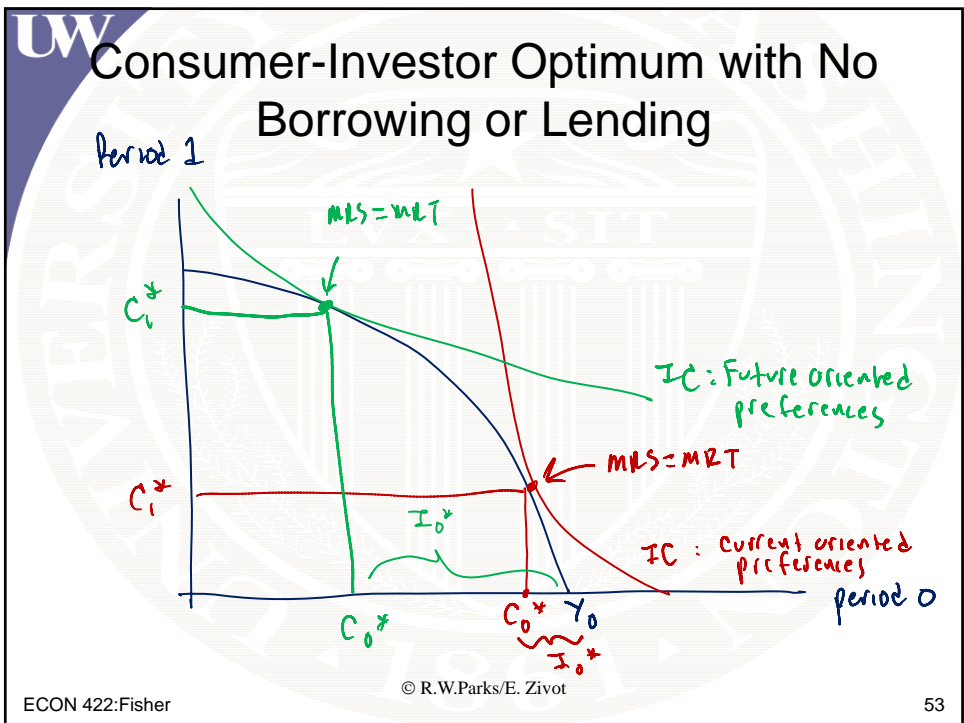
From Production Function to Fisher Diagram

- Reverse the production function around the vertical axis
- Drag the origin to the endowment point of the Fisher diagram
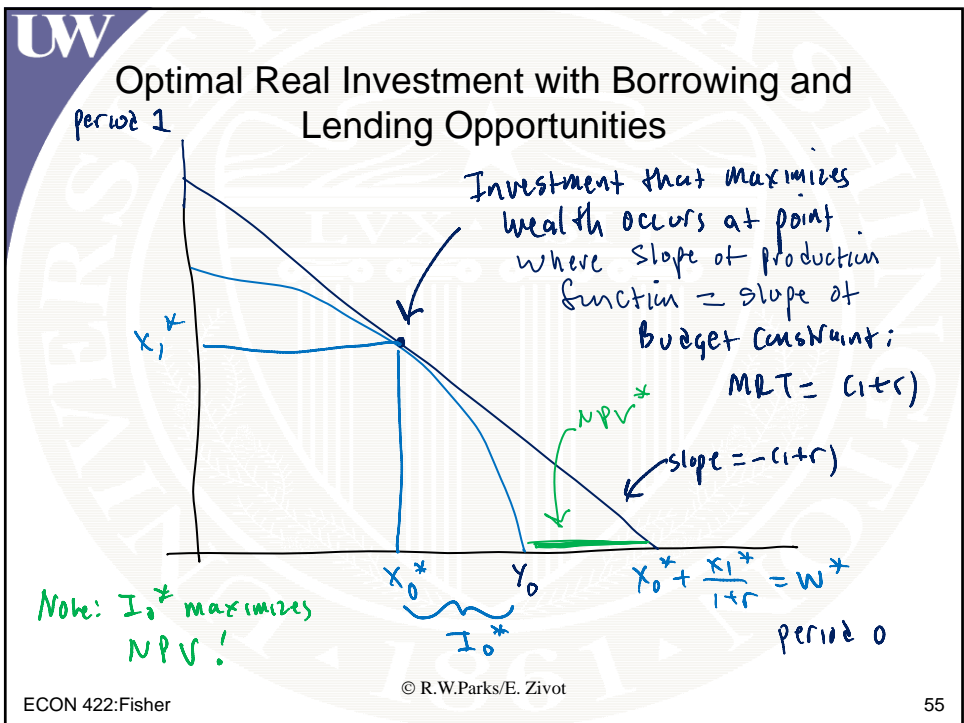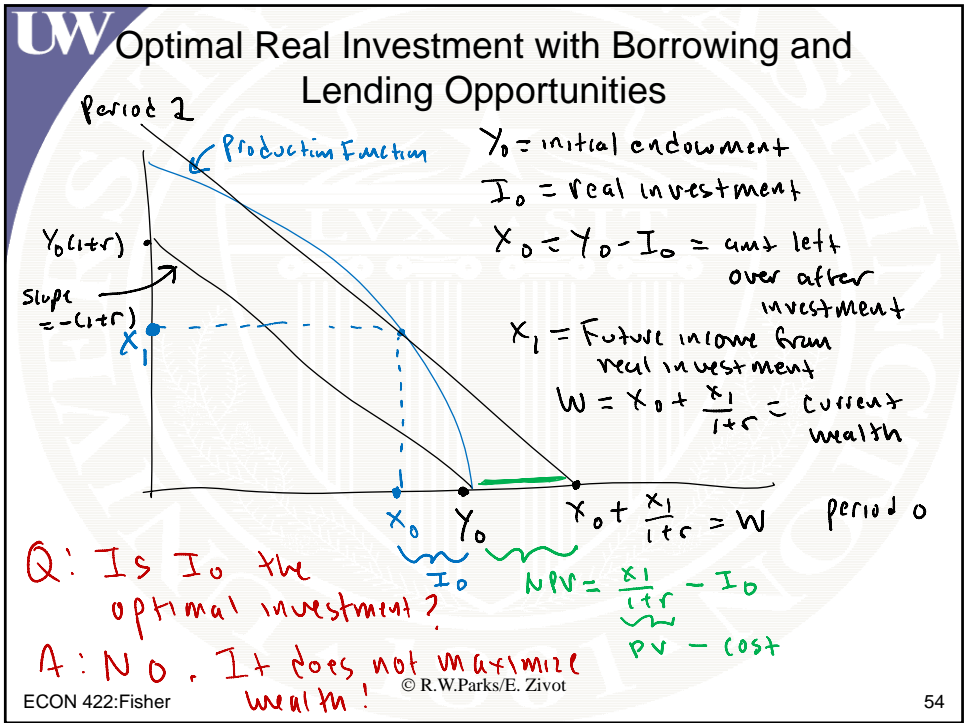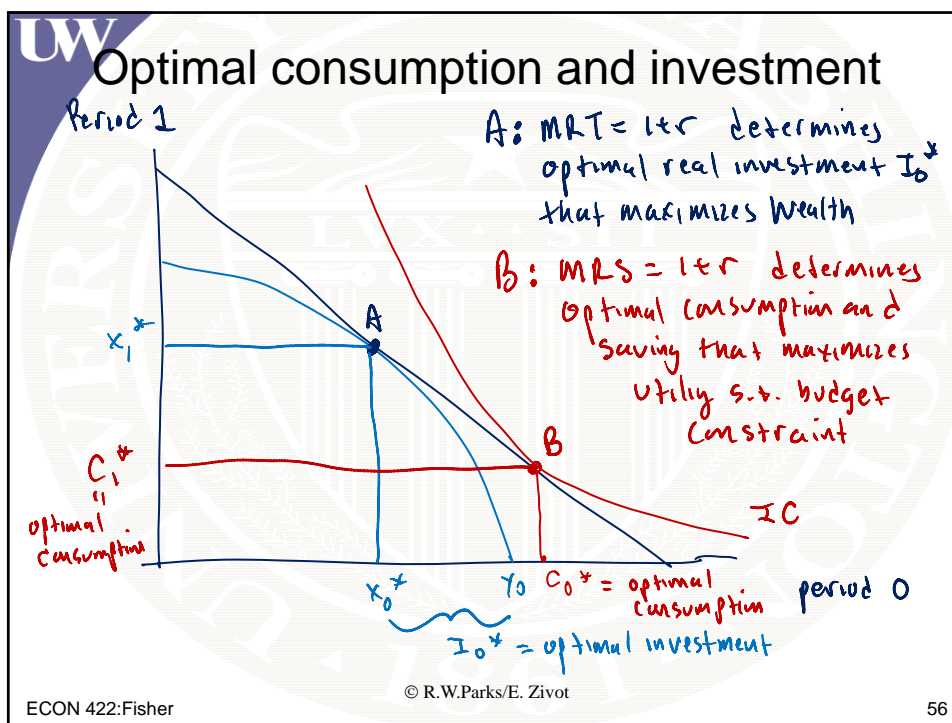
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Production Function to Fisher Diagram

Production Function   Reversed

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Consumer-Investor Optimum with No Borrowing or Lending

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Optimal Real Investment with Borrowing and Lending Opportunities

Period 2

Production Function

$Y_0(1+r)$

Slope $= -(1+r)$

X_1

$Y_0 = $ initial endowment

$I_0 = $ real investment

$X_0 = Y_0 - I_0 = $ amt left over after investment

$X_1 = $ Future income from real investment

$W = X_0 + \dfrac{X_1}{1+r} = $ current wealth

X_0

Y_0

I_0

$X_0 + \dfrac{X_1}{1+r} = W$ period 0

$NPV = \dfrac{X_1}{1+r} - I_0$

$PV - cost$

Q: Is I_0 the optimal investment?

A: No. It does not maximize wealth!

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Optimal Real Investment with Borrowing and Lending Opportunities

Period 1

Investment that maximizes wealth occurs at point where slope of production function = slope of Budget Constraint:

$MRT = (1+r)$

X_1^*

NPV^*

slope $= -(1+r)$

Note: I_0^* maximizes NPV!

X_0^*

Y_0

I_0^*

$X_0^* + \dfrac{X_1^*}{1+r} = W^*$

period 0

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Optimal consumption and investment

Period 1

A: MRT = 1+r determines optimal real investment I_0^* that maximizes Wealth

B: MRS = 1+r determines optimal consumption and saving that maximizes utility s.t. budget constraint

x_1^*

C_1^*

optimal consumption

IC

x_0^* Y_0 $C_0^* =$ optimal consumption   period 0

$I_0^* =$ optimal investment

The Fisher Separation Theorem

- The consumer - investor's two-fold problem of determining the optimal level of investment and the optimal consumption stream can be separated into two steps:
  - » First choose the investment level that maximizes wealth. This choice does <u>not</u> depend on preferences.
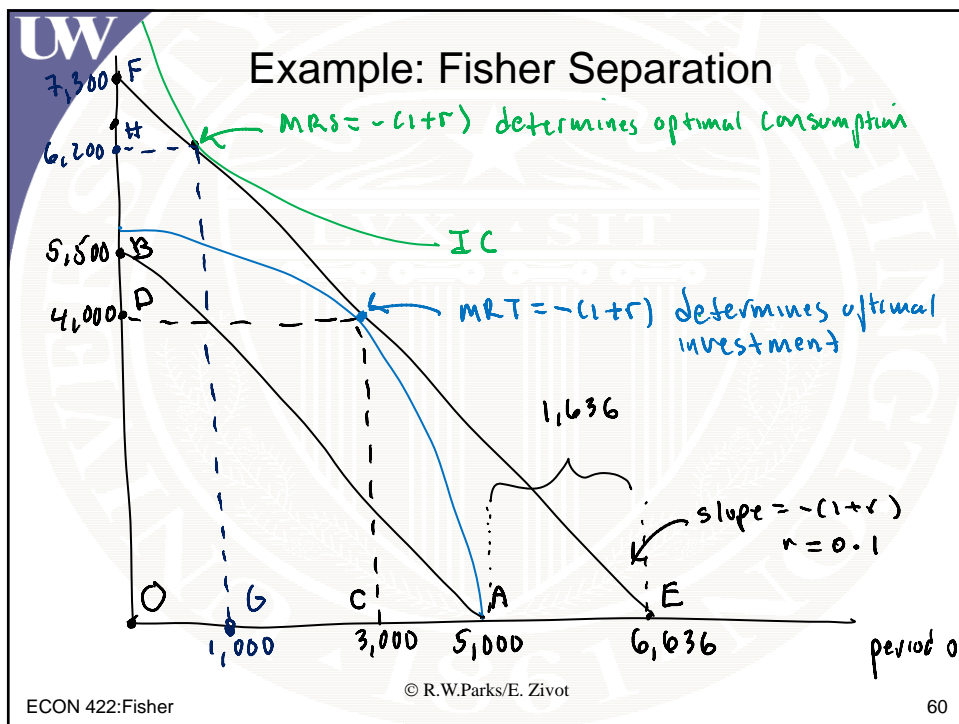 - » Next determine the optimum consumption stream, based on the maximized wealth.

The Fisher Separation Theorem (continued)

- The choice of optimal investment can be separated from the choice of optimal consumption. i.e it does not depend on investor preferences.

- A necessary condition for utility maximization is wealth maximization.

- Note: The separation result depends on the existence of a perfect capital market. Investors can borrow or lend at the market rate r.
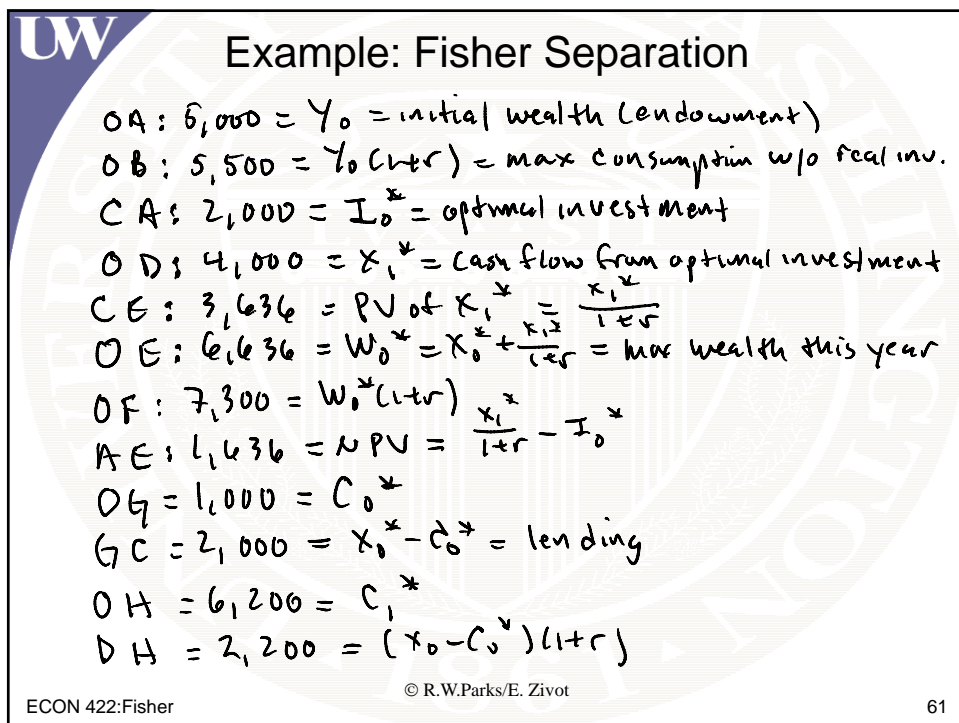
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Optimal Investment and NPV

- Optimal investment maximizes current wealth

- Equivalently, optimal investment maximizes Net Present Value (NPV)

  » NPV = PV of investment cash flows – initial investment cost

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Example: Fisher Separation

$MRS = -(1+r)$ determines optimal consumption

IC

$MRT = -(1+r)$ determines optimal investment

1,636

$slope = -(1+r)$
$r = 0.1$

7,300 F
6,200 H
5,500 B
4,000 D
O   G   C   A   E
1,000   3,000   5,000   6,636

period 0

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Example: Fisher Separation

$OA: 6,000 = Y_0 = $ initial wealth (endowment)

$OB: 5,500 = Y_0(1+r) = $ max consumption w/o real inv.

$CA: 2,000 = I_0^* = $ optimal investment

$OD: 4,000 = X_1^* = $ cash flow from optimal investment

$CE: 3,636 = PV$ of $X_1^* = \dfrac{X_1^*}{1+r}$

$OE: 6,636 = W_0^* = X_0^* + \dfrac{X_1^*}{1+r} = $ max wealth this year

$OF: 7,300 = W_0^*(1+r)$

$AE: 1,636 = NPV = \dfrac{X_1^*}{1+r} - I_0^*$

$OG = 1,000 = C_0^*$

$GC = 2,000 = X_0^* - C_0^* = $ lending

$OH = 6,200 = C_1^*$

$DH = 2,200 = (X_0 - C_0^*)(1+r)$

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Determinants of the Level of Real Interest Rates Revisited

- Societal Preferences
  - » The more present oriented are societal preferences, the higher the market r
    - Shifts Borrowing curve out
- Societal Endowments
  - » The more present oriented are societal endowments, the lower the market r
    - Shifts lending curve out
- Productive Opportunities
  - » The more productive are the opportunities for converting present into future resources through real investment (i.e. production), the higher the market r.

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