Determinants of the Level of Real Interest Rates

- Societal Preferences
  - The more present oriented are societal preferences, the higher the market rate
    - Shifts borrowing curve out

- Societal Endowments
  - The more present oriented are societal endowments, the lower the market rate
    - Shifts lending curve out

- Productive Opportunities [see later]

II. Intertemporal Production (Real Investment) and Exchange: Outline

A. Individual optima
B. The Fisher Separation Theorem
C. Market equilibrium
An Intertemporal Production Function Shows the Relationship Between Inputs Today and Outputs in the Future

Output

\[ Q_0 \]

\[ Q_1 \]

\[ F(I) \]

Note: slope of \( F(I) \) is called the *marginal rate of transformation* (MRT)

From Production Function to Fisher Diagram

- Reverse the production function around the vertical axis
- Drag the origin to the endowment point of the Fisher diagram
Production Function to Fisher Diagram

Consumer-Investor Optimum with No Borrowing or Lending
Optimal Real Investment with Borrowing and Lending Opportunities

Period 1

Investment that maximizes wealth occurs at point where slope of production function = slope of budget constraint:

\[ MRT = -(1+r) \]

Note: \( I_0^* \) maximizes NPV!

\[ X_0^* + \frac{X_1^*}{1+r} = W^* \]

Period 2

Production Function

\[ Y_0(1+r) \]

Slope = \( -\frac{1}{1+r} \)

\[ Y_0 = \text{initial endowment} \]

\[ I_0 = \text{real investment} \]

\[ X_0 = Y_0 - I_0 = \text{ amt left over after investment} \]

\[ X_1 = \text{future income from real investment} \]

\[ W = Y_0 + \frac{X_1}{1+r} = \text{current wealth} \]

Q: Is \( I_0 \) the optimal investment?

A: No, it does not maximize wealth.

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The Fisher Separation Theorem

- The consumer - investor’s two-fold problem of determining the optimal level of investment and the optimal consumption stream can be separated into two steps:
  - First choose the investment level that maximizes wealth. This choice does not depend on preferences.
  - Next determine the optimum consumption stream, based on the maximized wealth.
The Fisher Separation Theorem (continued)

- The choice of optimal investment can be separated from the choice of optimal consumption, i.e., it does not depend on investor preferences.
- A necessary condition for utility maximization is wealth maximization.
- Note: The separation result depends on the existence of a perfect capital market. Investors can borrow or lend at the market rate $r$.

Optimal Investment and NPV

- Optimal investment maximizes current wealth
- Equivalently, optimal investment maximizes Net Present Value (NPV)
  
  » NPV = PV of investment cash flows – initial investment cost
Example: Fisher Separation

OA: \( W_0 = y_0 = \text{initial wealth (endowment)} \)
OB: \( 5,500 = y_0 (1+r) = \text{max consumption w/o real inv.} \)
CA: \( 2,000 = I_0^* = \text{optimal investment} \)
OD: \( 4,100 = x_i^* = \text{cash flow from optimal investment} \)
CE: \( 3,436 = \text{PV of } x_i^* = \frac{x_i^*}{1+r} \)
OE: \( 6,436 = W_0^* = X_0^* + \frac{x_i^*}{1+r} = \text{max wealth this year} \)
OF: \( 7,300 = W_0^* (1+r) = \frac{x_i^*}{1+r} \)
AG: \( 1,000 = C_0^* \)
GC: \( 2,000 = X_0^* - C_0^* = \text{lending} \)
OH: \( 6,200 = C_1^* \)
DH: \( 2,200 = (X_0^* - C_0^*) (1+r) \)
Determinants of the Level of Real Interest Rates Revisited

- Societal Preferences
  - The more present oriented are societal preferences, the higher the market rate
    - Shifts Borrowing curve out

- Societal Endowments
  - The more present oriented are societal endowments, the lower the market rate
    - Shifts lending curve out

- Productive Opportunities
  - The more productive are the opportunities for converting present into future resources through real investment (i.e. production), the higher the market rate.