Euler, Darwin, and Population Statistics

By Dominic Klyve

This year marks the 200th anniversary of Charles Darwin's birth, and Darwin has been getting a lot of press. I've actually been somewhat distressed by how far the celebrations, conferences, encomia, and magazine covers have surpassed those of Euler's 300th birthday in 2007. Nevertheless, this broad focus on the history of science is good for all of us in the profession, and we wish the Darwin scholars well (even if we remain a touch jealous). My jealousy is mitigated, however, by the opportunity this anniversary presents for the Euler community, in that it gives us a chance to ask a rarely asked question: what is the connection between Darwin and Euler?

It is not well known that Darwin explicitly cites Euler in one of his books. On page 131 of *The Descent of Man* [D], Darwin writes

> Civilised populations have been known under favourable conditions, as in the United States, to double their number in twenty-five years; and according to a calculation by Euler, this might occur in a little over twelve years.[51]
> ...
> [51] See the ever memorable 'Essay on the Principle of Population,' by the Rev. T. Malthus, vol. i. 1826, p. 6, 517.

There are many wonderful reasons we remember Euler's work today, but no short list I've seen has ever included Euler calculating a possible population doubling time. In fact, this calculation is so elementary that it seems silly to cite anyone for it, let alone one of the greatest mathematicians of all time. It seems that Darwin may be giving Euler credit for something he doesn't especially deserve. Why, the discerning reader may ask, does Darwin cite Euler here?

We first note that Darwin himself was not a good mathematician. In his collected writings, he uses the word "repugnant" only twice, and of one these refers to mathematics: "I attempted mathematics, … but I got on very slowly. The work was repugnant to me, chiefly from my not being able to see any meaning in the early steps in algebra."
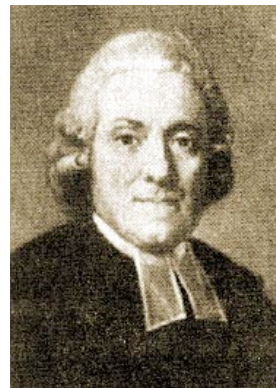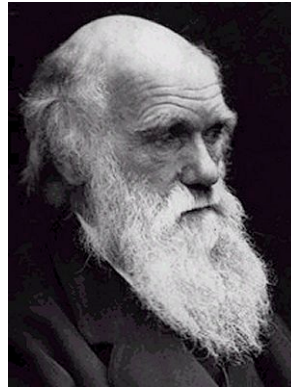
Clockwise from upper left: Charles Darwin (1809-1882), Thomas Malthus (1766-1834), Leonhard Euler (1707-1783), Johann Süssmilch (1707-1767).

We can therefore assume that, given the chance, he would much prefer to crib calculations from another source than to work them himself. Thus, while discussing population increase over time, he turned to the source that helped inspire his ideas about natural selection in the first place, Thomas Malthus's *Essay on the Principle of Population* [M].

It's not at all clear that Darwin knew who Euler was—he may simply have taken Malthus's attribution of the calculation to Euler and repeated it himself. All this is to say that if we want to understand how Euler got in Darwin's book, we need to turn to Malthus.

Malthus is best known today for his dire predictions of future famine and starvation. Malthus stated that human population would increase geometrically, while

The Euler Line

Dispatches from the world of Euler scholarship

Euler and Logarithms. Erik Koelink and Walter Van Assche have published their paper *Leonhard Euler and a q-analogue of the logarithm* in the Proceedings of the American Mathematical Society (Vol. 137, No. 5).

Calinger on Fellman on Euler. Ron Calinger reviewed a recent translation of Emil Fellmann's biography on Euler (translation by Erika Gautchi and Walter Gautschi) in a recent issue of *Isis* (Vol. 100, No. 2). According to Calinger (and many others), Fellman's book is the best introduction to the life and work of Euler available in English.

Tercentenary in Brunswick. As part of its Euler Tercentenary celebrations, the Brunswick State Museum (Braunschweigisches Landesmuseum) published a collection of essays on Euler. The collection is titled *Leonhard Euler, 1707-1783 : Mathematiker, Mechaniker, Physiker*, and was published in 2008.

Among its many essays is John Glaus's paper, "Das Heilige und das Weltliche" (The Holy and the Profane). This work examines instructions that Euler wrote to a French church in Berlin of which he was a member. An appendix includes the original French text by Euler and its German translation, as well as a photocopy of the first page.

When Euler Met l'Hôpital. William Dunham's latest article on Euler, "When Euler met L'Hôpital," appeared this year in Mathematics Magazine (Vol. 82, No. 1).

The Quartic Equation. Dick Nickalls of Nottingham University Hospitals in the United Kingdom has published his paper, "The quartic equation: invariants and Euler's solution revealed" in the Mathematical Gazette (Vol. 93, pp. 66-75). This paper is based in part on Euler's paper, *De formis radicum aequationum cuiusque ordinis coniectatio* [E30].

our ability to grow more food would (Malthus claimed) grow only arithmetically. Thus at some point more people will be alive than can be fed, leading to starvation (and, for Darwin, helping usher in the idea of the survival of only the fittest). In order to establish that population growth was geometric, Malthus needed data. And while that data came from a variety of sources, one of his favorites was *Die Göttliche Ordnung* by Johann Peter Süssmilch. On page 493 of his *Essay*, for example, Malthus included a short note: "I subjoin two tables from Süssmilch, calculated by Euler, which I believe are very correct" [M]. This leaves us only to wonder: who is Süssmilch, and what does he have to do with Euler?

Johann Süssmilch, born four months and 400 miles from Euler (in Berlin), is known as the father of German demography for his almost-singlehanded transformation of the field from a loose set of ideas into a codified, quantitative field of study based on careful assimilation of data (see [ESS] for more on Süssmilch's work). Süssmilch believed that the multiplication of mankind was a divine command (the "Be fruitful and multiply" from the first chapter of Genesis), and tried to show that the growth of population was a divinely ordained statistical law, on the order of the divinely ordained laws of Kepler or Newton.

Contribute to the *Opusculus*

The success of this newsletter is dependent on your contributions! If you or someone you know has done something of note in the world of Euler scholarship, please contact us at the *Opusculus*. Short items can be as worthwhile as longer pieces. Consider writing to us about any of the following:

Seminars and Colloquia on Euler or 18th century science.

Translations and new availability of translations.

Publications, both books and articles.

Photographs are always welcome.

Any items that you deem worthy can be sent by e-mail to Erik Tou (etou@carthage.edu). Please include "Opusculus" in the subject line.

Euler and Darwin, continued from page 5

Süssmilch probably met Euler shortly after his election to the Berlin Academy of Sciences (class of philology) in 1745, in recognition of his work on Oriental languages. They apparently talked about demographic matters at some length, resulting in Euler's co-authoring chapter eight of Süssmilch's magnum opus, the second edition of *Die Göttliche Ordnung* [S], published in 1762. (In fact, some authors ([Sh], for example) have suggested that Euler may have participated in writing the entire book.) This is an easy work for Euler scholars to miss, as it doesn't have an Eneström number, and doesn't fit easily into our standard schemes. Happily, Du Pasquier included this chapter in Series 1, Volume 7 of the Opera Omnia, so it is now quite accessible. As Ed Sandifer points out [S2], Euler's work *Recherches générales sur la mortalité et la multiplication du genre humain* [E334] probably arose directly from his work with Süssmilch.

On the whole, it seems that Süssmilch, as later with Darwin, was not much of a mathematician, and it's safe to assume that Euler's influence led directly to the rigor and careful analysis that went into *Die Göttliche Ordnung*. We can therefore give Euler some of the credit for creating the work which would help lead Malthus, and later Darwin, to their great theories. In light of this, Darwin's brief mention of Euler, rather than being overly generous, seems like scant praise indeed.

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[D] Darwin, C. The descent of man, and selection in relation to sex. Volume 1. 1st edition. (1871).

[D2] Darwin, C. *Recollections of the development of my mind and character*. Published in "The life and letters of Charles Darwin including an autobiographical chapter edited by his son Francis Darwin" (1887).

[E334] Euler, L. *Recherches générales sur la mortalité et la multiplication du genre humain.* Memoires de l'academie des sciences de Berlin 16, (1769), 1767, pp. 144-164. Reprinted in Opera Omnia: Series 1, Volume 7, pp. 79 - 100 Available online at www.eulerarchive.org.

[ESS] *Süssmilch, Johann Peter* (article) Encyclopedia of Statistical Sciences, John Wiley and Sons (2006).

[M] Malthus, T. An essay on the principle of population; or, a view of its past and present effects on human happiness; with an inquiry into our prospects respecting the future removal or mitigation of the evils which it occassions. (1826).

[S] Süssmilch, J. *Die Göttliche Ordnung* (full title translation: The Divine Order in the Transformations of the Human Race as Demonstrated through Birth, Death, and the Multiplication of the Same)

[S2] Sandifer, E. *St. Petersburg Paradox,* How Euler Did It, MAA Online, July 2007.

[Sh] Shynin, O. B. On the Mathematical Treatment of Observations by L. Euler. Archive for History of Exact Sciences, Volume 9, Number 1 / January, 1972.

On the Euler Circuit

Recent seminars and colloquia on Euler
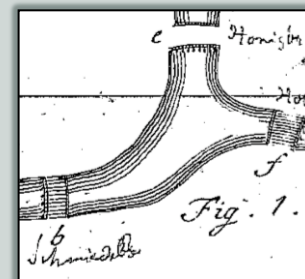
Dominic Klyve was invited to the University of Houston to give an address "The Life, Legacy, and Lost Library books of Leonhard Euler" on April 15 (Euler's 302nd birthday).

In addition to an examination of Euler's life and work, this talk described the creation of the Euler Archive, and the Archive's work in finding old and misplaced Euler volumes.

Present at the talk were John Lienhard and Andrew Boyd, whose "Engines of Our Ingenuity" program in the History of Science Klyve first learned about in a *How Euler Did It* column by Ed Sandifer.

Boyd then used Klyve's talk as an impetus for a new program on D'Albembert, which can be found at http://www.uh.edu/engines/epi2485.htm.

Translation and Archive Update

This section covers translations and other descriptive works that have been completed or updated since the last printing of the *Opusculus*.

[E31] *Solution to differential equations of the form $ax^n\, dx = dy + y^2\, dx$*

Ian Bruce has recently completed another translation of one of Euler's early works.

[E47] *Finding the sum of any series from a given general term*

Jordan Bell's translation is available on Arxiv.org, recently added to the Euler Archive.

[E115] *Method of determining the longitude of places by observing occultations of fixed stars by the Moon*

Jennifer Grabowski and Jeff Meyer, working under Erik Tou of Carthage College, have translated E115 and provided a historical and technical analysis of the longitude problem with special focus on Euler's contributions.

[E149] *Reflections on Space and Time*

Michael Saclolo and Peter Wake of St. Edward's University in Austin, Texas, have recently completed a translation.

[E523] *On three square numbers of which the sum and the sum of the products will be a square*

Emily Dietrich, Jacqueline Huynh, Jessica Murray (students) and Abdul Hassen and Olcay Ilicasu (faculty advisors) from Rowan University have completed a translation and synopsis.

[E745] *On the Continued Fractions of Wallis*

Kristin Masters and Christopher Tippie, working under Thomas Osler of Rowan University, have completed a translation and synopsis for this article.

New documents available on Google Books

There are two works of Euler that have been added recently to Google Books.

- **[E343/344]** An 1842 French printing of *Lettres à une Princesse d'Allemagne* (Volumes 1 and 2) is now available.

- **[E387]** An 1807 French translation of *Vollständige Anleitung zur Algebra* (printed in two volumes) is now available. This particular edition was translated by J. G. Garnier, a professor at l'École Polytechnique in Paris.

Links to all documents mentioned above can be found through the Euler Archive: **www.eulerarchive.org**