State-space models for PVA

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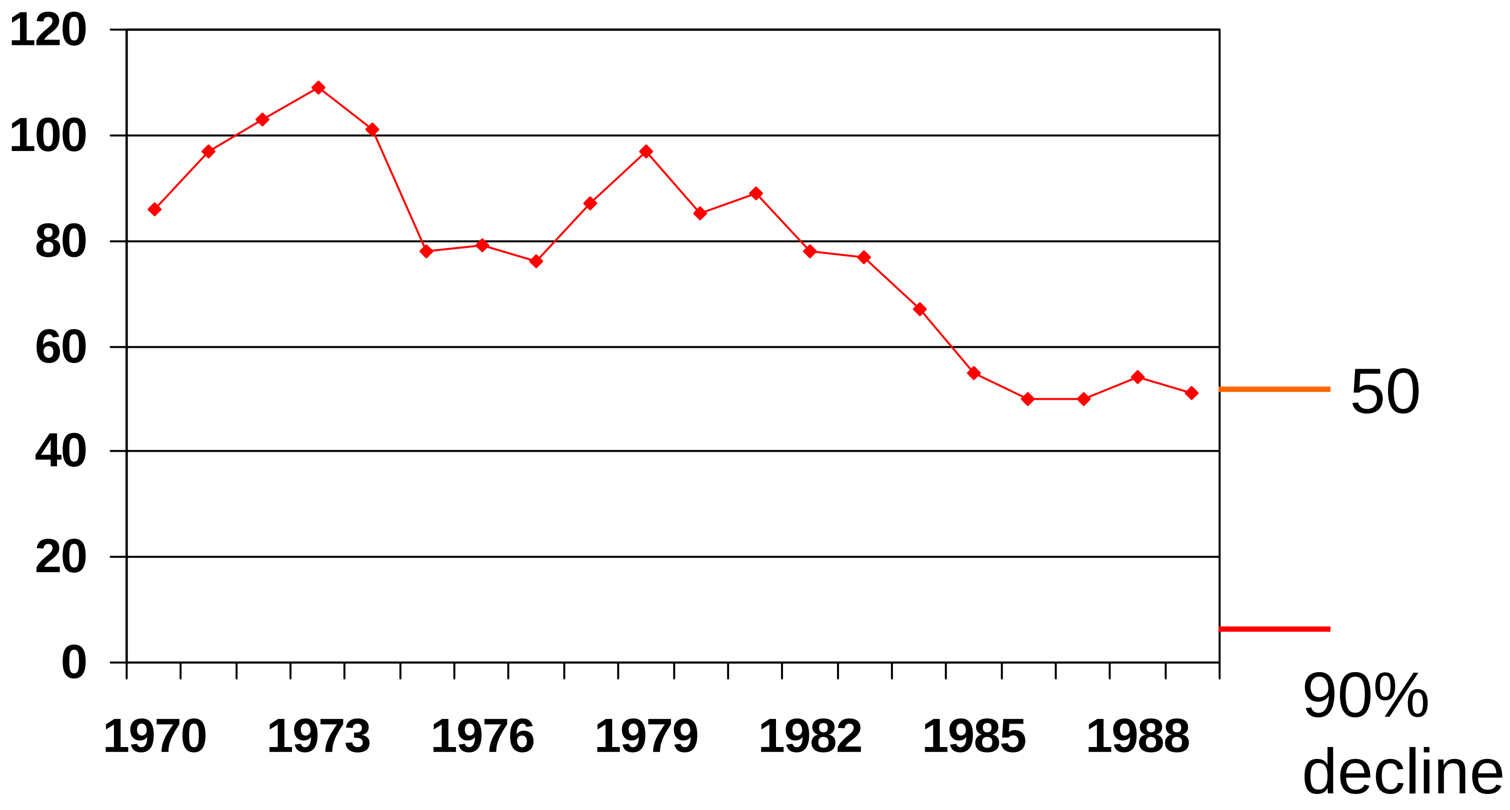
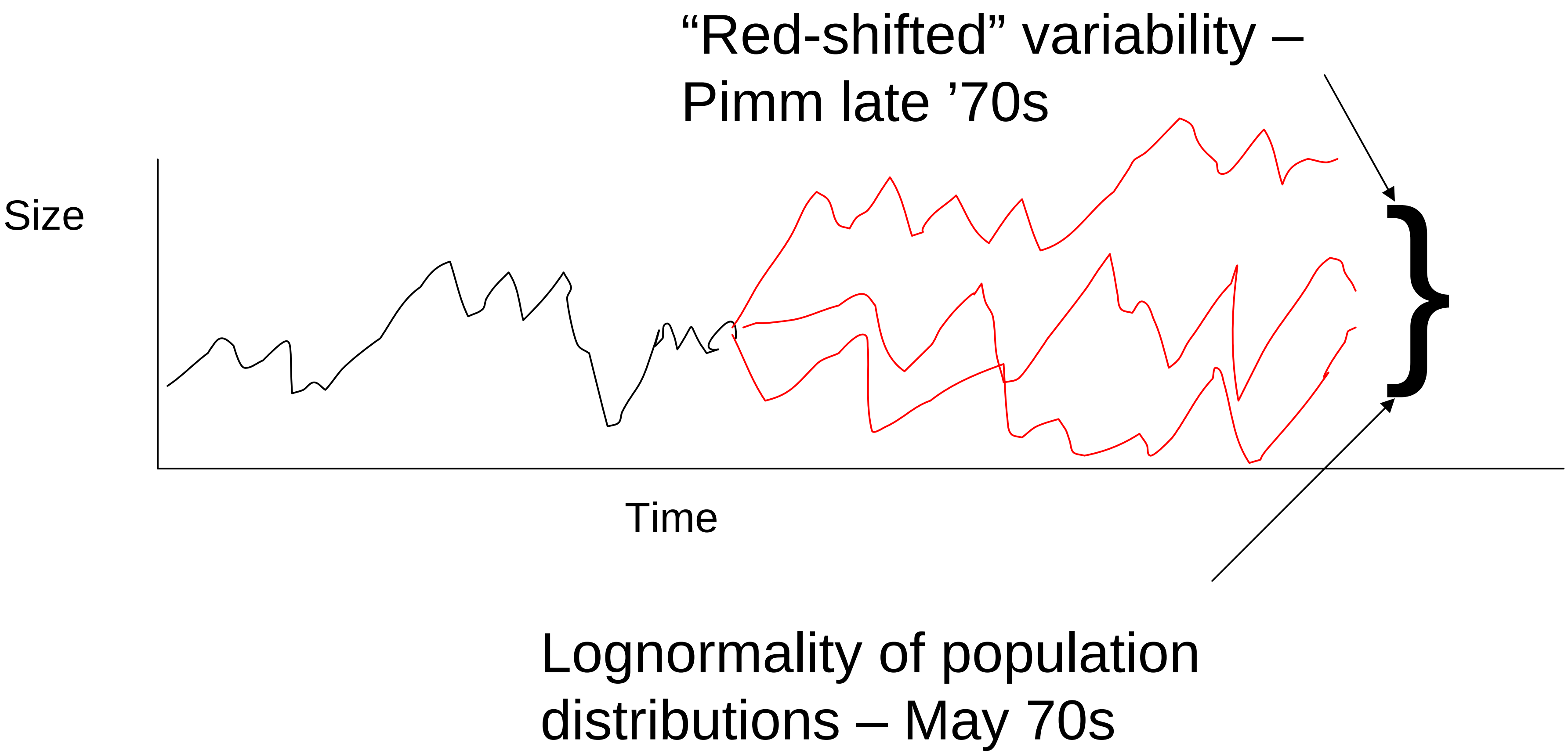
IUCN Red List Criteria

- Criteria A2: “A reduction of at least xx%, projected or suspected to be met within the next xx years….”
- Criteria C1: “Population estimated to number less than xx and an estimated continuing decline of at least xx% within xx years....”
- Criteria E: “Quantitative analysis showing the probability of extinction in the wild is at least xx% within xx years...”

Quasi-extinction risk



The nature of variability in population trajectories



What might produce these patterns?

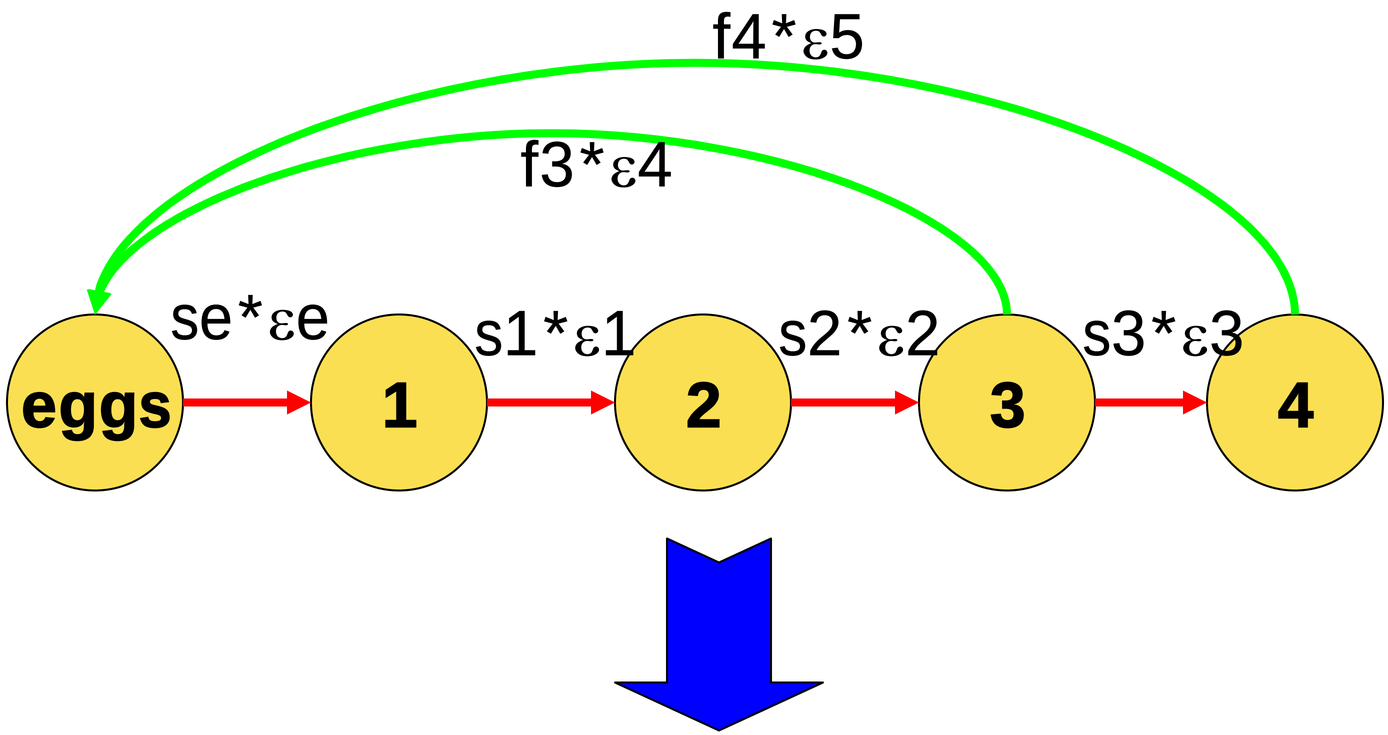
- Extrinsic forces: Increased variability due to accumulation of rare events with time. Populations are tightly regulated by density-dependence; variance doesn't increase with time due to intrinsic forces.
- Intrinsic forces: Year-to-year variability in growth rates caused increased variability with time – not extrinsic forces. Populations are regulated by density-dependence, but in a stochastic process, the effects of year-to-year variability dominate.

Using the Global Population Dynamics Database to study these patterns

- Inchausti and Halley 2003 “On the relation between temporal variability and persistence time in animal populations”
- Akcakaya, Halley, Inchausti 2003 “Population-level mechanisms for reddened spectra in ecological time series”

A simple model of annual variability in growth rate + measurement error or annual variability + weak density-dependence can explain the “red-shift” pattern in the GPDD

stochastic age-structured models: Tuljapurkar 1980s



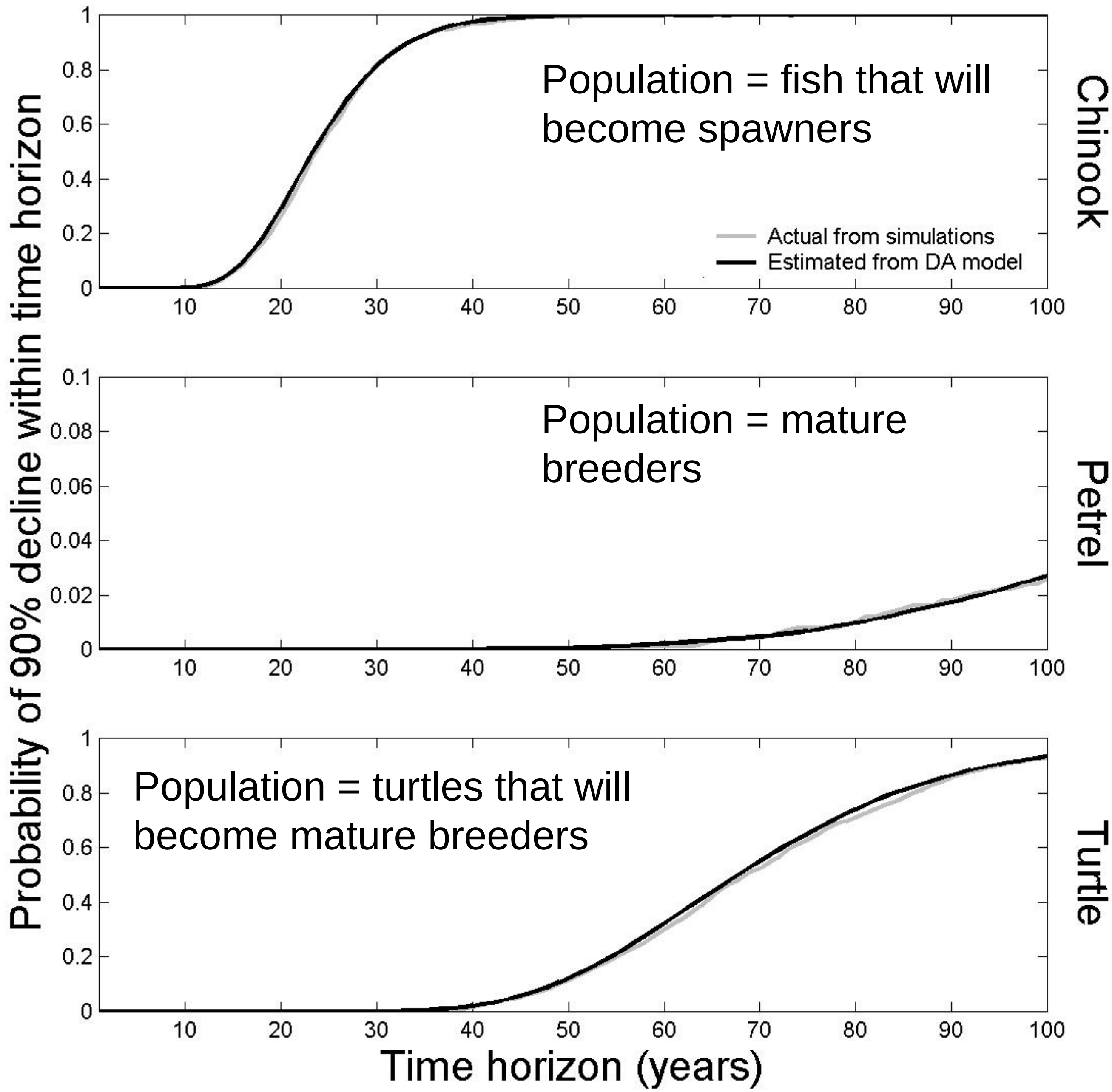
$N_{t+\tau} = N_t * \exp(\mu\tau + \varepsilon)$ where

$\varepsilon \sim N(0, \sigma\ \mathrm{sqrt}(\tau))$

Dennis, Munholland, Scott 1991: How this all applies to conservation biology

- A really simple diffusion approximation for the stochastic exp model can predict quasi-extinction in age-structured models (Lande and Orzack 1988)
- Lots of nifty risk metrics can be calculated using this approximation
- A maximum-likelihood approach for estimating the 2 parameters from time series data

- Salmon:
  - Leslie matrix model of Snake R. spr/sum chinook
  - Has density dependence
  - Has environmental autocorrelation
- Petrel:
  - Leslie matrix model of the Hawaiian Dark-rumped Petrel (Simons 1984);
  - Long-lived; census is of mature breeders
  - environmental autocorrelation
- Sea Turtle:
  - Leslie matrix model of the Loggerhead Sea Turtle (Crowder et al. 1994);
  - Long-lived; census is of eggs which is highly variable
  - environmental autocorrelation



Population = fish that will become spawners
Chinook
Actual from simulations
Estimated from DA model
Population = mature breeders
Petrel
Population = turtles that will become mature breeders
Turtle
Probability of 90% decline within time horizon
Time horizon (years)

Linear gaussian state-space model

Strength of dens.dep.

Exp. growth

$$\mathbf{x}_t = \mathbf{T}\mathbf{x}_{t-1} + \mathbf{c} + \mathbf{R}\eta_{t-1}$$

$$\mathbf{y}_t = \mathbf{Z}\mathbf{x}_t + \mathbf{d} + \varepsilon_t$$

Things we're interested in tracking. Log(N_t)

Measurement of x

Gaussian errors

Gaussian errors

Process error (aka random walk)

$$\mathrm{var}(\log(a_{t+\tau} / a_t)) \propto \tau$$

Non-process error (aka white noise)

$$\mathrm{var}(\log(a_{t+\tau} / a_t)) = \mathrm{constant}$$

Model of measurement error

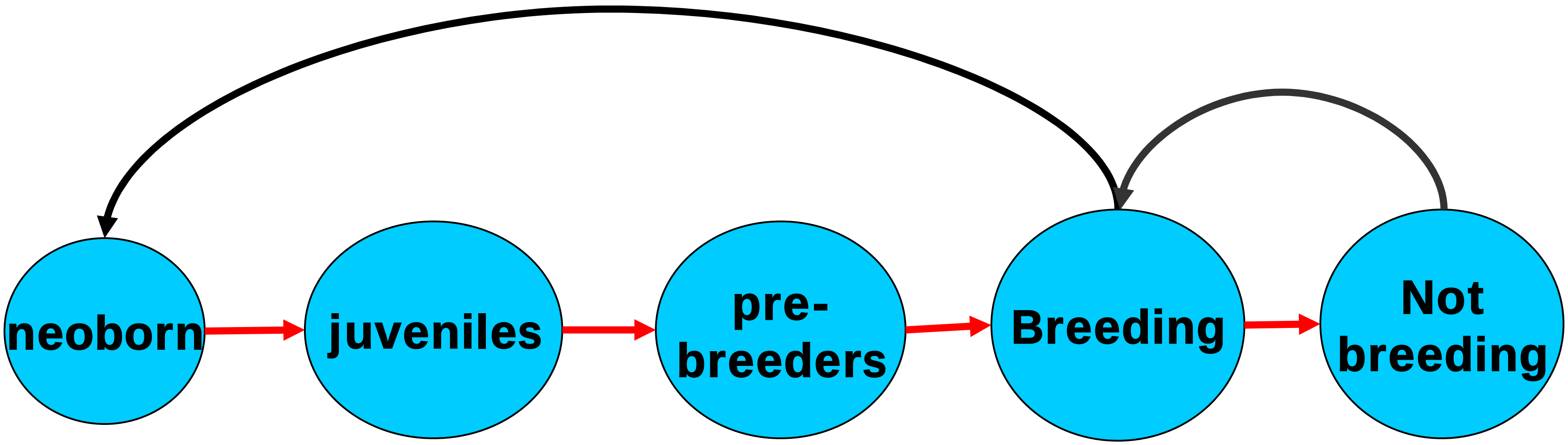
$$\log(N_{t+1}) = b\log(N_t) + \varepsilon_{t,p}$$

$$\log(y_{t+1}) = \log(N_{t+1}) + \varepsilon_{t+1,np}$$

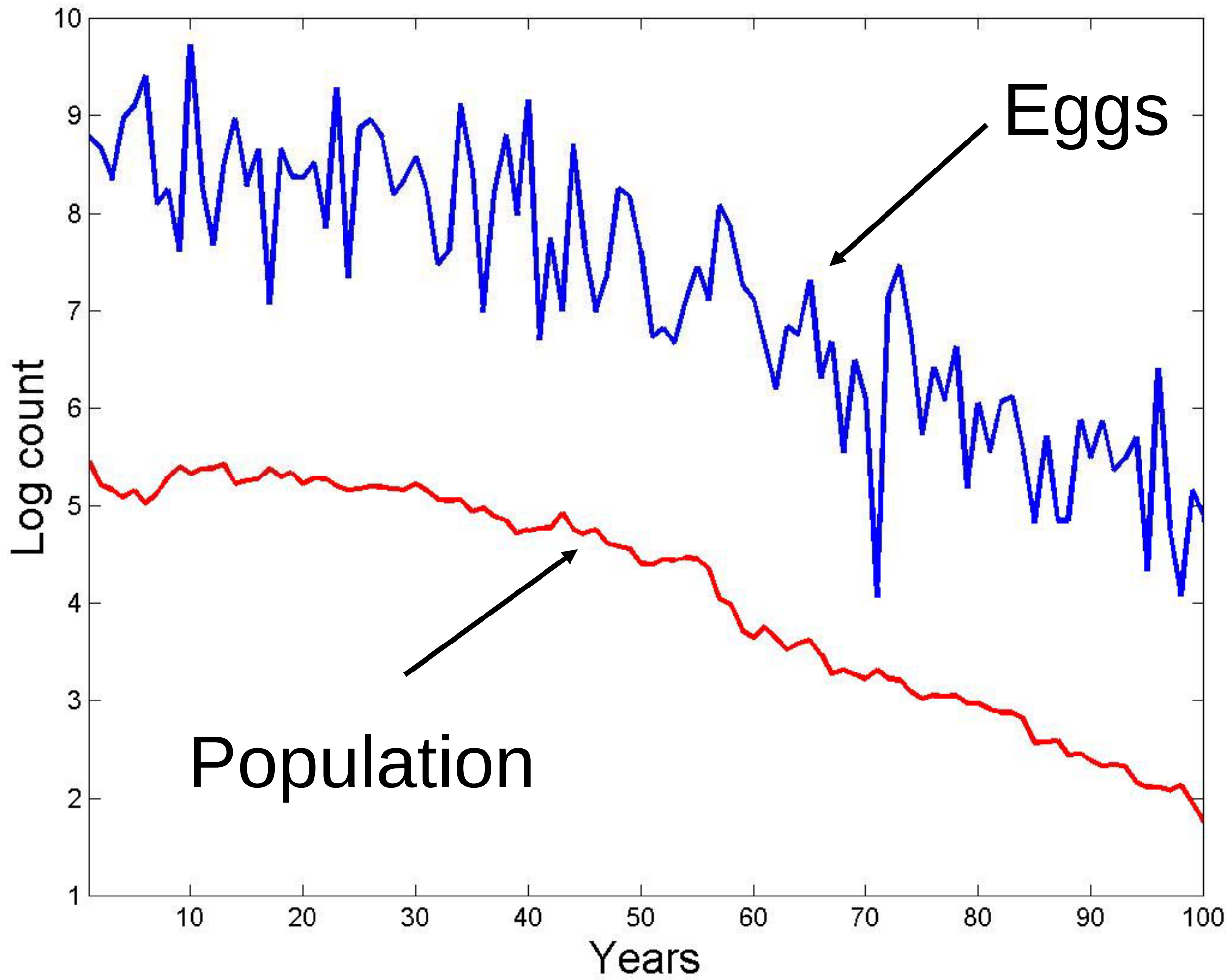
$$\varepsilon_{t,p} \sim Normal(\mu, \sigma_p^2)$$

$$\varepsilon_{t,np} \sim f(\beta, \sigma_a^2)$$

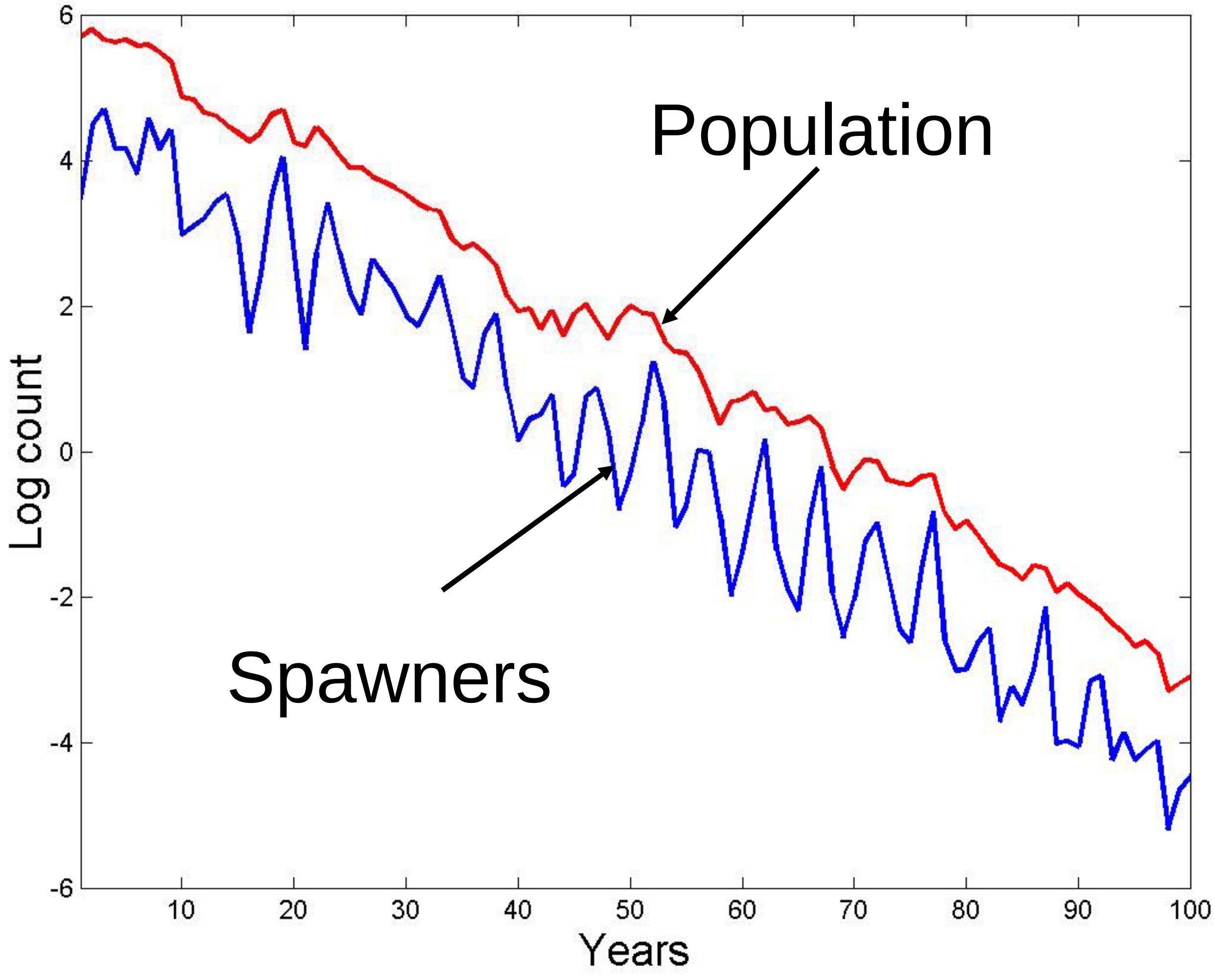
Monitoring data is often stage specific

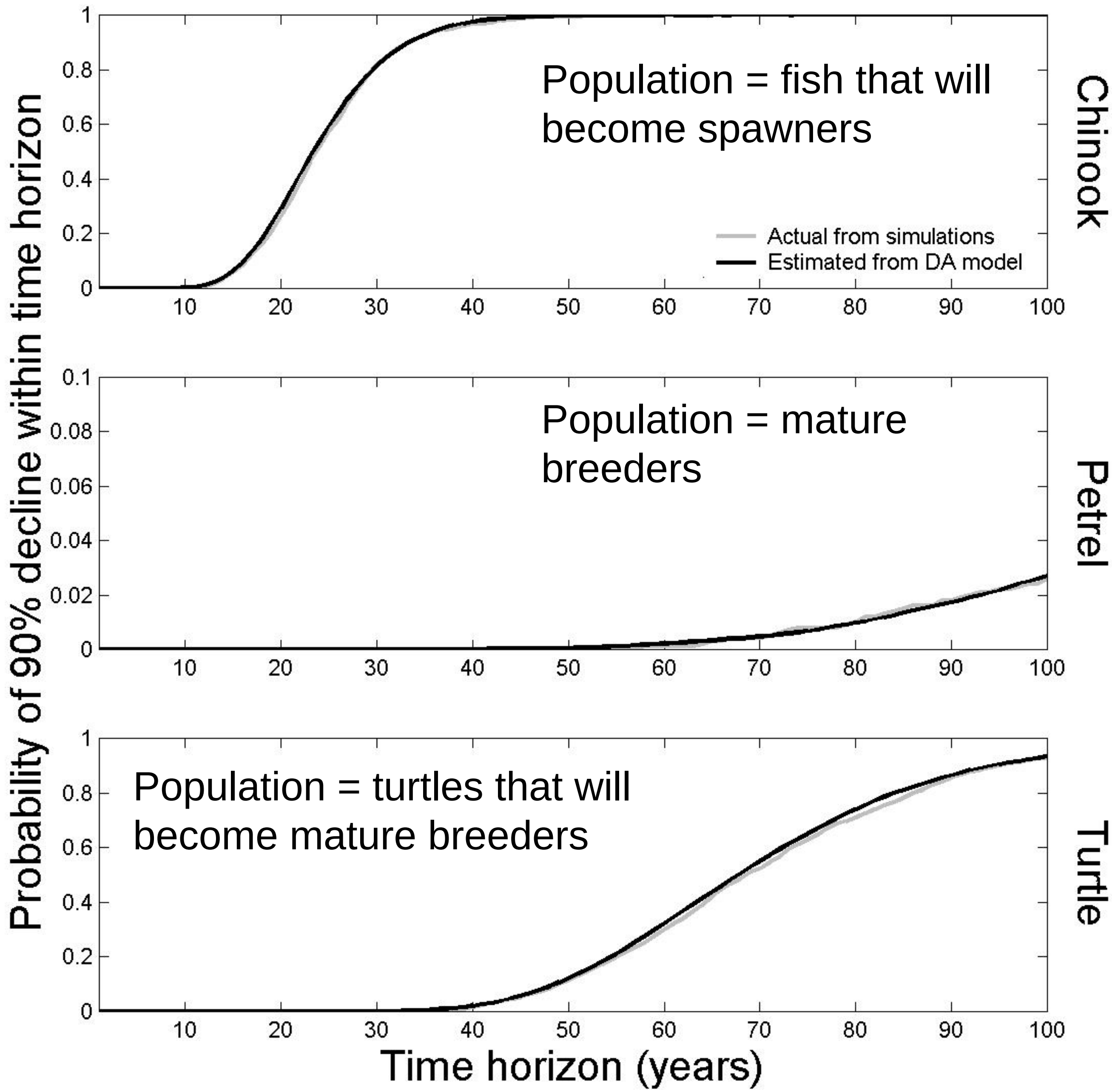


Example with sea turtles

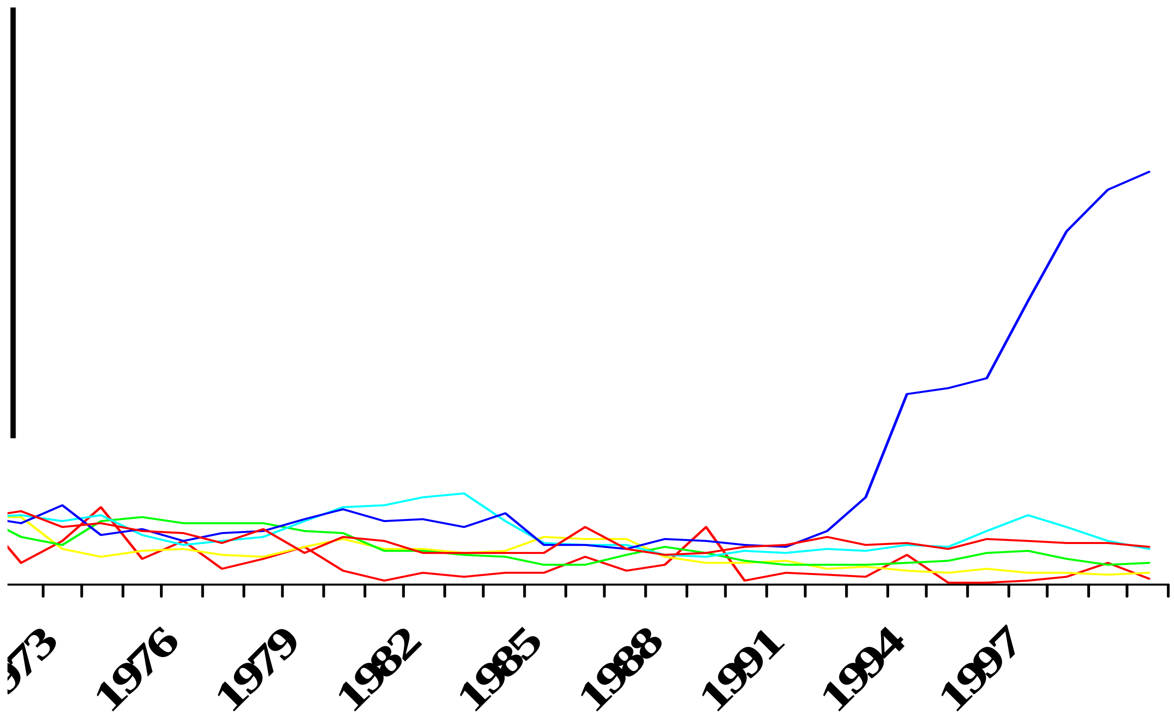


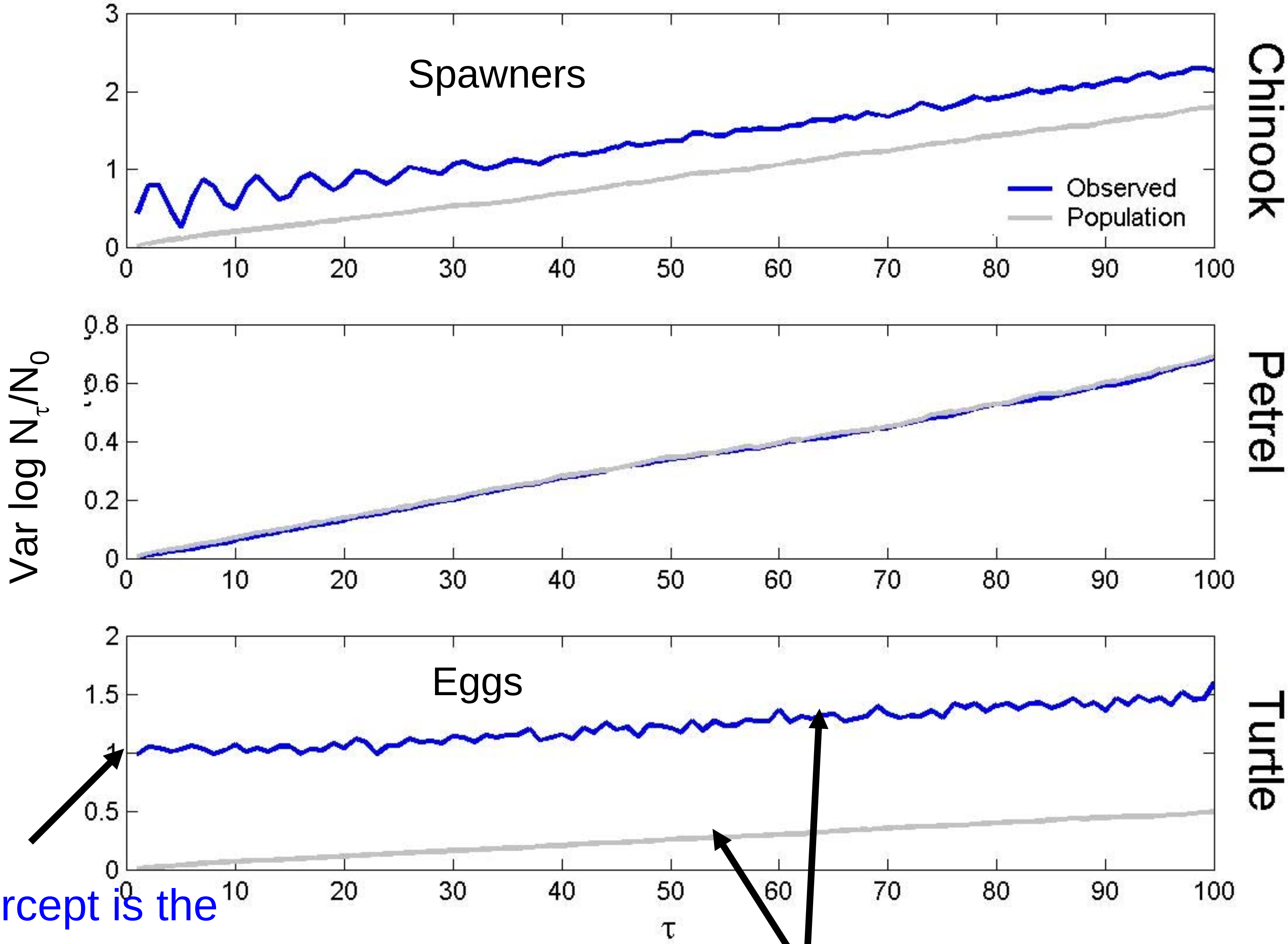
Example with salmon




Population = fish that will become spawners
Chinook
Actual from simulations
Estimated from DA model
Population = mature breeders
Petrel
Population = turtles that will become mature breeders
Turtle
Probability of 90% decline within time horizon
Time horizon (years)
0
0.2
0.4
0.6
0.8
1
0.02
0.04
0.06
0.08
0.1
10
20
30
40
50
60
70
80
90
100

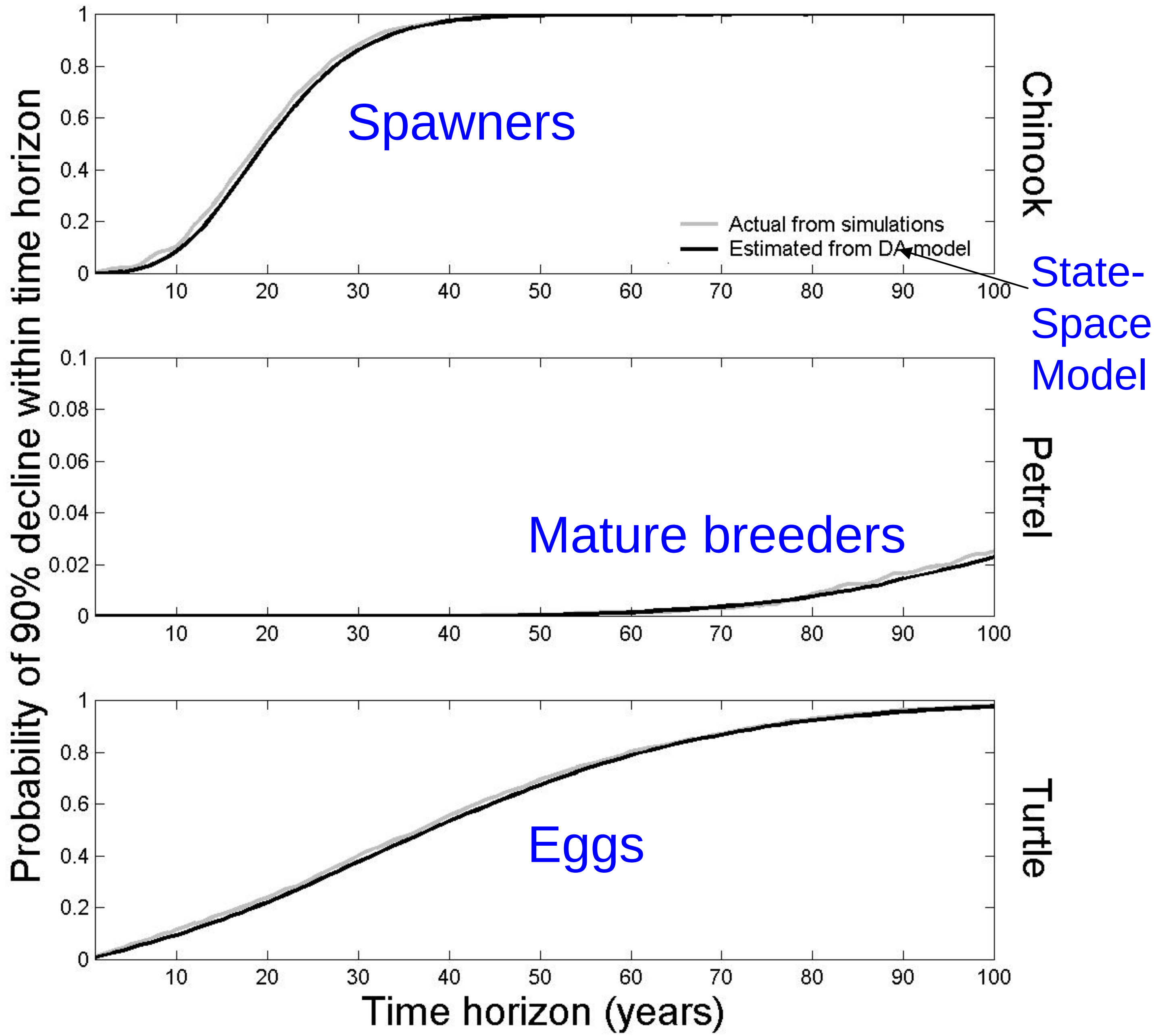
Does a state-space model exist for the age-specific counts?





Intercept is the non-process error parameter

Slope is the process error parameter



Spawners
Actual from simulations
Estimated from DA model
Chinook
State-Space Model
Mature breeders
Petrel
Eggs
Turtle
Probability of 90% decline within time horizon
Time horizon (years)

Linear gaussian state-space model

Strength of dens.dep.

Exp. growth

$$\mathbf{x}_t = \mathbf{T}\mathbf{x}_{t-1} + \mathbf{c} + \mathbf{R}\eta_{t-1}$$

$$\mathbf{y}_t = \mathbf{Z}\mathbf{x}_t + \mathbf{d} + \varepsilon_t$$

Things we're interested in tracking. Log(N_t)

Gaussian errors

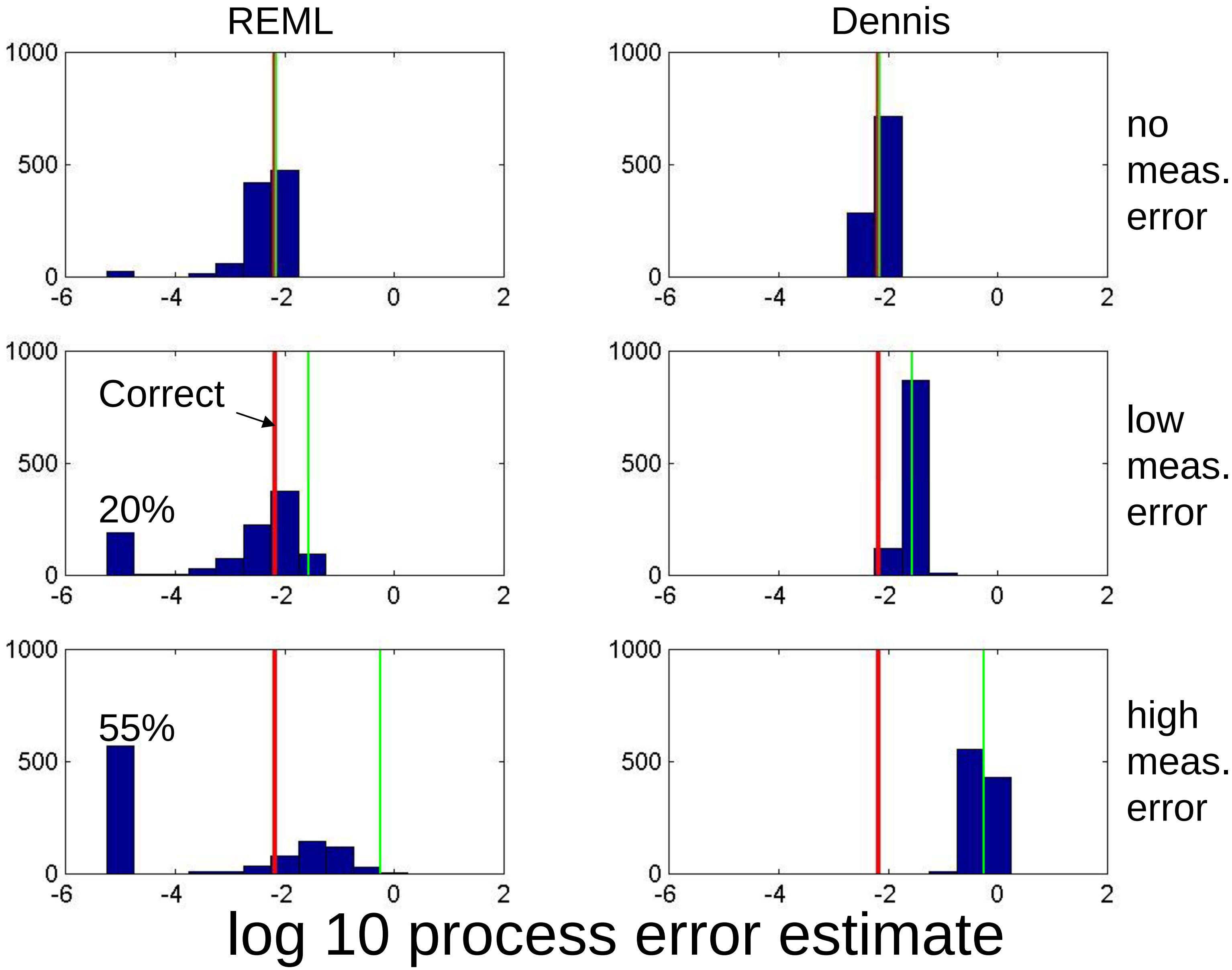
Measurement of x

Gaussian errors

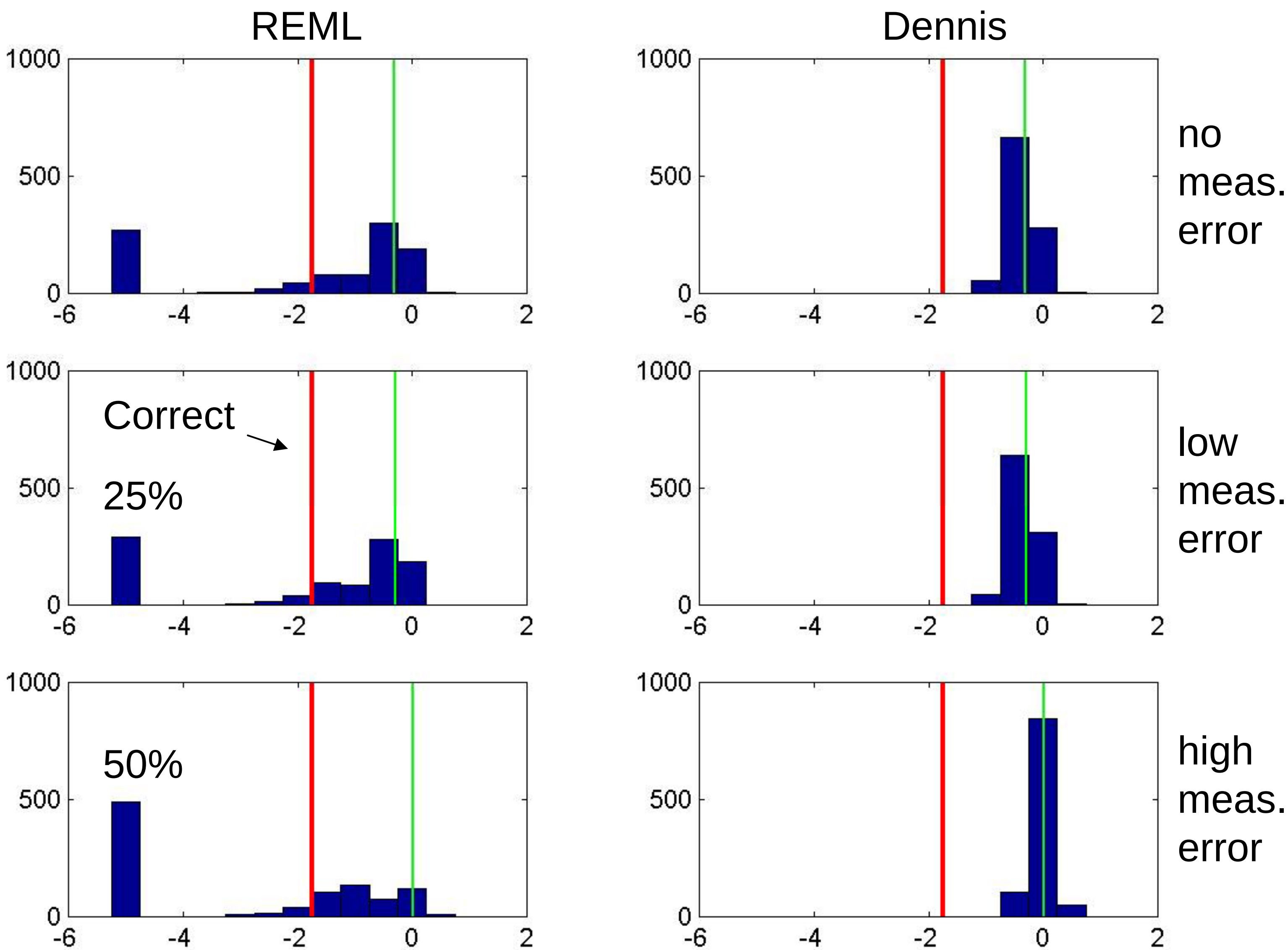
Estimation of σ^2 (process error) + σ_{np}^2 (non-process error) using the state-space model

- Regression estimating the increase in variance in log $N_{t+\tau}/N_t$ with τ (Holmes 2001)
- Kalman filter (Lindley 2003)
- “REML”: Restricted ML estimation (Staples et al. 2004)

Petrel: pretty good…

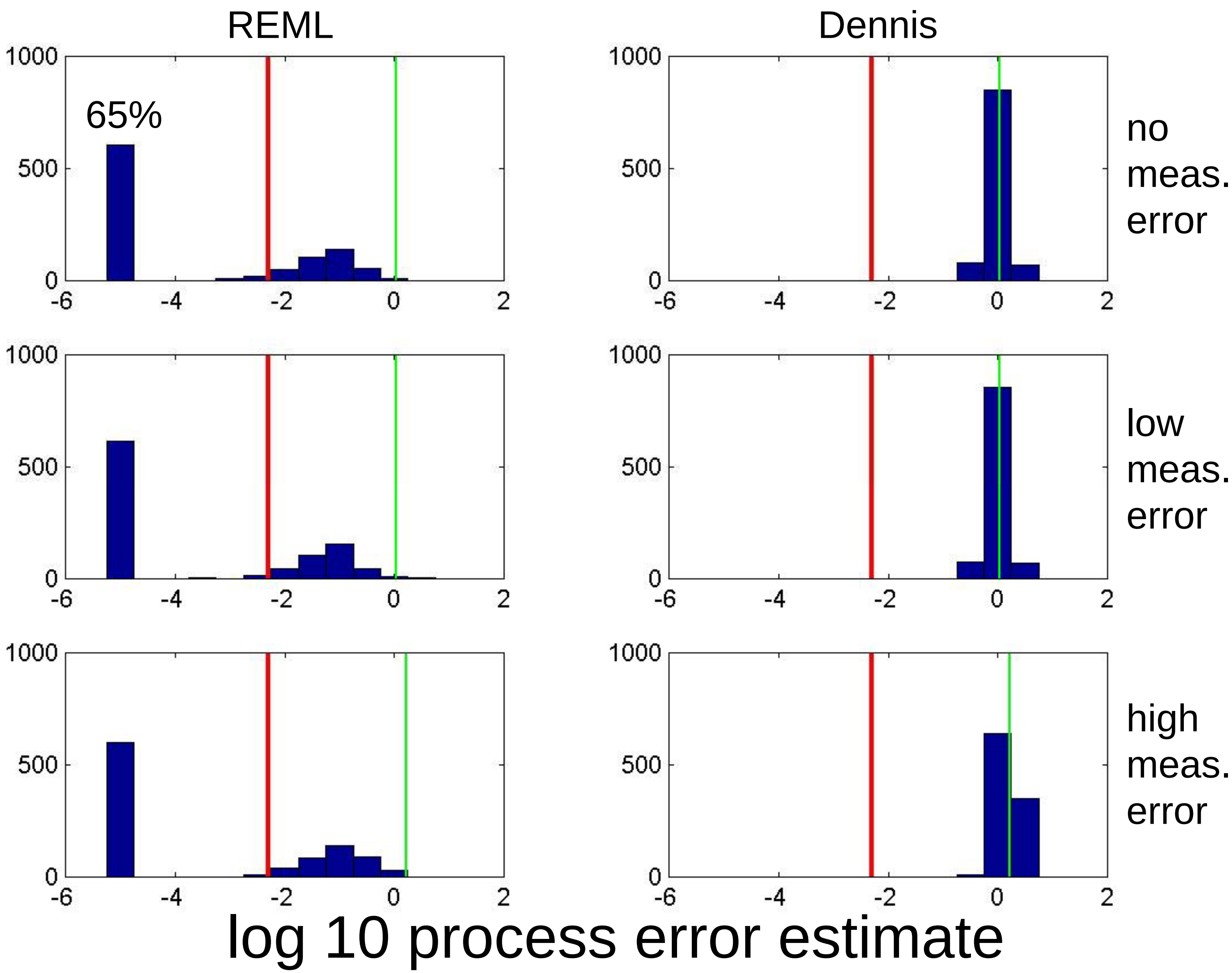


Salmon: struggling…



log 10 process error estimate

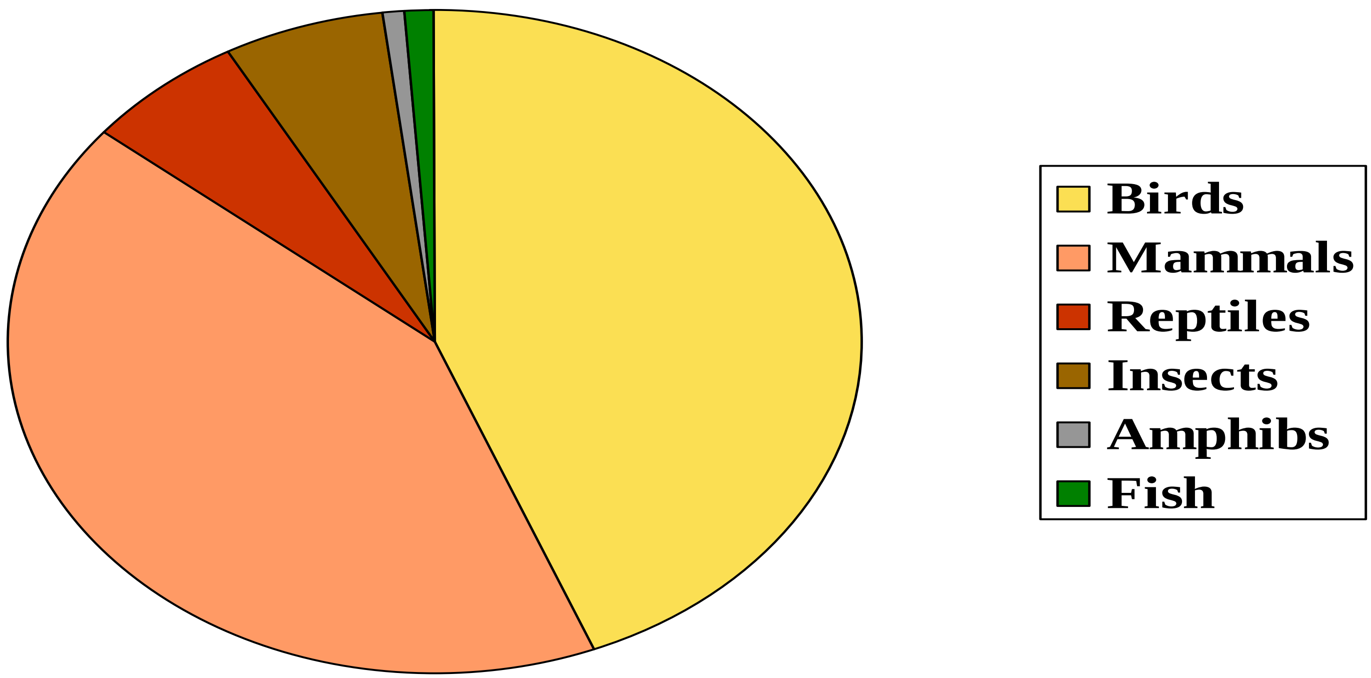
Turtle: more struggling…



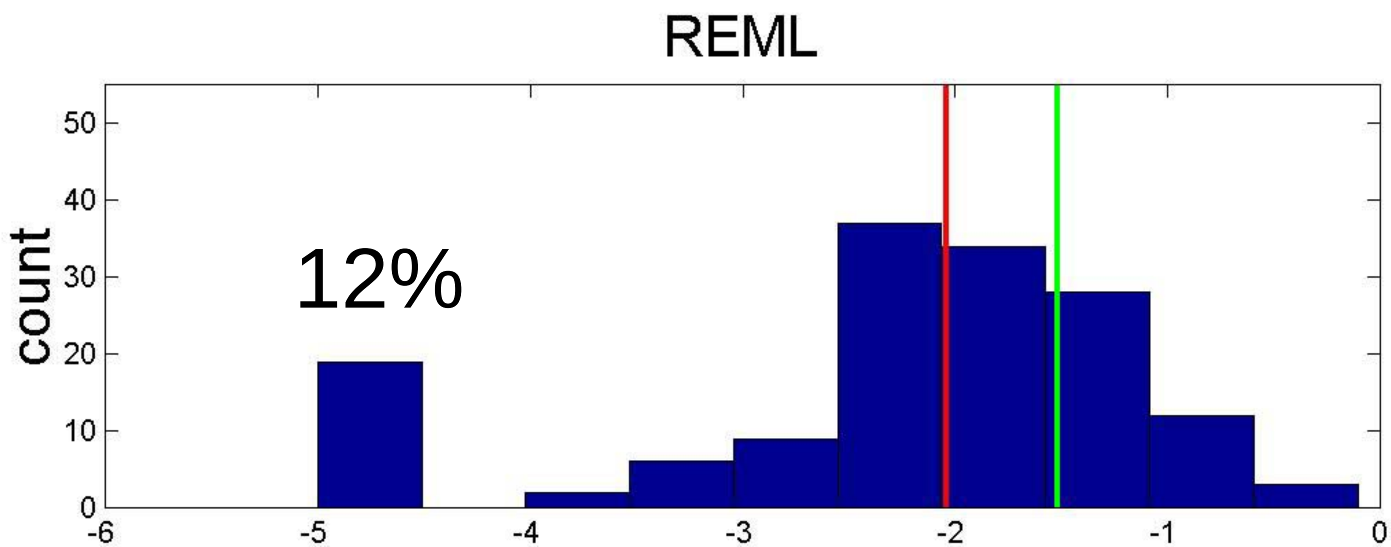
Estimating parameters might be challenging for some species

- What does real data tell us about the performance of these methods? Are most data “petrel-like” or “sea turtle-like”?

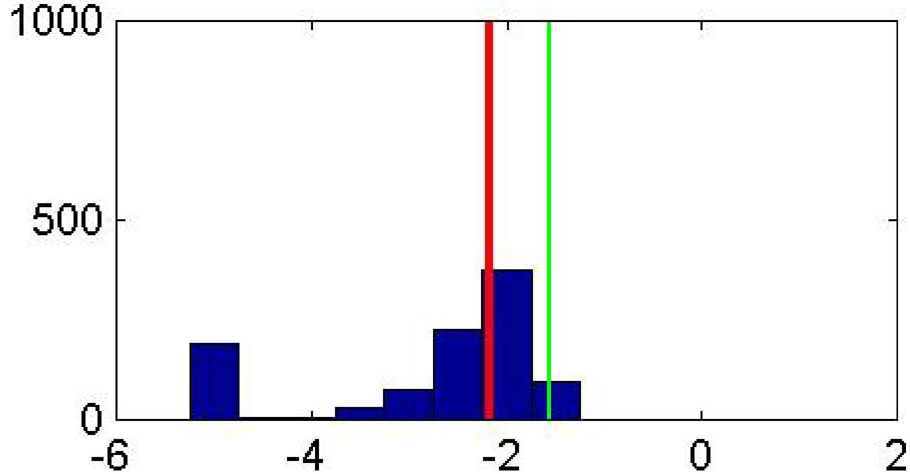
117 Time series 20-50 yrs long
72 are listed species



Distribution of process error estimates



DATA



PETREL SIMS with low non-process error

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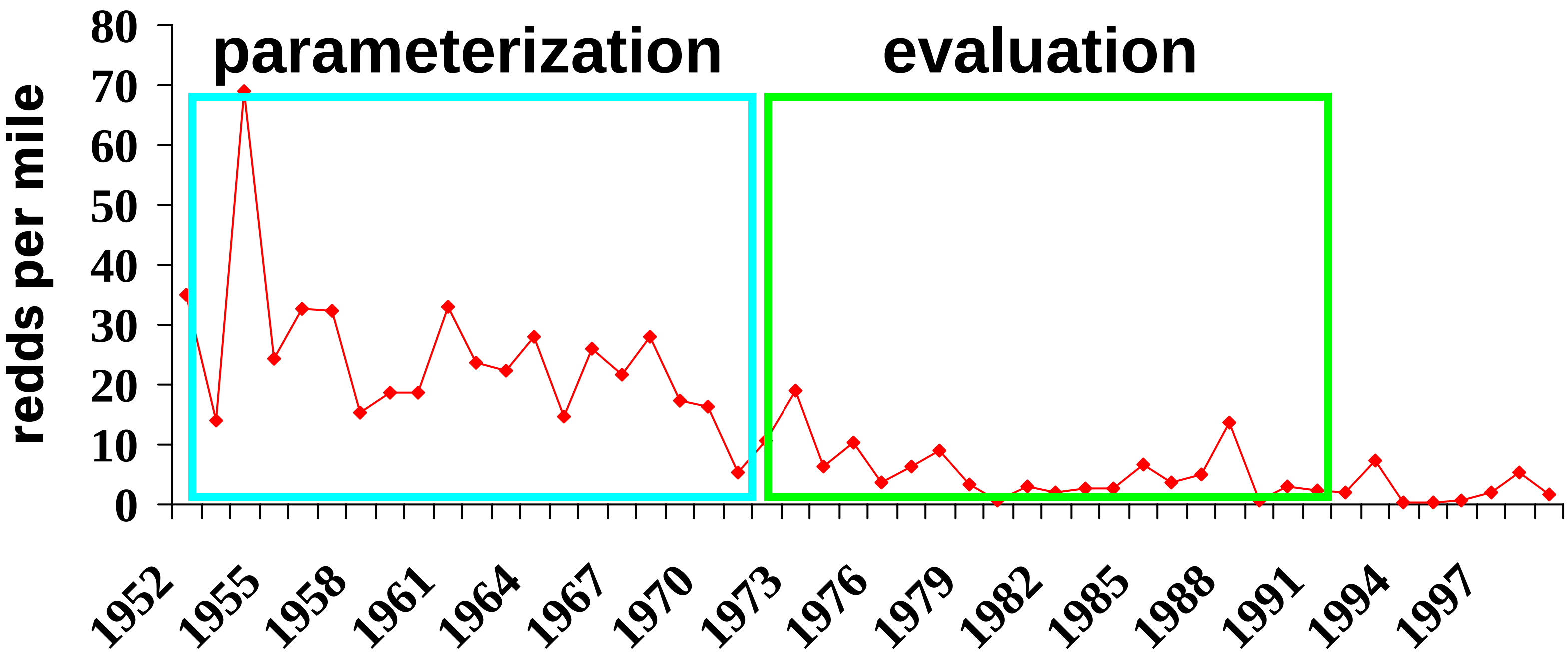
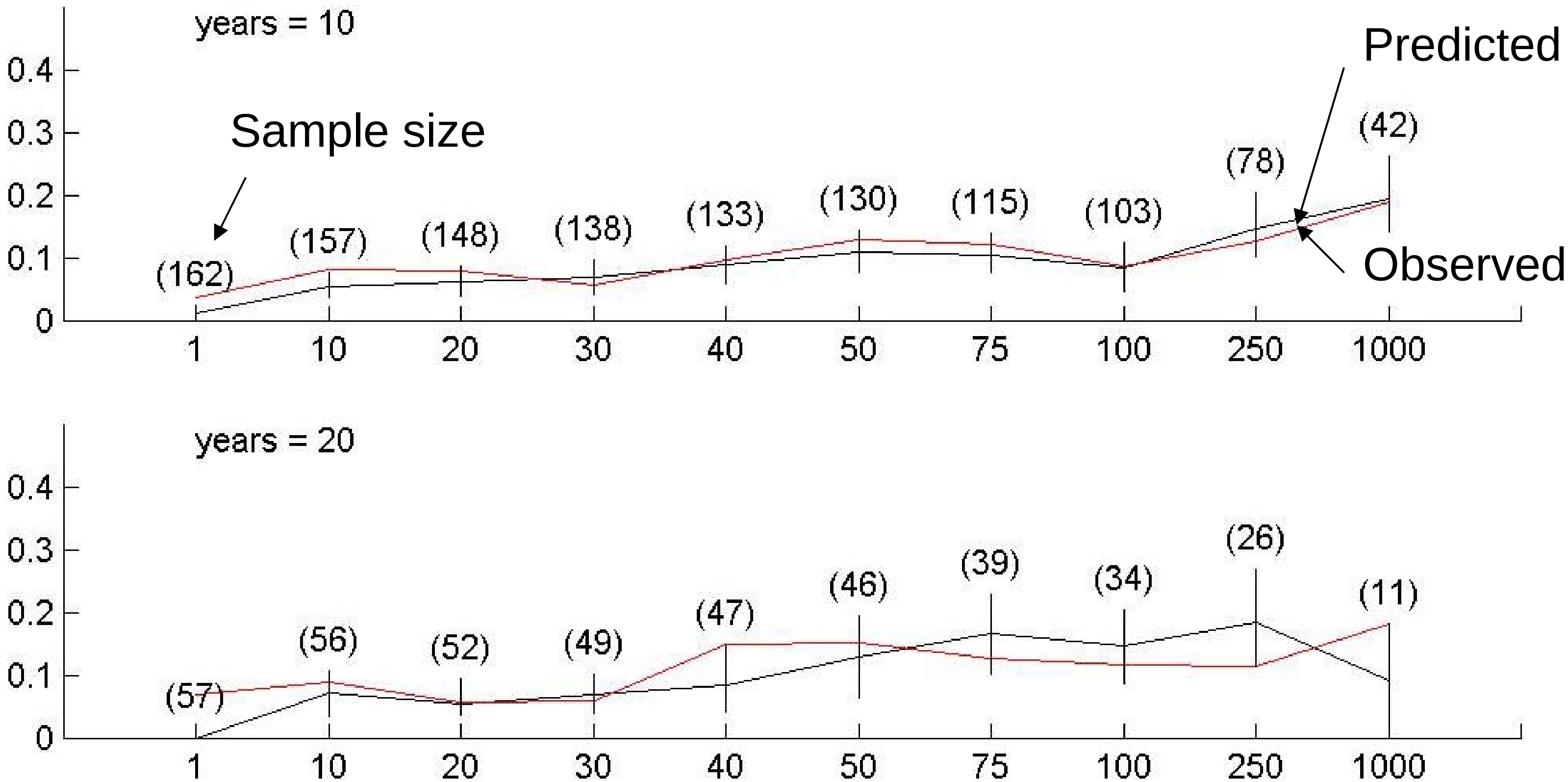
Cross-validation

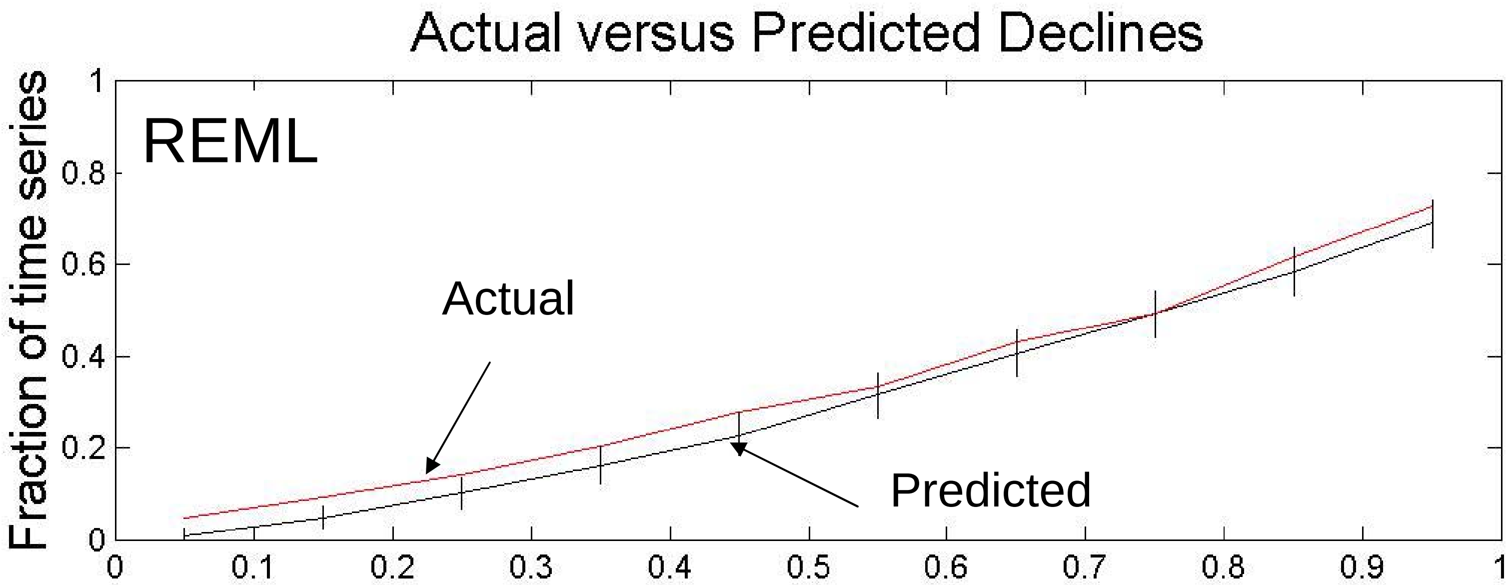


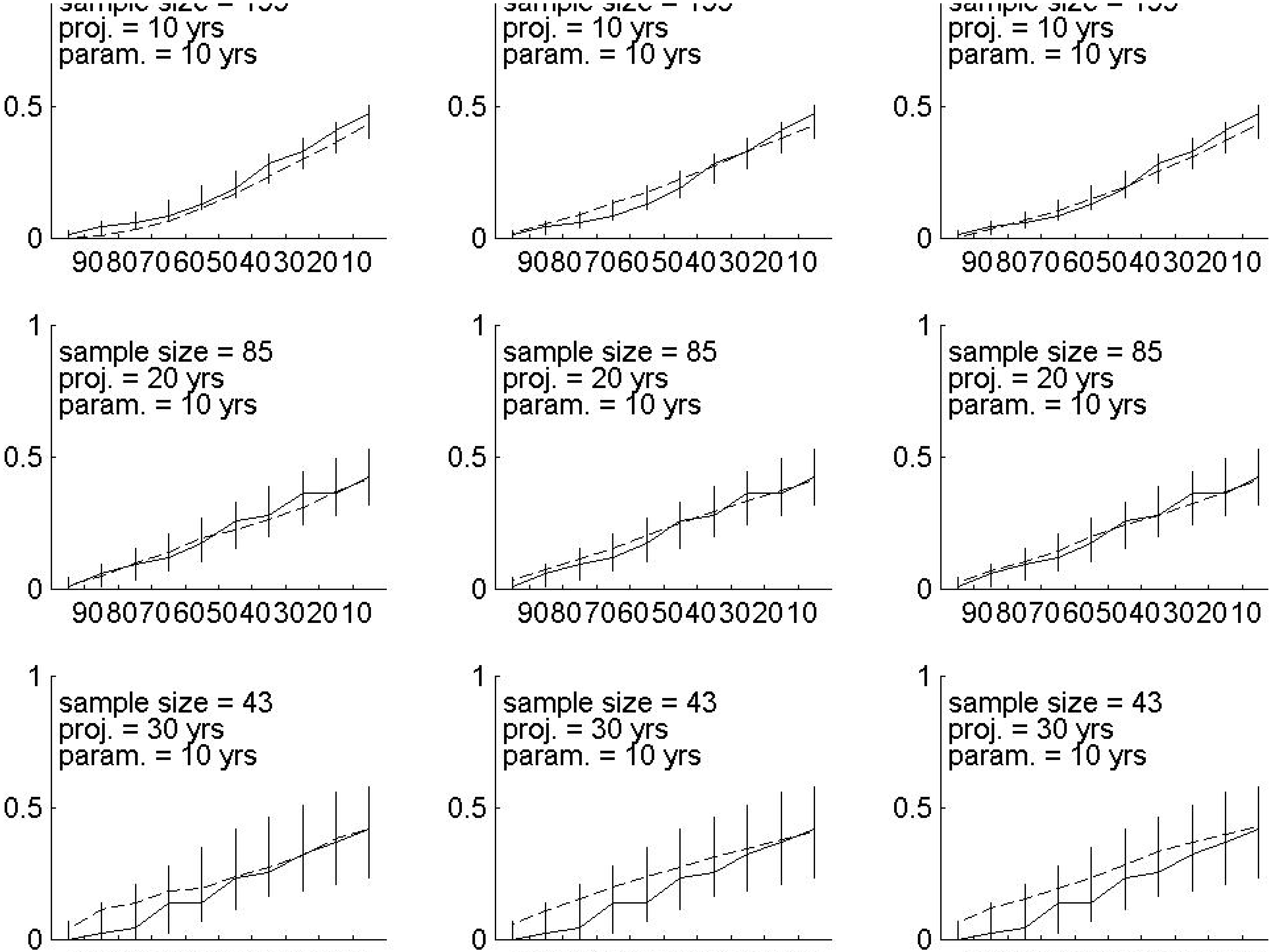
Expected vs Observed Freq. of Hitting Particular Thresholds



Quasi-extinction level

Proportional Declines

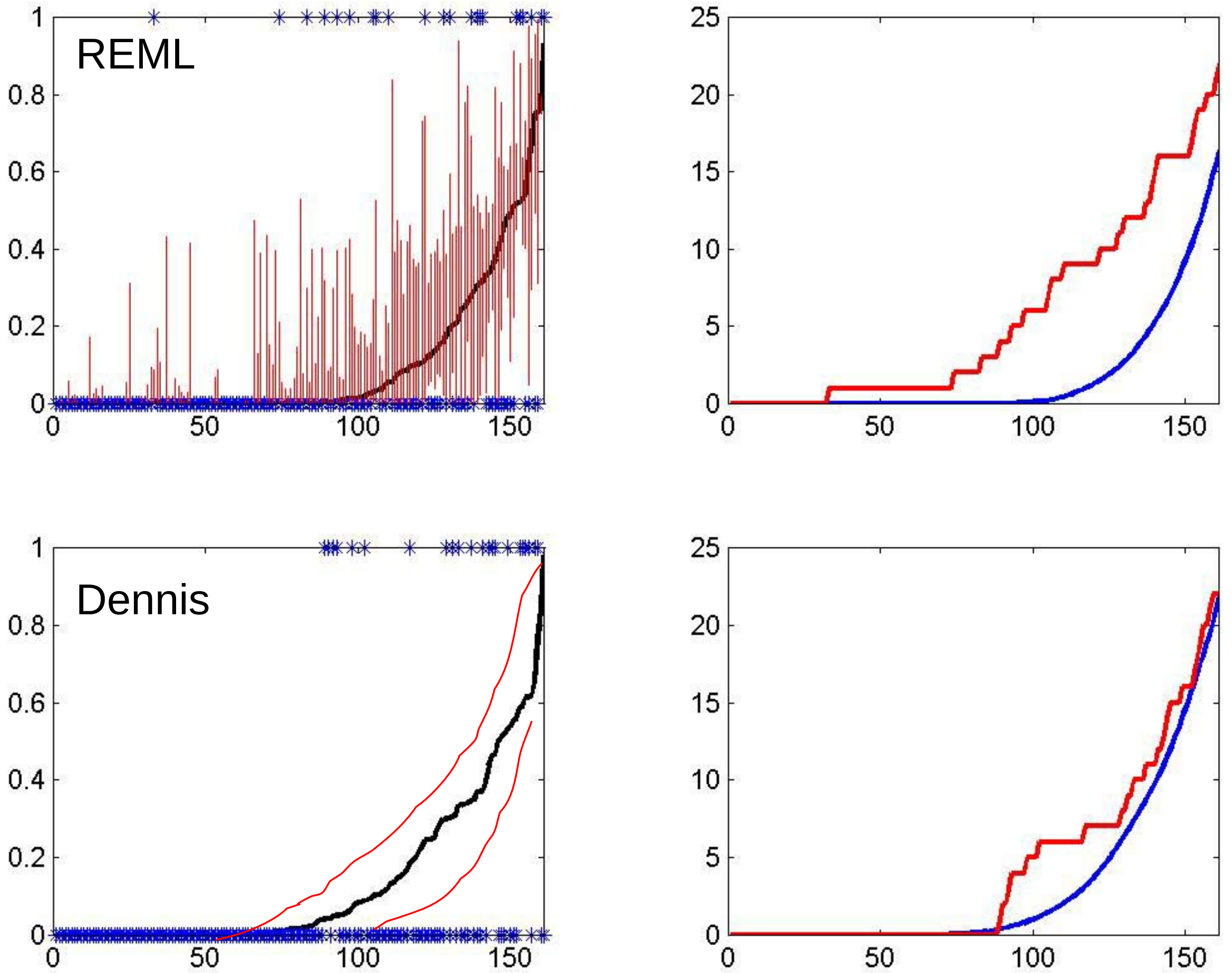




proj. = 10 yrs
param. = 10 yrs
0.5
0
90807060504030201 0
proj. = 10 yrs
param. = 10 yrs
0.5
0
90807060504030201 0
proj. = 10 yrs
param. = 10 yrs
0.5
0
90807060504030201 0
1
sample size = 85
proj. = 20 yrs
param. = 10 yrs
0.5
0
90807060504030201 0
1
sample size = 85
proj. = 20 yrs
param. = 10 yrs
0.5
0
90807060504030201 0
1
sample size = 85
proj. = 20 yrs
param. = 10 yrs
0.5
0
90807060504030201 0
1
sample size = 43
proj. = 30 yrs
param. = 10 yrs
0.5
0
1
sample size = 43
proj. = 30 yrs
param. = 10 yrs
0.5
0
1
sample size = 43
proj. = 30 yrs
param. = 10 yrs
0.5
0

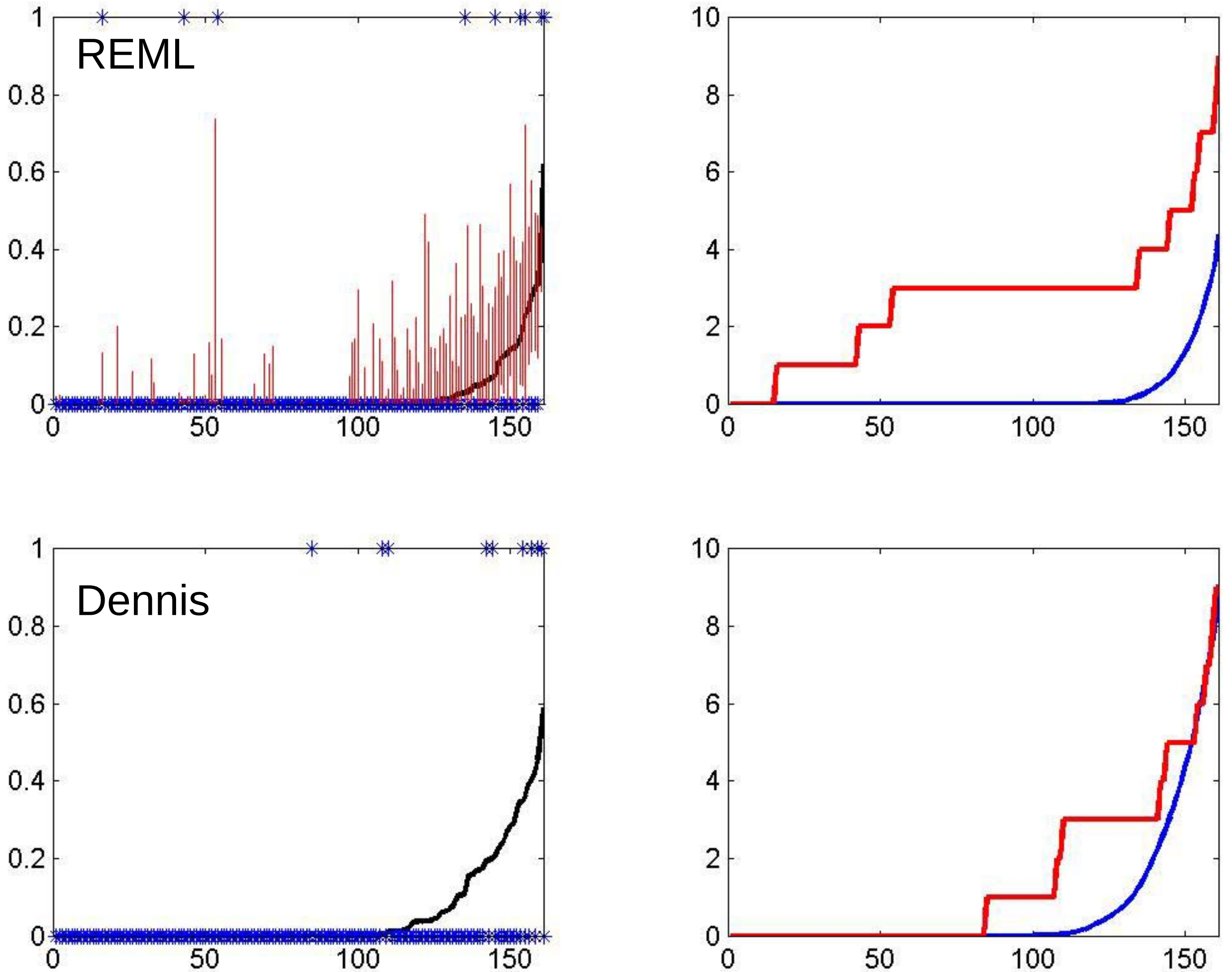
Estimated 75% decline risk vs actual 75% decline

Predicted accumulated 75% declines versus actual



Estimated 90% decline risk vs actual 90% decline

Predicted accumulated 90% declines versus actual



In conclusion….

- This is research in progress...

- This problem does not appear pervasive in data on species of conservation concern
- There appears to be a trade-off between precision of estimates versus bias