Lecture 9 – Introduction to Rock Strength

David Hart
University of Wisconsin
*ecow.engr.wisc.edu/cgi-bin/get/gle/474/hartdavid/notes/****lecture9strength****intro.ppt* -

ROCK STRENGTH

- Shear fracture is the dominant mode of failure for rocks under all but the lowest confining stress.

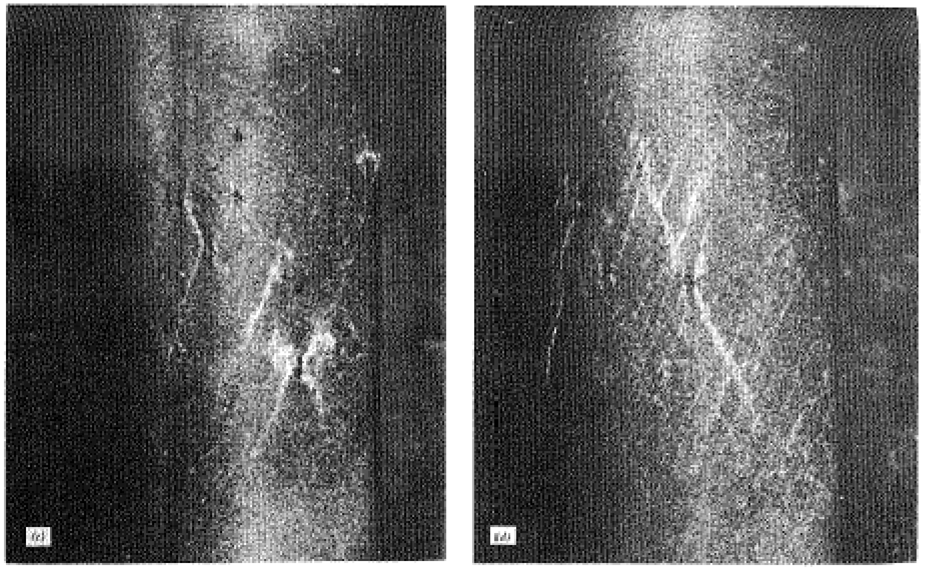
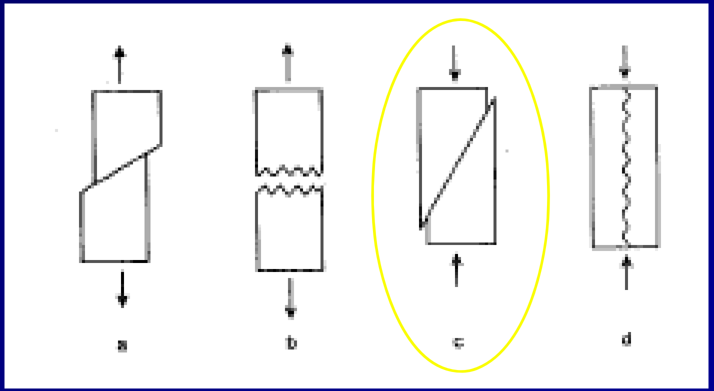


Figure 3.7 Deformation under increasing deviatoric stress, with constant mean stress (hypothetical curves). (*c*) at the peak axial stress; (*d*) just after the peak showing linkup of fractures to form a rupture surface. *c* and *d* Fractures formed in diabase during triaxial compression at 500 psi, reproduced from Wawersik and Brace (1971) with permission.

Example from Goodman, <u>Intro to Rock Mechanics</u>

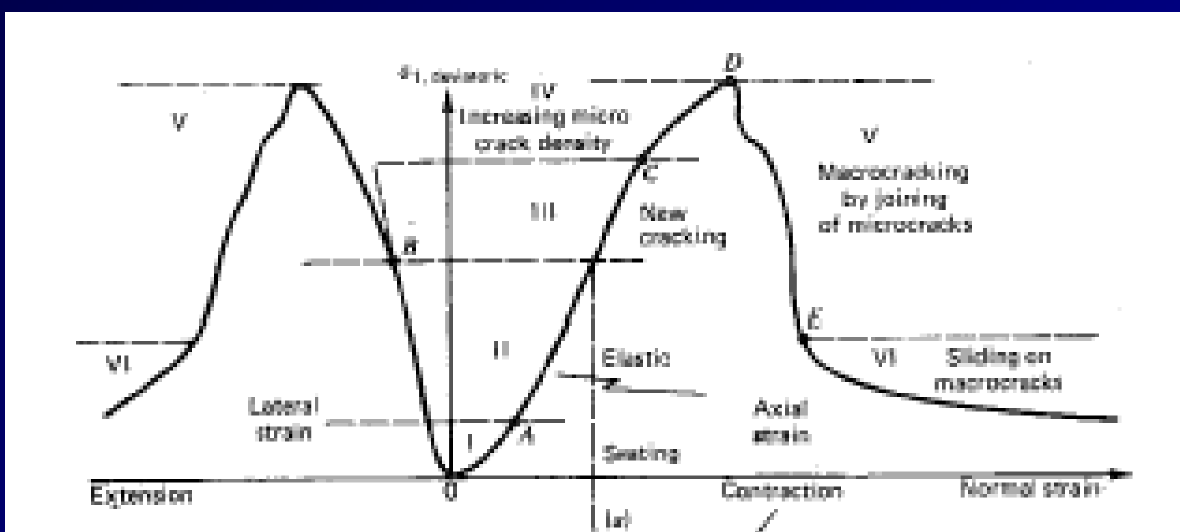


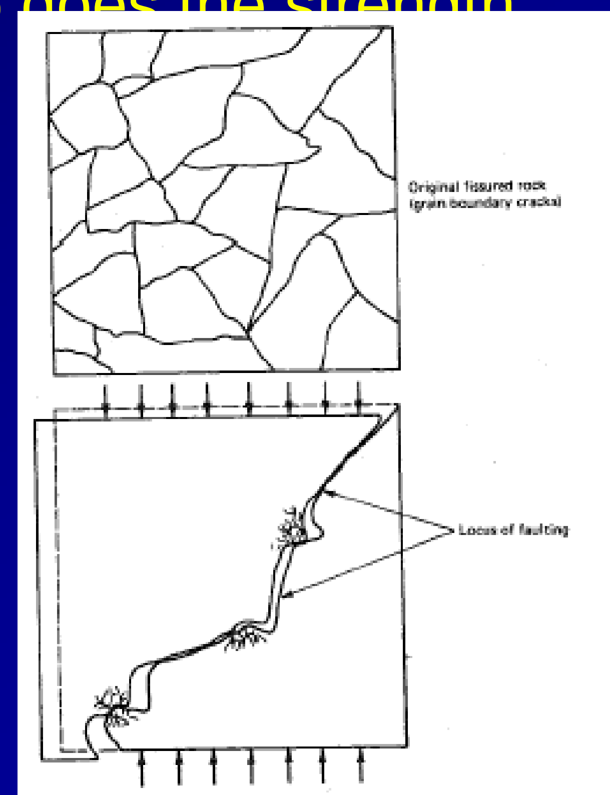
Extension Compression

Paterson, <u>Experimental Rock Deformation – The Brittle Field</u>

ROCK STRENGTH

- The peak stress is the strength of the rock.
  - It may fail catastrophically if the load frame is “soft”. Example below is for a “stiff” frame.
- The compressive strength of rock is a function of the confining pressure.
- As the confining pressure increases so does the strength





Goodman, Intro to Rock Mechanics

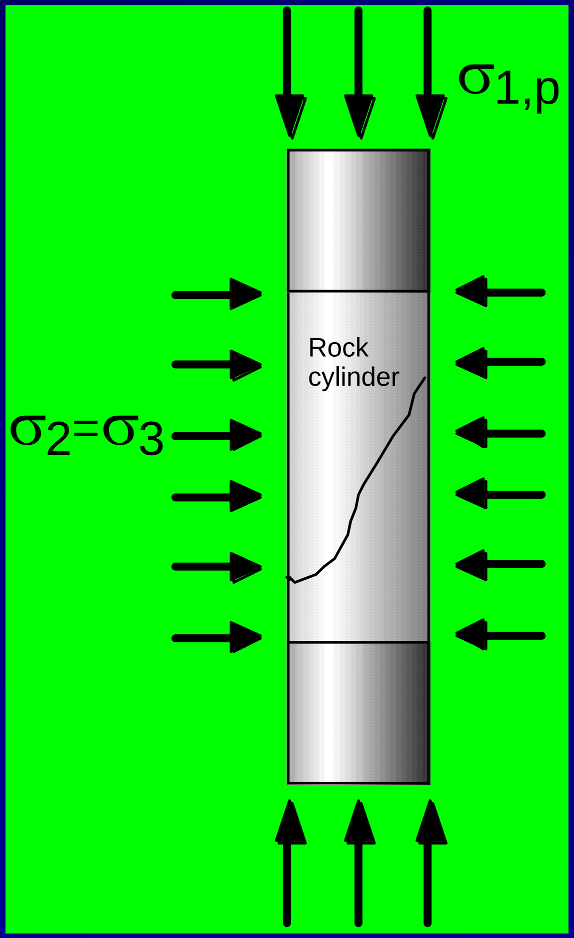
ROCK STRENGTH

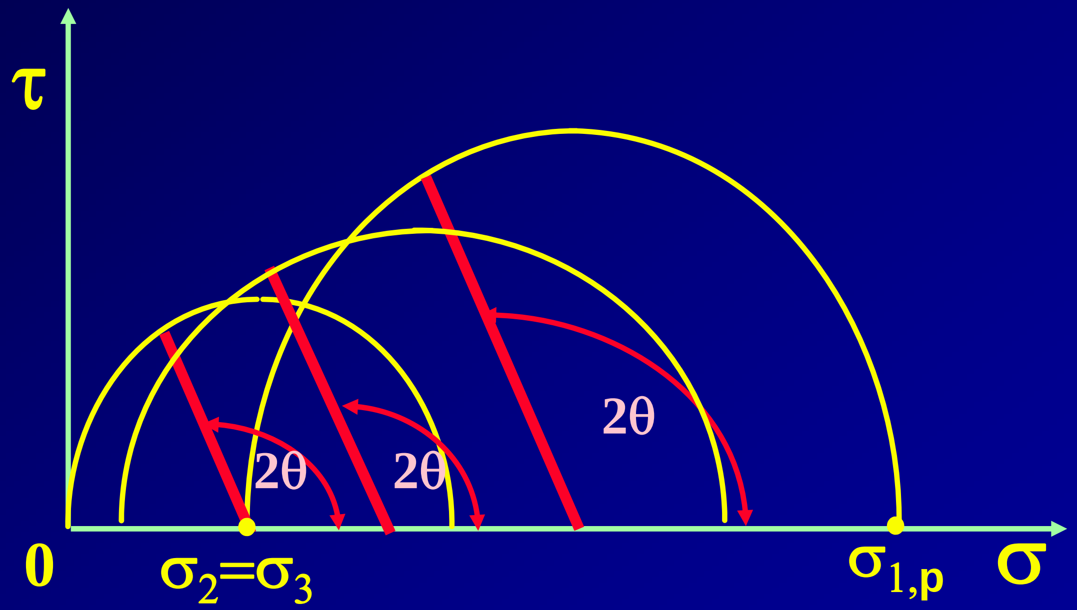
- **The variation of peak stress $\sigma_{1,\ peak}$ (at which failure occurs) with the confining pressure (for which $\sigma_2 = \sigma_3$) is referred to as the rock *CRITERION OF FAILURE*.**

- The simplest and the best known failure criterion of failure is the *MOHR-COULOMB* (M-C) criterion: the linear approximation of the variation of peak stress $\sigma_{1,\ peak}$ with the confining pressure.

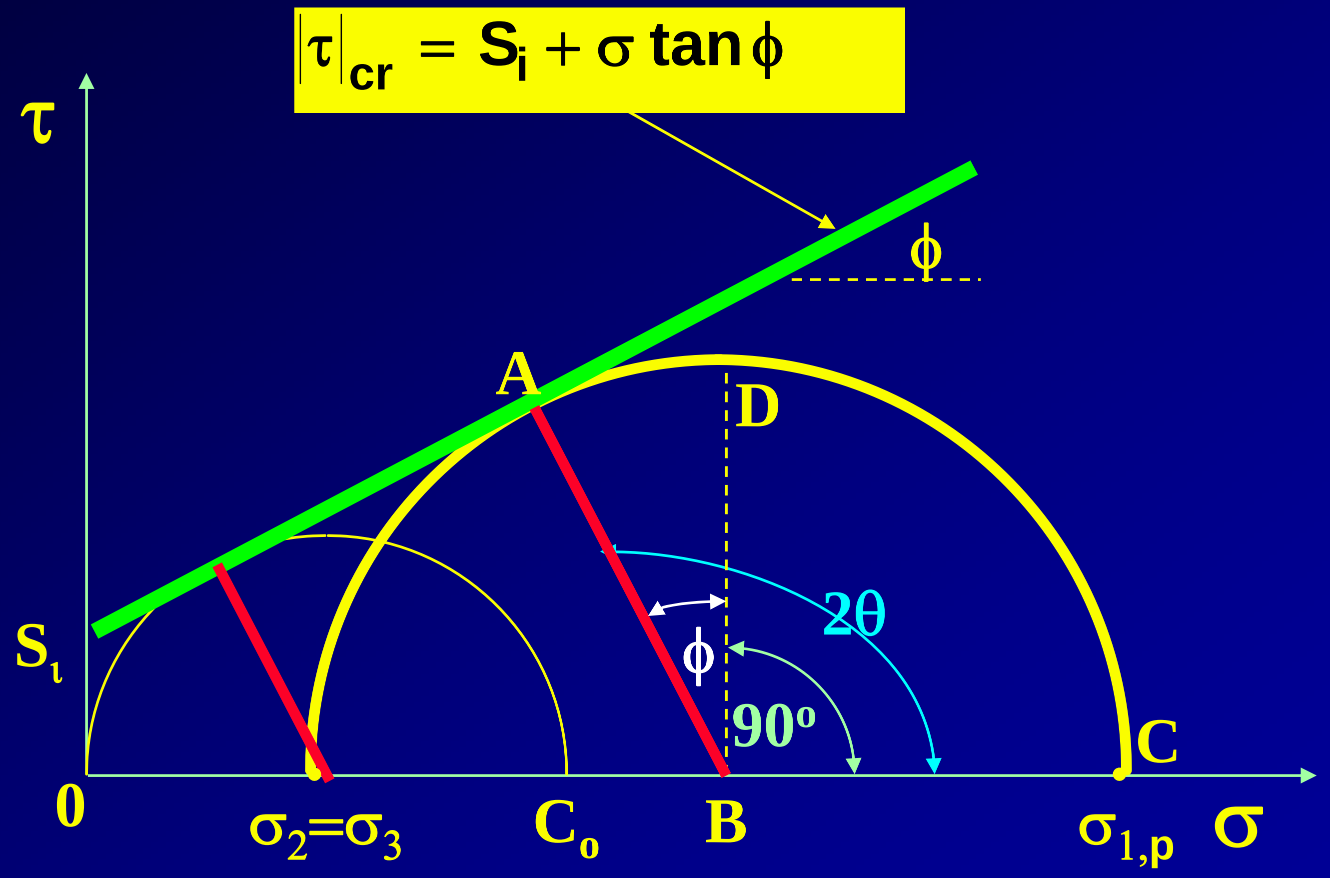
MOHR-COULOMB Criterion of Failure

- **It has been established that rock fails in compression by shearing along a 'failure' plane oriented at an angle θ with respect to σ_1 that is specific for a particular rock.**
- **The M-C linear strength criterion implies that θ stays the same regardless of the confining pressure applied.**





MOHR-COULOMB Criterion



MOHR-COULOMB Criterion in terms of shear and normal stress on the plane of failure

$$\left|\tau\right|_{cr} = S_i + \sigma \tan\phi$$

where $|\tau|_{cr}$ is the shear strength, S_i (cohesion) is the intercept with the τ axis of the linear envelope, and ϕ ('angle of friction') is the slope angle of the linear envelope of failure.

The relationship between the angle θ (the angle between the normal to the plane of failure -point A- and the direction of the max. principal stress) and ϕ is

$$2\theta = \phi + \frac{\pi}{2} \quad \text{or} \quad \theta = 45^{o} + \frac{\phi}{2}$$

MOHR-COULOMB Criterion

The Mohr-Coulomb criterion can also be given in terms of the principal stresses. To do this we recall the following relationships:

$$\left|\tau\right| = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta$$

$$\sigma = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$$

Plug these relationships into the Mohr-Coulomb criterion:

$$\left|\tau\right|_{cr} = S_i + \sigma \tan\phi$$

and you get:

$$\frac{\sigma_{1,p} - \sigma_2}{2} \sin 2\theta = S_i + \left(\frac{\sigma_{1,p} + \sigma_2}{2} + \frac{\sigma_{1,p} - \sigma_2}{2} \cos 2\theta \right) \tan\phi$$

MOHR-COULOMB Criterion in Terms of Principal Stresses

By separating all the terms containing σ_1 from the rest of the expression, and recalling the above relationship between θ and ϕ (and also performing some trigonometric manipulations), we finally obtain the Mohr-Coulomb criterion in terms of the principal stresses only:

$$\sigma_{1,p} = 2S_i\, tan\left(45° + \frac{\phi}{2}\right) + \sigma_2\, tan^2\left(45° + \frac{\phi}{2}\right)$$

or:

$$\sigma_{1,p} = C_0 + \sigma_2\, tan^2\left(45° + \frac{\phi}{2}\right)$$

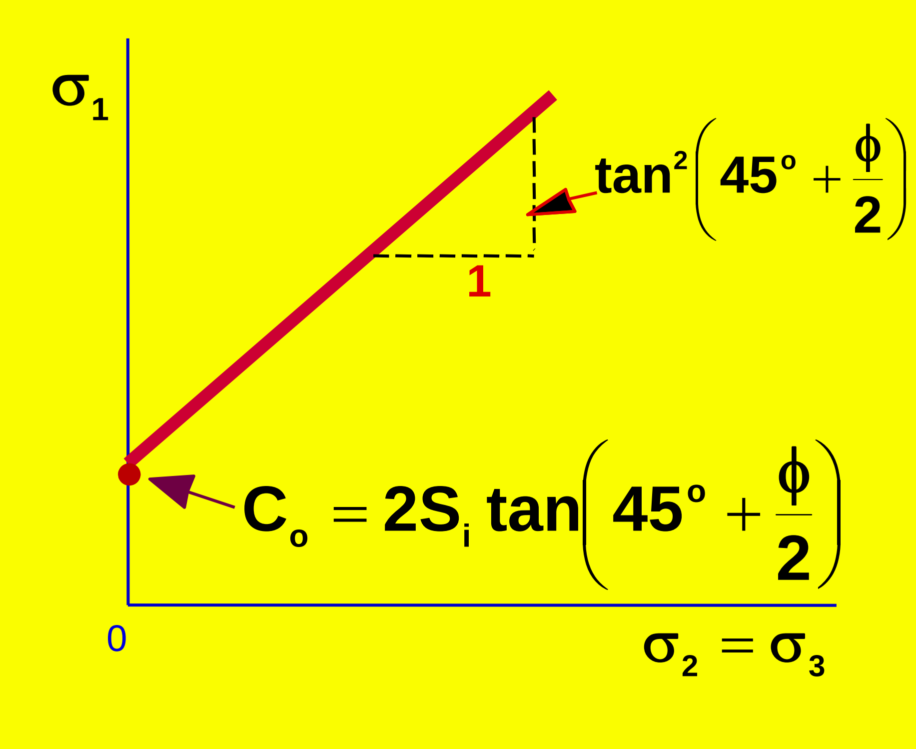
MOHR-COULOMB Criterion in Terms of Principal Stresses

$$\sigma_{1,p} = 2S_i\, tan\left(45° + \frac{\phi}{2}\right) + \sigma_2\, tan^2\left(45° + \frac{\phi}{2}\right)$$

or:

$$\sigma_{1,p} = C_0 + \sigma_2\, tan^2\left(45° + \frac{\phi}{2}\right)$$

Remember: the angle $45+\phi/2$ is equal to θ, where θ is the angle between the direction of σ_1 and the normal to the plane of failure.



Example

The criterion of failure of a limestone is given by (in MPa):

$$|\tau|_{cr} = 10 + 0.8\sigma$$

Represent the same criterion in terms of principal stresses.

Solution

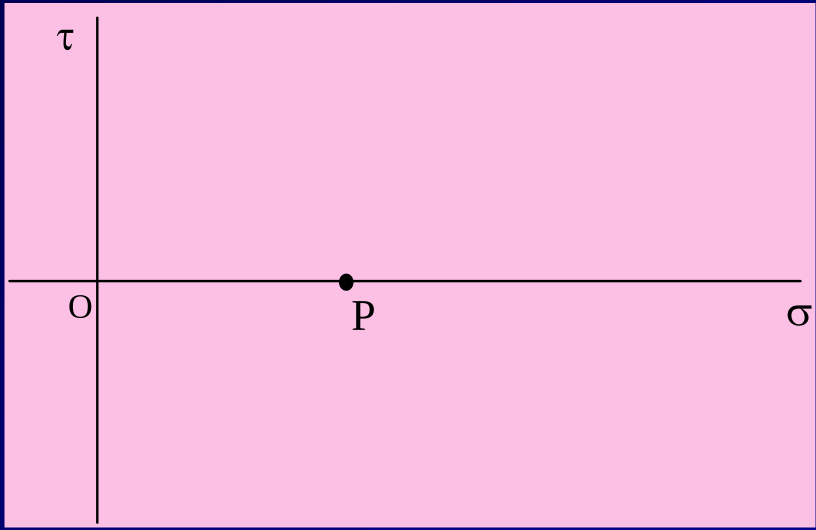
From the given equation we deduce that:
S_i = 10 MPa, and tan $\phi = 0.8$, or $\phi = 38.7°$.
The criterion of failure in terms of principal stresses is:

$$\sigma_{1,p} = 2S_i \tan\left(45° + \frac{\phi}{2}\right) + \sigma_2 \tan^2\left(45° + \frac{\phi}{2}\right)$$

By plugging the given values in the above equation we obtain (in MPa):

$$\sigma_{1,p} = 2 \times 10 \tan\left(45° + \frac{38.7°}{2}\right) + \sigma_2 \tan^2\left(45° + \frac{38.7°}{2}\right)$$

EXAMPLES OF MOHR-COULOMB CRITERION:



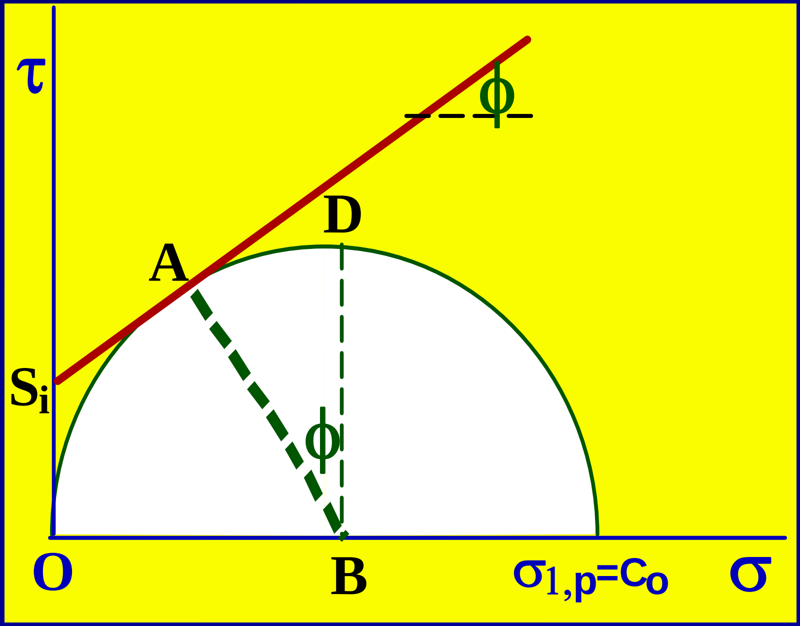
1. ***Hydrostatic state of stress*:**

$\sigma_1 = \sigma_2 = \sigma_3 = P$

When does rock fail under this condition?

2. ***Uniaxial compression:***

$$\sigma_1 \neq 0 \; ; \sigma_2 = \sigma_3 = 0$$



Failure occurs when σ_1 attains its peak value C_o. Failure plane (point A) normal is at $\theta = (45 + \phi/2)$ to σ_1 direction.

MOHR-COULOMB Criterion- Tensile Stresses

Question: What if the *normal stress* on the plane of failure is tensile?

This implies that there is no contact between the two created surfaces and hence there is no frictional resistance. Thus, the Mohr-Coulomb (M-C) criterion loses its validity.

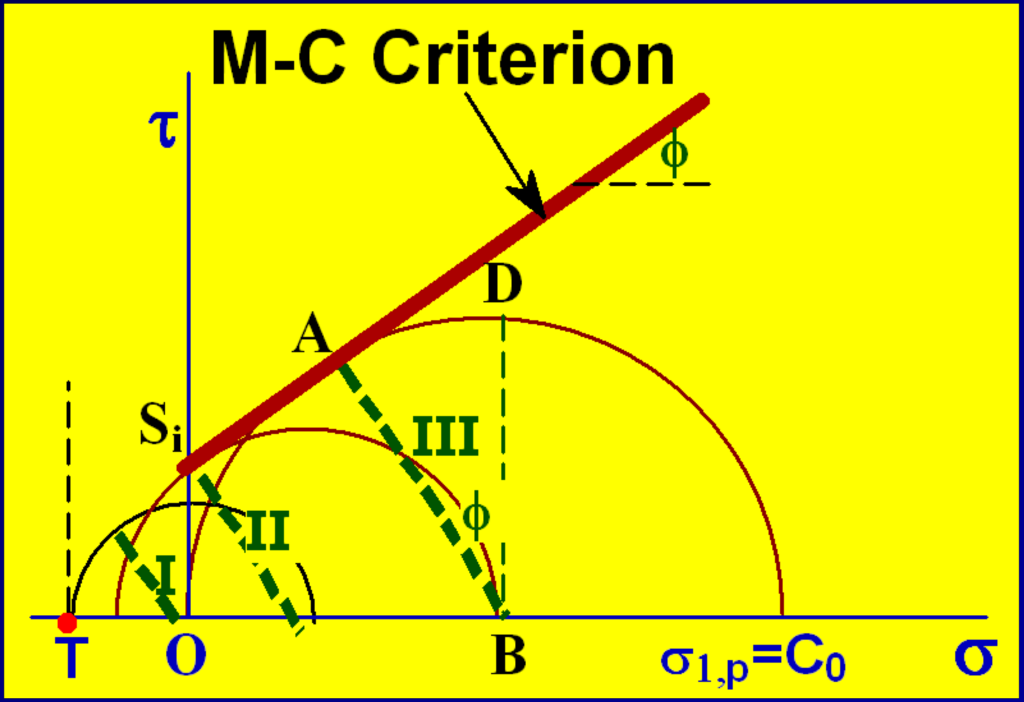
Note, however, that σ_2 can be tensile as long as the normal stress σ remains compressive: Mohr-Coulomb criterion will still be valid. Remember that σ_2=σ_3 can never be less than the tensile strength of the rock T, since T implies tensile failure.

MOHR-COULOMB Criterion- Tensile Stresses

Case I: σ_2 and σ are both tensile, so M-C criterion does not apply. Specimen fails in tension when $\sigma_2 = T$.

Case II: σ_2 is tensile, but σ is zero or larger, so M-C criterion DOES apply!

Case III: All stresses are compressive so M-C does apply.



NOTE:
Typically $T = 0.1C_o$ to $0.05C_o$

MOHR-COULOMB CRITERION IN TERMS OF THE PRINCIPAL STRESS RATIO K

Engineers/practitioners like to to work with principal stress ratio K = σ_2 / σ_1 (typically σ_2 represents the horizontal stress σ_h, and σ_1 represents the vertical stress σ_v).

It is thus more convenient on occasion to express the Mohr-C criterion in terms of K.

MOHR-COULOMB CRITERION IN TERMS OF THE PRINCIPAL STRESS RATIO K

Given:

$$\sigma_{1,p} = C_o + \sigma_2 \tan^2\left(45° + \frac{\phi}{2}\right)$$

We divide both sides by $\sigma_{1,p}$:

$$1 = \frac{C_o}{\sigma_{1,p}} + \frac{\sigma_2}{\sigma_{1,p}} \tan^2\left(45° + \frac{\phi}{2}\right)$$

Using K and rearranging terms, we obtain:

$$\frac{C_o}{\sigma_{1,p}} = 1 - K \tan^2\left(45° + \frac{\phi}{2}\right) \rightarrow \sigma_{1,p} = \frac{C_o}{1 - K \tan^2\left(45° + \frac{\phi}{2}\right)}$$

MOHR-COULOMB CRITERION IN TERMS OF THE PRINCIPAL STRESS RATIO K

With the M-C criterion written this way we can determine a limit for the ratio K at failure. It is

$$\mathbf{K} < \cot^2\left(\mathbf{45^\circ} + \frac{\boldsymbol{\phi}}{\mathbf{2}}\right)$$

This condition makes the criterion feasible. Otherwise the failure stress is infinite or negative, which is not physically possible.

MOHR-COULOMB CRITERION IN TERMS OF THE PRINCIPAL STRESS RATIO K

This condition for K is a useful tool in determining stability.

For example if the angle ϕ = 40° then K < 0.22

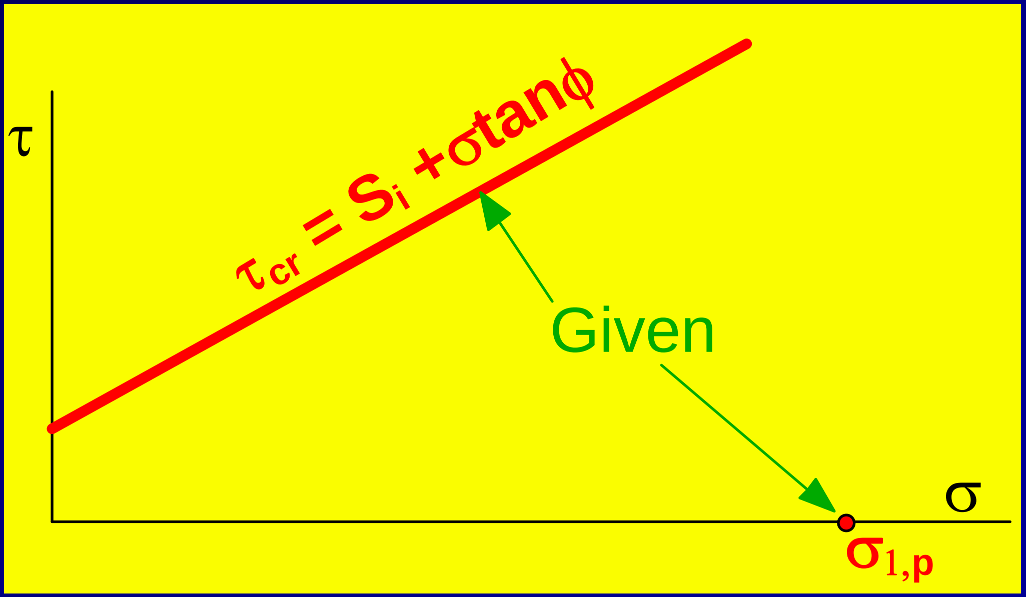
Or $\sigma_2 < 0.22\,\sigma_1$.

CLASS: What if the angle ϕ = 0°?

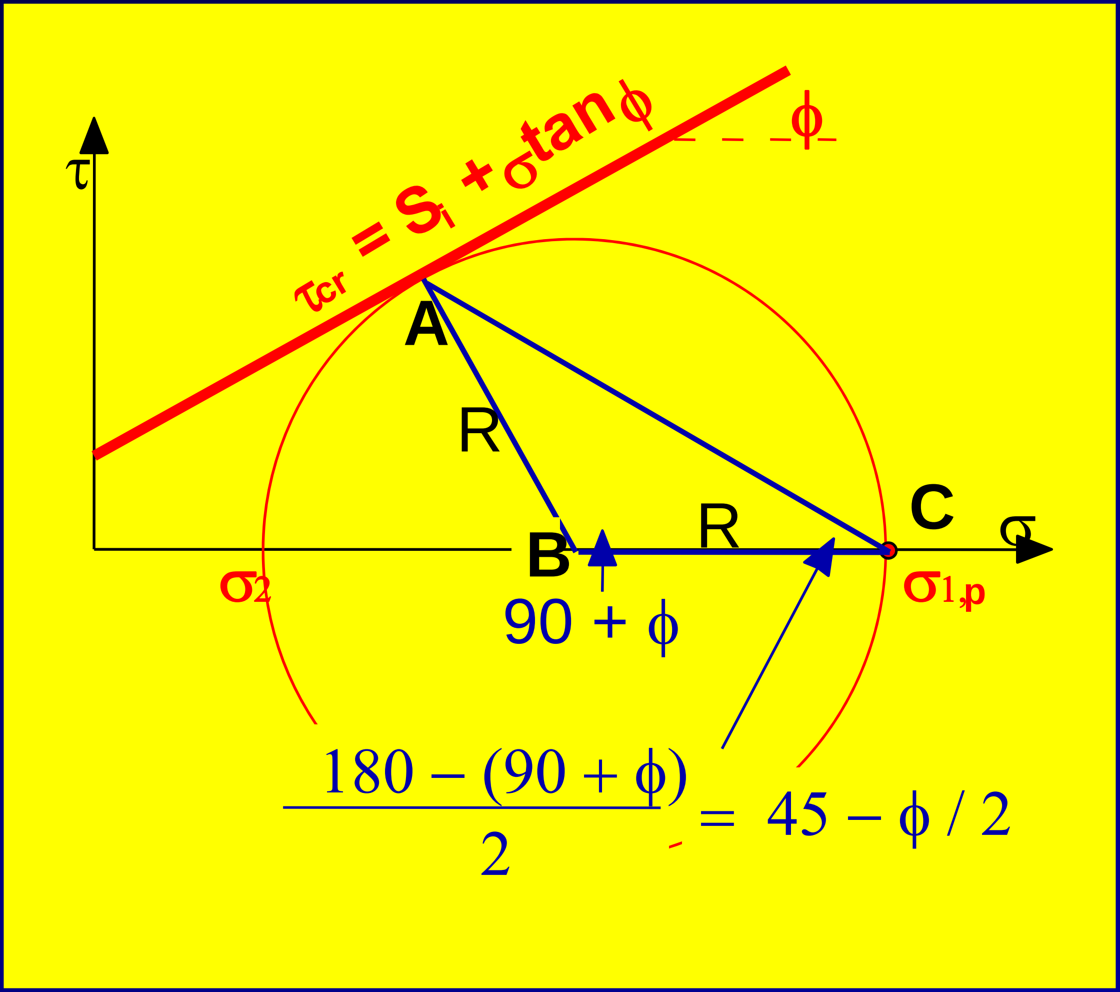
Determining the *Critical State of Stress* at a Point in a Rock Formation When the M-C Criterion of Failure and One of the Principal Stresses Are Known

In this example the strength material properties of the rock S_i and ϕ are given, and we also know the $\sigma_{1,p}$ at failure.

What is then the complete state of stress at the point in question?

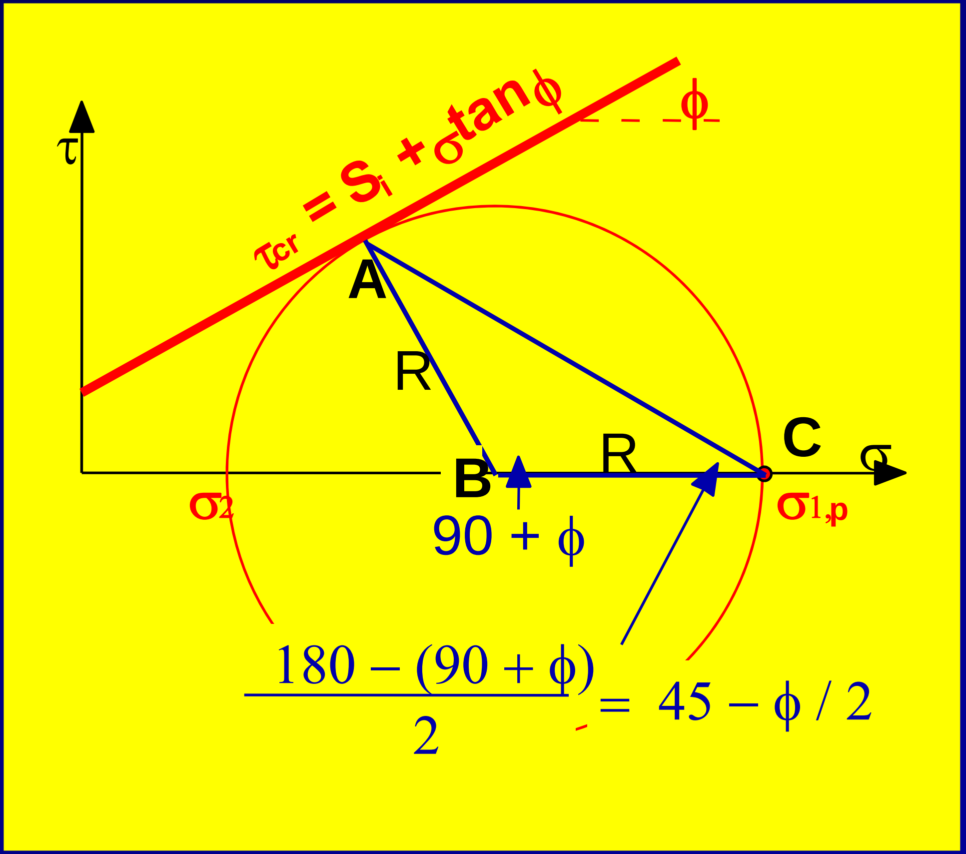


Let’s imagine first that we have solved the problem, i.e. we have found the CRITICAL Mohr circle denoting the state of stress at failure.

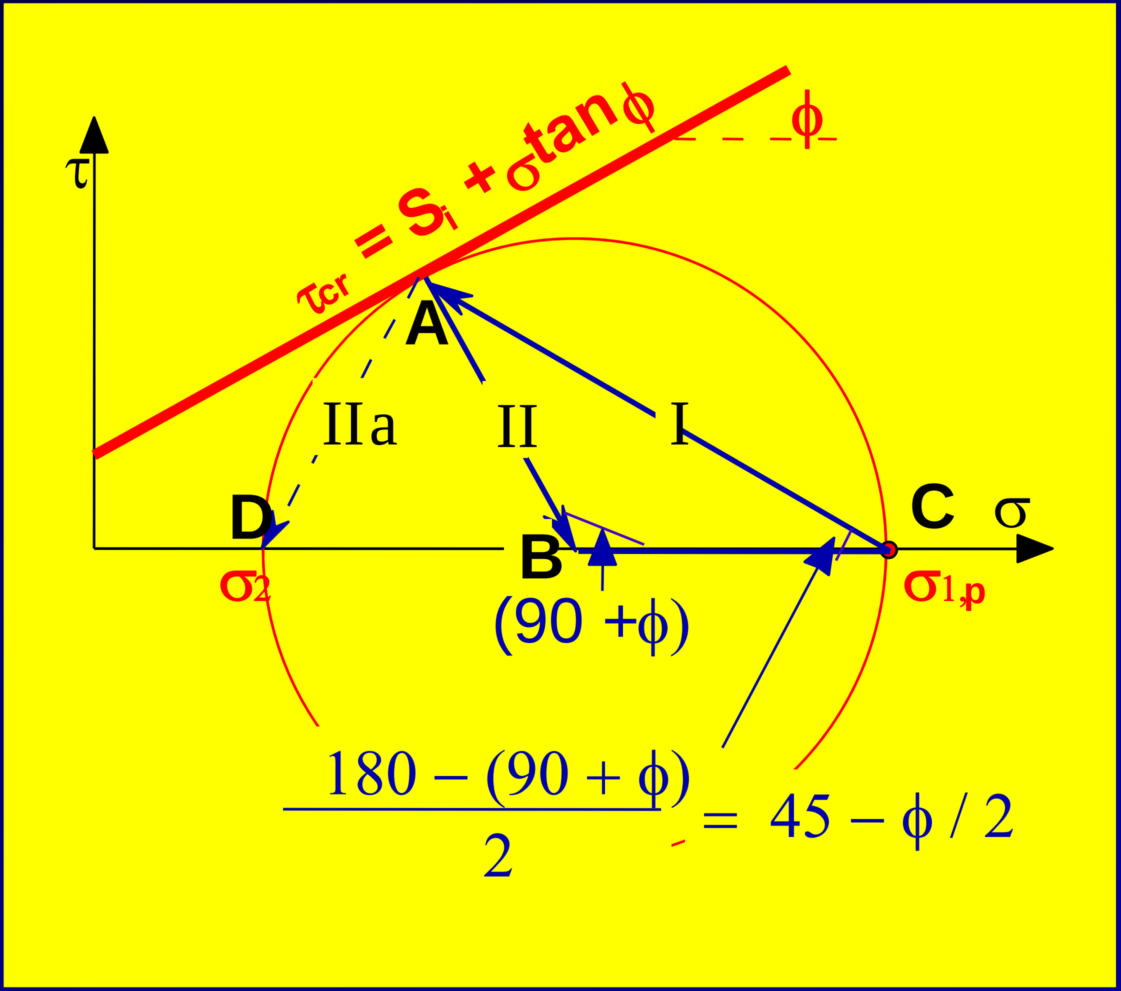


We notice that in the triangle ABC two of the sides are equal to the radius R. Also, recall the value of the angle ABC. Hence the angle ACB is 45° - ϕ/2.

Thus, in order to construct the critical Mohr circle we fist plot the M-C criterion and the point representing $\sigma_{1,p}$.



Then, we construct the angle ACB and draw the line CA, where A intersects the M-C criterion. Now we construct an identical angle CAB and draw AB. Point B on the σ axis is the center of the critical Mohr circle.



An alternate way is to construct at point A a 90° angle and draw line AD. Point D has to be equal to σ_2.

OTHER EMPIRICAL MOHR ENVELOPES

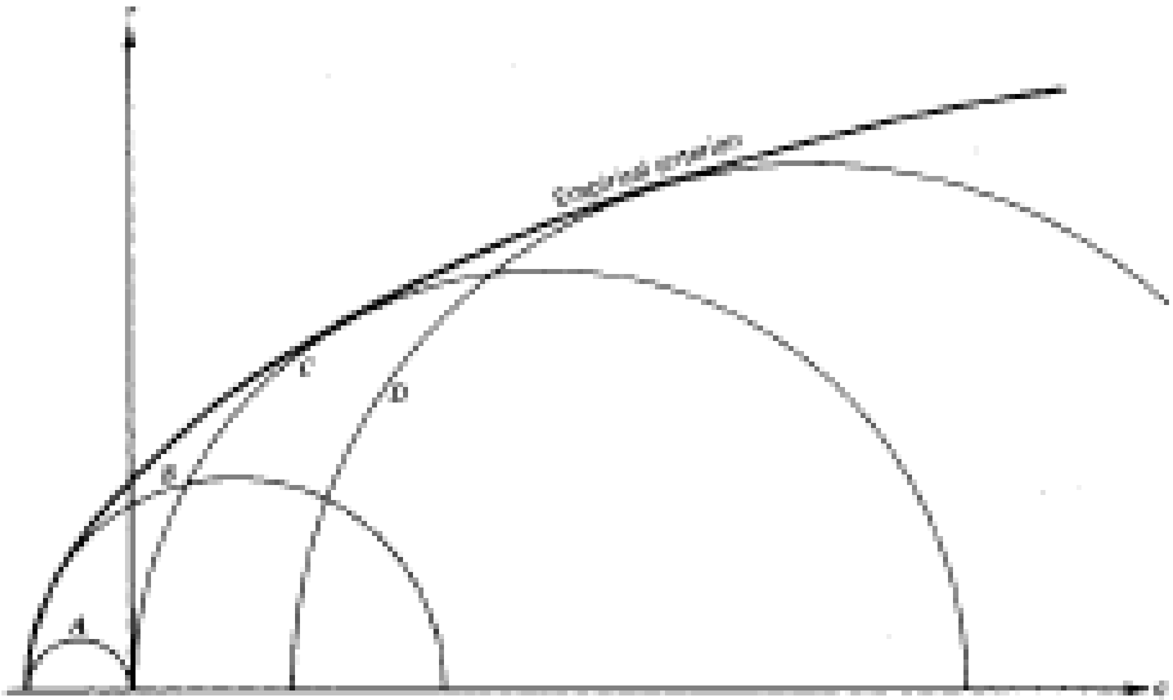


Figure 3.19 An empirical criterion of failure defined by the envelope to a series of Mohr's circles: *A*, direct tension; *B*, Brazilian; *C*, unconfined compression; *D*, triaxial compression.

Goodman, Intro to Rock Mechanics

A straight line doesn't always fit the data well
Mohr-Coulomb is an approximation

OTHER EMPIRICAL MOHR ENVELOPES

Several examples of other expressions defining the Mohr envelope in rocks for which the Coulomb linear relationship does not appear to be representative, can be found in the literature. These are all EMPIRICAL relationships. The Hoek and Brown criterion is particularly popular now:

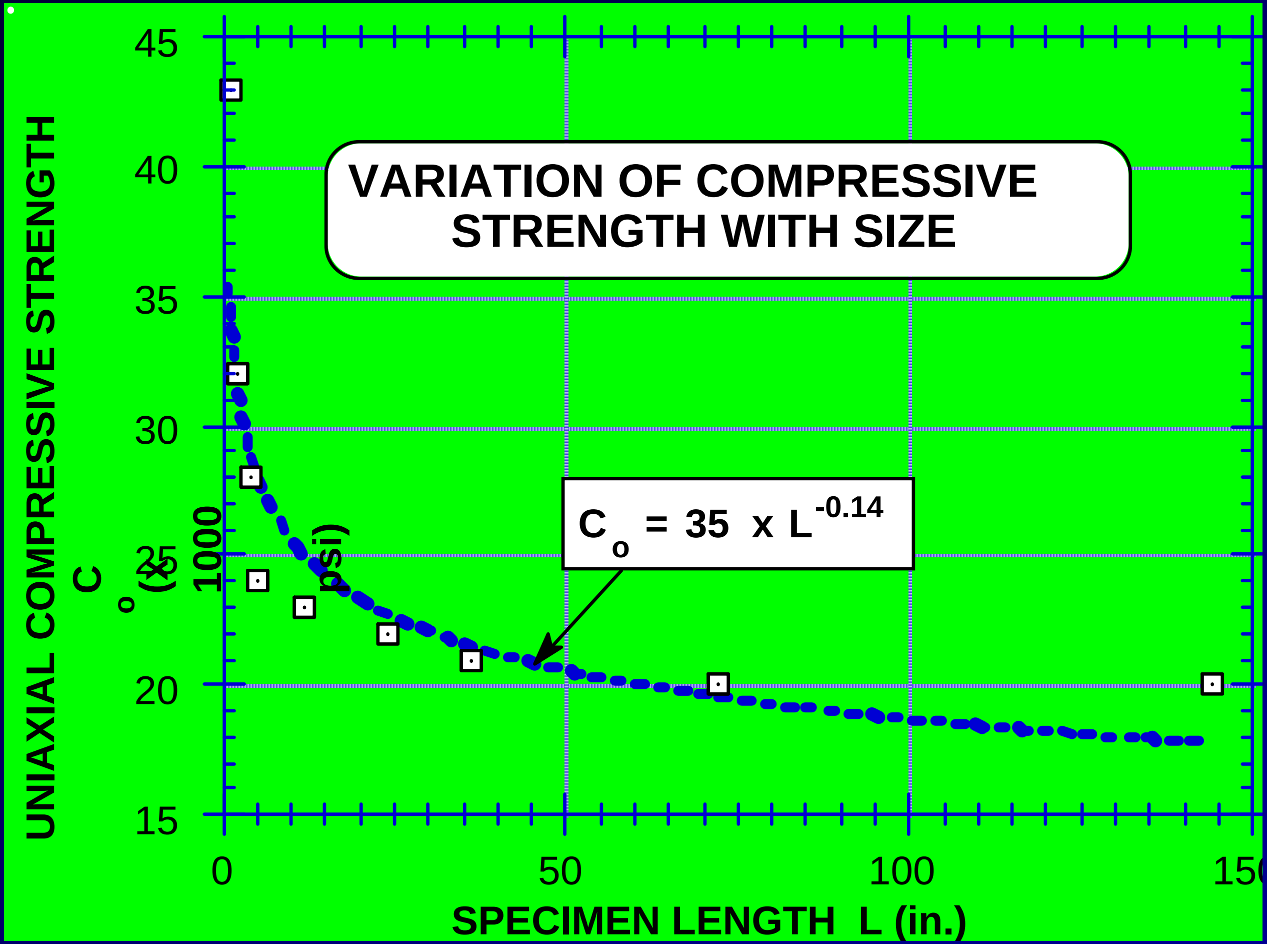
$$\frac{\sigma_{1,p}}{C_o} = \frac{\sigma_2}{C_o} + \sqrt{m\frac{\sigma_2}{C_o} + s}$$

where m and s are constants of the material and are determined empirically.

SIZE EFFECT ON COMPRESSIVE STRENGTH

It has been found through experimentation that rock strength is affected by the SIZE of the specimen tested. This can be explained simply as being the result of the many flaws found in rock in the form of fissures, pores, cleavage, bedding planes, foliation, etc.

The larger the size of the specimen tested the higher the probability that a weak flaw will be available, capable of initiating failure.



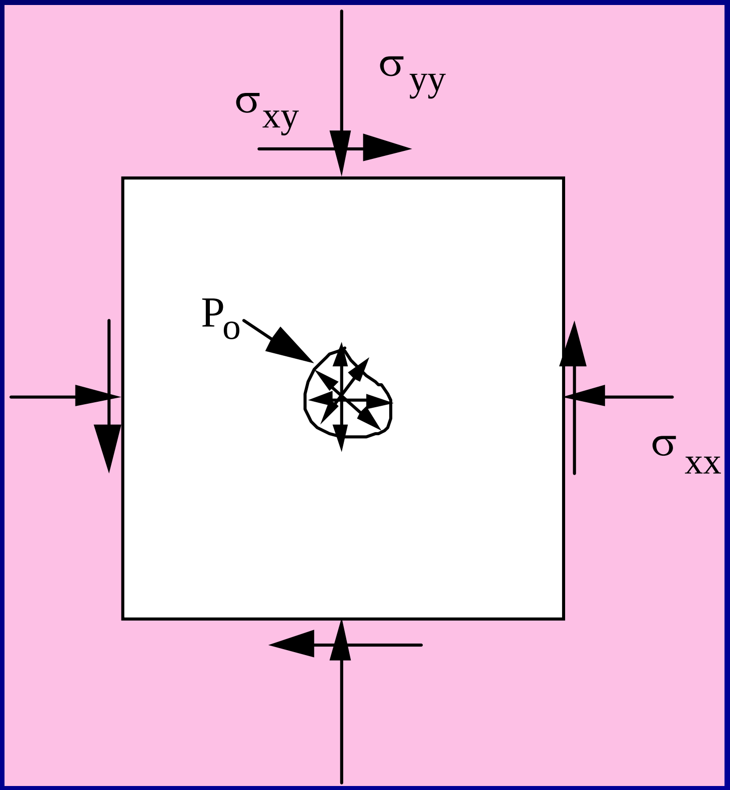
In this plot of the size effect on strength, a power function was used to best-fit the experimental data. NOTE, however, that beyond L=36 in. the size effect ceases to be of much consequence.

THE EFFECT OF PORE PRESSURE ON STRESSES IN ROCK

Rock is a porous material. If these pores are occupied by water (or other fluids such as oil) then they are subjected to some fluid pressure. This is what we call *'pore pressure'*.

THE EFFECT OF PORE PRESSURE ON ROCK STRESS AND STRENGTH

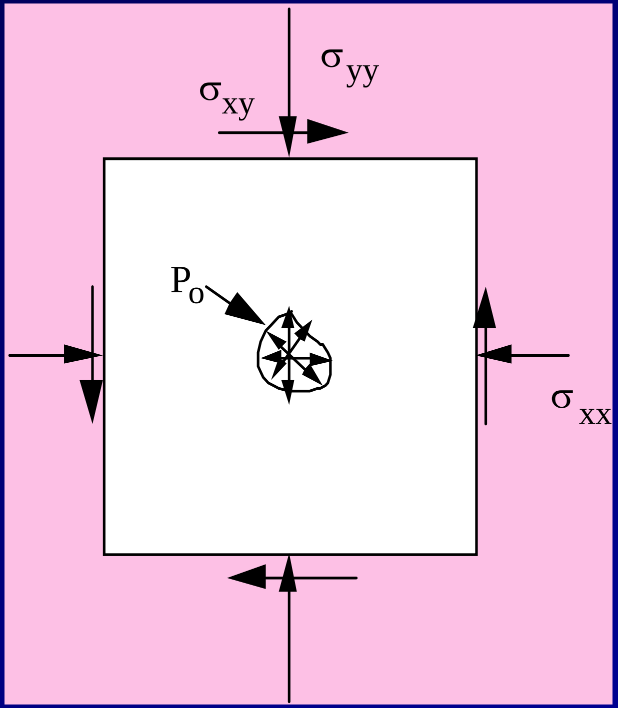
Recognizing that a typical rock is pervaded by a network of interconnected pores and fissures, we can represent an incremental volume of it by a solid cube of rock with a hole in it representing the accumulated pore volume in that cube.



We call the EFFECTIVE STRESS (σ_{eff} or σ') the net stress acting in the rock as a whole. Its value can be obtained straight from this cartoon:

$$\sigma'_i = \sigma_i - P_0 \quad \text{where } \sigma_i = \sigma_{xx}, \sigma_{yy}, \sigma_{zz}$$

$$\tau'_{ij} = \tau_{ij} \quad \text{where } \tau_{ij} = \sigma_{xy}, \sigma_{yz}, \sigma_{zx}$$



It has been determined empirically that ROCK STRENGTH is a direct function of the EFFECTIVE STRESS, i.e.:

$$|\tau|_{cr} = S_i + (\sigma - P_0)\tan\phi$$

$$\sigma_{1,p} - P_0 = C_0 + (\sigma_2 - P_0)\tan^2\left(45° + \frac{\phi}{2}\right)$$

CRITICAL PORE PRESSURE

It is obvious that pore pressures REDUCES the net normal stress on the eventual plane of failure, and thus brings about failure at principal stress levels <u>that would be stable otherwise.</u>

CRITICAL PORE PRESSURE

It is often required to calculate the *CRITICAL PORE PRESSURE P_{cr}* that would cause failure under a given state of stress (i.e. solid or total stress). All we have to do is to isolate $P_o = P_{cr}$ in the above equations:

$$\left|\tau\right| = S_i + \sigma \tan\phi - P_{cr} \tan\phi$$

$$P_{cr} = \frac{S_i + \sigma \tan\phi - \left|\tau\right|}{\tan\phi} \Rightarrow P_{cr} = \sigma + \frac{S_i - \left|\tau\right|}{\tan\phi}$$

CRITICAL PORE PRESSURE

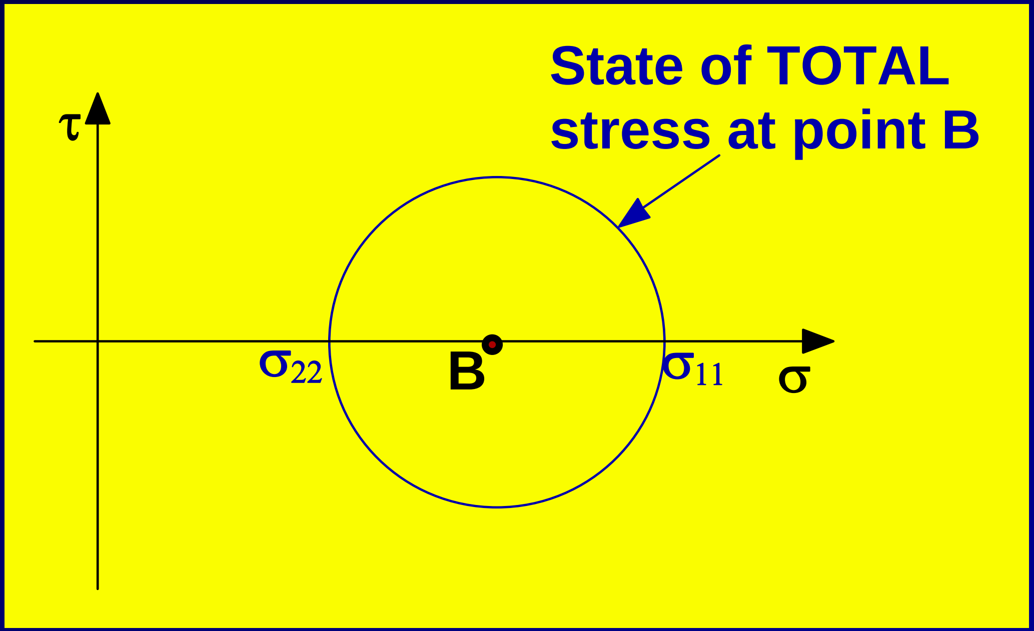
A similar derivation of the M-C equation in terms of effective principal stresses can isolate the critical pressure P_{cr} leading to compressive failure for a given set of otherwise stable principal stresses:

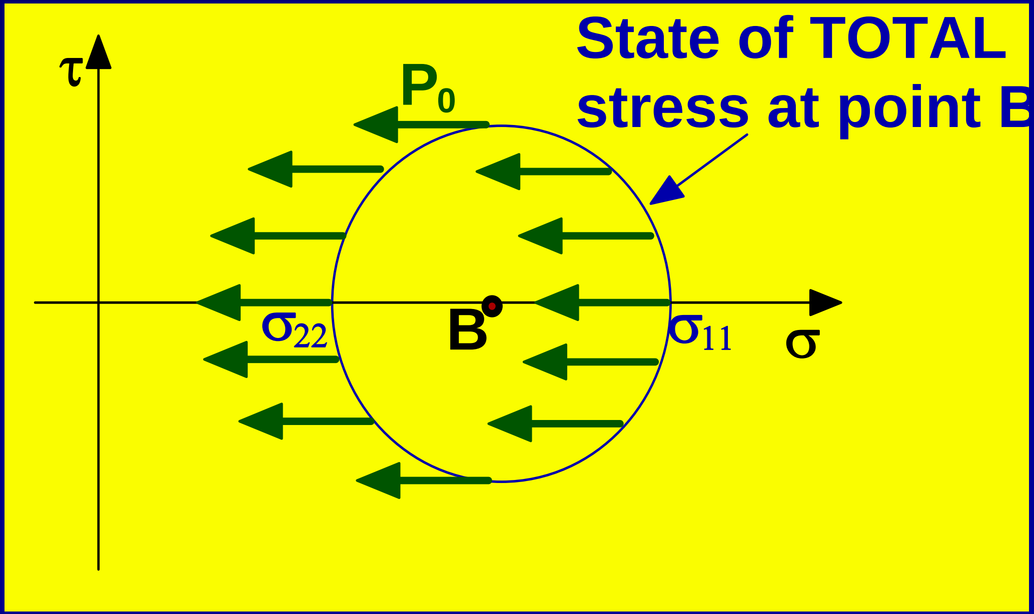
$$\sigma_1 - P_{cr} = C_0 + (\sigma_2 - P_{cr}) tan^2\left(45° + \frac{\phi}{2}\right)$$

or:

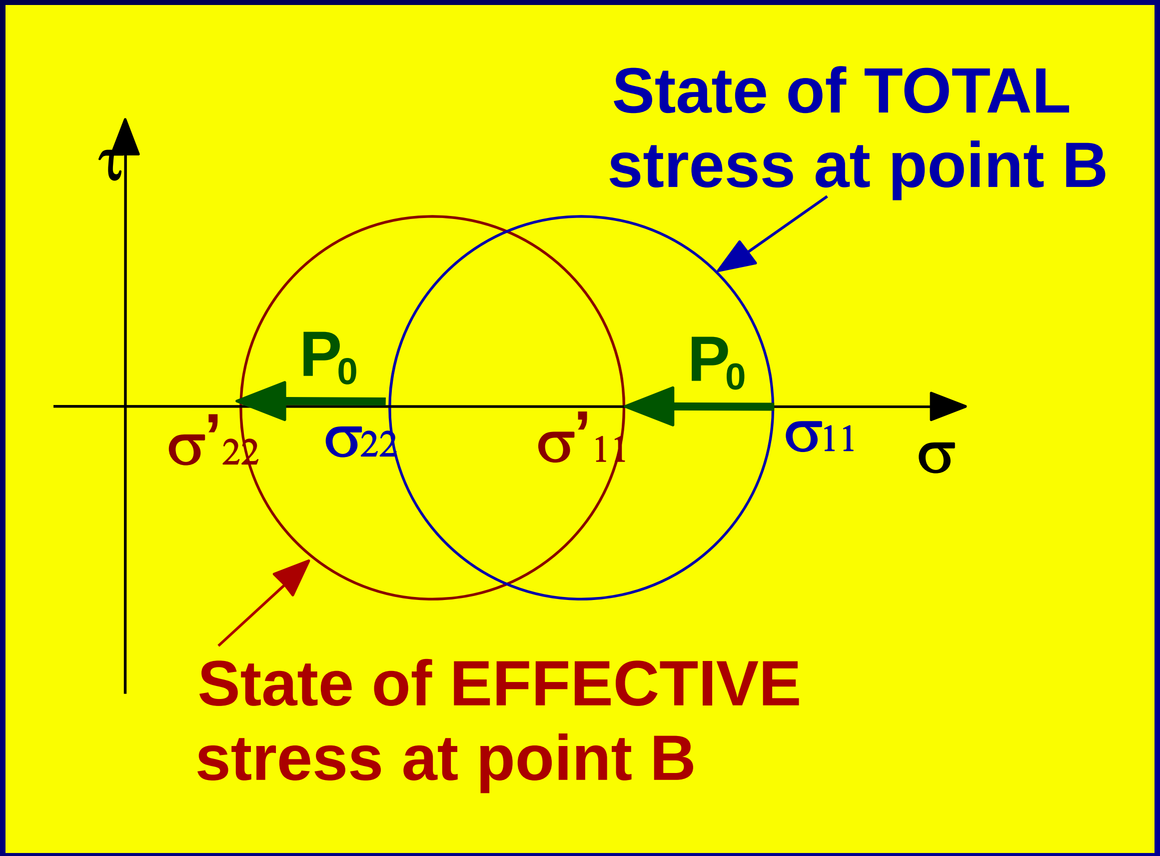
$$P_{cr} = \sigma_2 - \frac{\sigma_1 - \sigma_2 - C_0}{tan^2\left(45° + \frac{\phi}{2}\right) - 1}$$

PORE PRESSURE EFFECT ON STRESSES IN A MOHR DIAGRAM





EFFECTIVE STRESSES IN A MOHR DIAGRAM



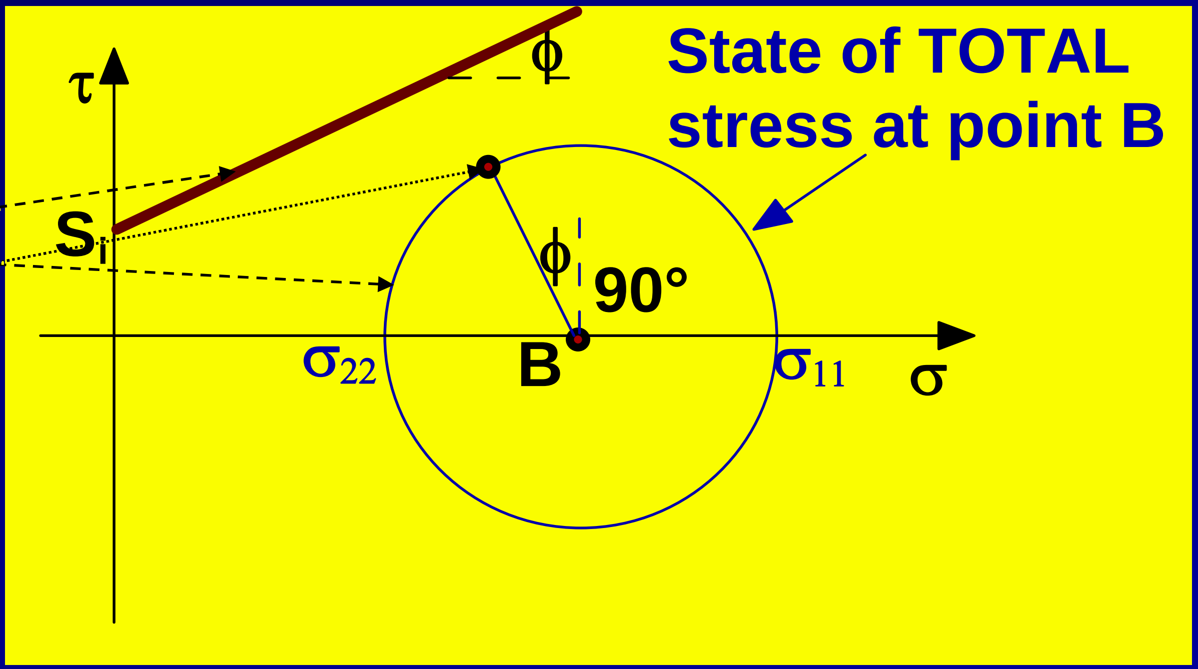
CRITICAL PORE PRESSURE USING MOHR DIAGRAM

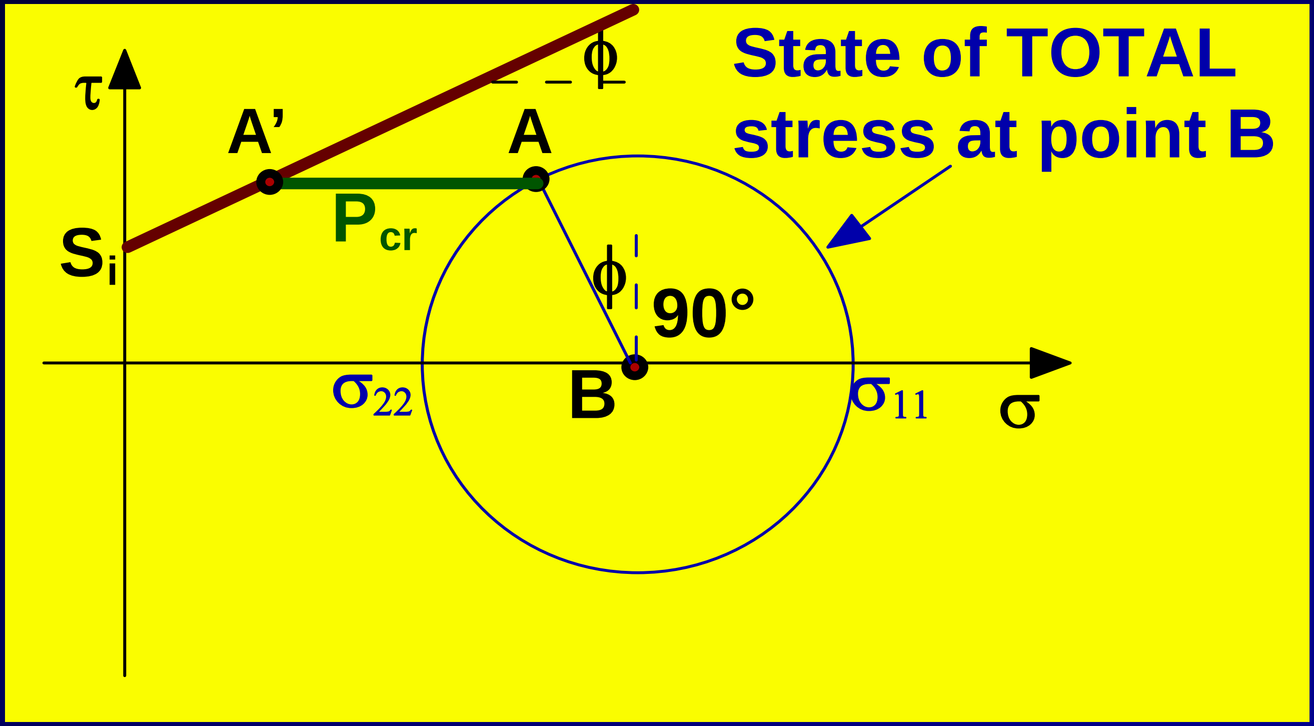
The Mohr circle can be very effectively used to obtain P_{cr} leading to failure for a state of total stress which is otherwise in a stable condition:

To construct P_{cr} we need to know the Criterion of Failure for the rock in question (material properties ϕ and S_i or C_0), and the State of TOTAL Stress (e.g. σ_1 and σ_2).

Given data:

Potential plane of failure

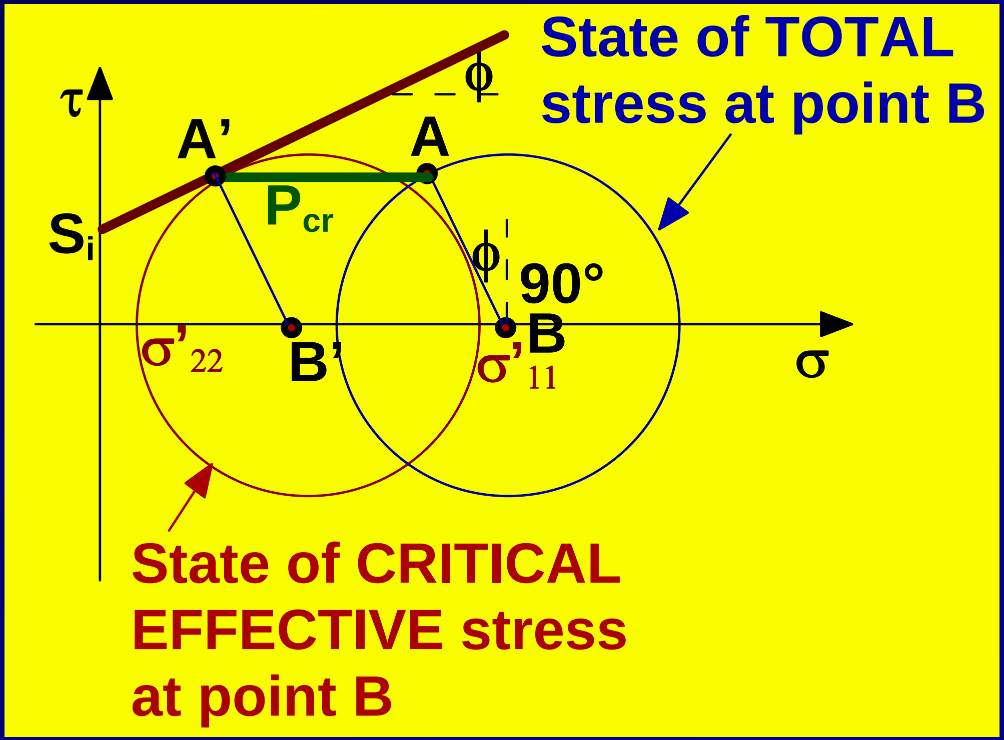




Construction steps: Determine the potential failure plane A, and construct a horizontal line from there which intersects the M-C Criterion of failure for the rock in question (A'). The length of this segment is the critical pore pressure P_{cr}.

CRITICAL PORE PRESSURE USING MOHR DIAGRAM

To complete the construction we can draw the Mohr circle representing the state of CRITICAL EFFECTIVE stress at point B. The circle is identical to the one for the TOTAL stresses, except it has shifted to the left by an amount equal to P_{cr} (BB' = AA').



Pore Pressure Effect - Example

1 - The state of stress on a fault plane is given by: σ = 50 MPa and τ = 50 MPa.

The strength properties of the rock are: S_i = 10 MPa, and ϕ = 45°. Is the fault in danger of failing (slipping)? Answer by calculation AND graphical construction.

2 - If the answer is NO, how much PORE PRESSURE will be necessary to build up in the fault zone in order for the fault to become unstable?

Answer