Statistics/EE 530, Spring Quarter 2018

Problem Set 6

Problem 19 (to be turned in – 5 points). What sequence of filtering operations would we need to perform on a time series $\mathbf{X}$ in order to obtain DWPT coefficients $\mathbf{W}_{j,n}$ that nominally represent fluctuations in $\mathbf{X}$ within the frequency interval $[\frac{19}{128}, \frac{20}{128}]$?

Problem 20 (to be turned in – 5 points). Figure 216 displays the contents of the binary-valued vectors $\mathbf{c}_{j,n}$ for the levels $j = 1, 2$ and 3. Determine these vectors for level $j = 4$. (This is part of Exercise [6.5], p. 253.)

Problem 21 (to be turned in – 10 points). As in Figures 225 and 226, apply the best basis algorithm to the DWPT coefficients in the WP table of Figure 224, but now use (a) the ℓ_p information cost functional with $p = 1$ and (b) the threshold cost functional with the threshold δ set to the median of the absolute values of all the coefficients in the WP table (including those at level $j = 0$, i.e., $\mathbf{W}_{0,0} = \mathbf{X}$; this method of setting δ is used in Bruce and Gao, 1996a). How do the best basis transforms picked out by these two cost functionals compare to what we found using the $-\ell^2 \log \ell^2$ norm? Note: please amend the two-part rule stated just below Figure 226 so that, if the parent node has the same cost as the sum of the costs of the children nodes, the parent node is marked. (This is Exercise [6.7], p. 253.)

Problem 22 (to be turned in – 10 points). Using a dictionary that consists of the vectors

$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

apply the matching pursuit algorithm to the two point time series $\mathbf{X} = [5, 3]^T$. How many steps m are needed to obtain a residual vector $\mathbf{R}^{(m)}$ with zero norm? (This is Exercise [6.13], p. 254.)

Self-Graded Problem XV (*not* to be turned in). Explain why the cost functional $m(|W_{j,n,t}|) = |W_{j,n,t}|^2$ is not of much use for selecting an 'optimal' orthonormal transform. (This is Exercise [223], p. 223.)

Self-Graded Problem XVI (*not* to be turned in). Show that $\langle \mathbf{X}^{(1)}, \mathbf{R}^{(1)} \rangle = 0$ and hence that

$$\|\mathbf{X}\|^2 = \|\mathbf{X}^{(1)}\|^2 + \|\mathbf{R}^{(1)}\|^2 = |\langle \mathbf{X}, \mathbf{d}_{\gamma_0} \rangle|^2 + \|\mathbf{R}^{(1)}\|^2.$$

(This is Exercise [240], p. 240.)

Solutions to Problems 19 to 22 are due Wednesday, May 9, at the beginning of the class.